KEY

TO

LIPPINCOTT'S

ELEMENTARY ALGEBRA

IN WHICH ARE GIVEN

SOLUTIONS IN FULL OR IN PART TO THE MORE DIFFICULT EXERCISES AND PROBLEMS

BY

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KEY TO
ELEMENTARY ALGEBRA.

Page 14.

1. $3+6\times2=3+12=15$.

2. $10-3\times3=10-9=1$.

3. $12\div3\times2=12\div6=2$.

5. $21\times3\div7+8\times3-33=63\div7+24-33=9+24-33=33-33=0$.

7. $32\div8\times4+49\div7-4=32\div32+49\div7-4=1+7-4=8-4=4$.

9. $1\times2\times3\div6-1+100=6\div6-1+100=1-1+100=0+100=100$.

10. $0\times2\times4\times5\div40+0=0\div40+0=0+0=0$.

Page 15.

1. $6+12\times4=6+48=54$.

3. $24\div6\times4=24\div24=1$.

5. $42\times6\div14+16\times6-66=252\div14+96-66=18+96-66=18+30=48$.

7. $144\div6\times8+18\div6-5=144\div48+18\div6-5=3+3-5=6-5=1$.

9. $2\times4\times6\div12-2+200=48\div12-2+200=4-2+200=2+200=202$.

10. $0\times4\times8\times10\div80\div2+0=0\div40+0=0+0=0$.

Page 16.

1. $(22+7-2)\times(12\div6)=(29-2)\times2=27\times2=54$.

3. $22+(7-2)\times(12\div6)=22+5\times2=22+10=32$.

5. $13\times(6+4)+8\times2=13\times10+16=130+16=146$.

7. $4\times5\times(9-3)+(9\div3+6)=4\times5\times6+9=120+9=129$.

9. $\overline{3+4\times9}-\overline{(3+6)\div3}=39-3=36$.

10. $327\times6\div109+52\times5-\dfrac{42\times5}{2\times7}=1962\div109+260-15=18+260-15=278-15=263$.

Problems. Page 16.

2. Let $x=$ the money B had left.
Then the equation is $x=82-3\times9$.
Removing $\times$, we have $x=82-27$.
Removing $-$, we have $x=55$.

3. Let $x=$ how many knives can be bought.
Then the equation is $x=1.00\div(6\div12)$, or $x=1.00\div\frac{6}{12}=1.00\div.50=2$.

4. Let x = what 50 pens would cost.
Then the equation is $x = (\$3.60 \div 720) \times 50$.
Removing the parenthesis, $x = .005 \times 50$.
Removing $\times$, we have $x = \$.25$.

Problems. Page 17.

7. Let x = the required dividend.
Equation, $x = 367 \times 445 + 189 = 163504$.

9. Let x = what remains for other expenses.
Since he spends one-third of his earnings, two-thirds remain.
Equation, $x = \frac{2}{3}\ (70 \times 7 + 82 \times 5)$.

10. Let x = how much they save.
Equation, $x = [16 \times 2 - (11 + 8)]\ 13$.

12. Let x = the required cost.
Equation, $x = 4 \times 14 + 13 \times (\frac{1}{2}$ of $4)$.
Solution, $x = 56 + 13 \times 2 = 56 + 26 = 82$.

13. Let x = what all earn in 11 days.
Equation, $x = (2 \times 9 + 1 \times 6)\ 11$.

14. Let x = the required sum.
Equation, $x = (98 \div 14)\ 5 + 5$.

15. Let x = what the histories cost apiece.
Equation, $x = \dfrac{500 - 40 \times 6}{5}$.

Exercises. Page 18.

2. If $a = 1$, $5a = 5 \times 1 = 5$.

3. If $a = 6$, $5a + a = 5 \times 6 + 6 = 30 + 6 = 36$.

4. If $a = 11$, $7a - 7a = 77 \times 11 - 7 \times 11 = 77 - 77 = 0$.

5. $2a + b = 2 \times 25 + 50 = 50 + 50 = 100$.

$$\frac{2a + 2b}{b} = \frac{2 \times 25 + 2 \times 50}{50} = \frac{50 + 100}{50} = \frac{150}{50} = 3.$$

8. $(3a + 4d)\ (2c - 3b) = (3 \times 1 + 4 \times 4)\ (2 \times 3 - 3 \times 2) = (3 + 16)\ (6 - 6) = 19 \times 0 = 0$.

9. $\dfrac{5a}{b+c} - \dfrac{2b}{c+a} - \dfrac{2c}{b+a} + \dfrac{2d}{a+c} = \dfrac{5 \cdot 1}{2+3} - \dfrac{2 \cdot 2}{3+1} - \dfrac{2 \cdot 3}{2+1} + \dfrac{2 \cdot 4}{1+3} = \frac{5}{5} - \frac{4}{4}$
$- \frac{6}{3} + \frac{8}{4} = 1 - 1 - 2 + 2 = 0$.

10. I. $(a - b)\ (8a - 10b) = (5 - 3)\ (8 \cdot 5 - 10 \cdot 3) = 2\ (40 - 30) = 20$.

II. $(4ab \div 2ab)\ (20ab \div 10b) = (4 \cdot 5 \cdot 3 \div 2 \cdot 5 \cdot 3)\ (20 \cdot 5 \cdot 3 \div 10 \cdot 3) = (60 \div 30)\ (300 \div 30) = 2 \times 10 = 20$.

III. $\dfrac{12ab}{3a+b} \times \dfrac{5b+a}{5b-a} = \dfrac{12 \cdot 5 \cdot 3}{3 \cdot 5 + 3} \times \dfrac{5 \cdot 3 + 5}{5 \cdot 3 - 5} = \dfrac{180}{18} \times \dfrac{20}{10} = 10 \times 2 = 20$.

11. $(a^2 + b^2)\ (a - b) = (5^2 + 3^2)\ (5 - 3) = (25 + 9)\ (2) = 34 \times 2 = 68$.

Exercises. Page 19.

12. $\sqrt{5^2+3^2+2}=\sqrt{25+9+2}=\sqrt{36}=6.$

Exercises. Page 20.

1. Equation, $2x-3=7.$
By axiom 1, $2x-3+3=7+3.$
Whence, $2x=7+3=10.$
By axiom 4, $x=5.$

Or
Equation, $2x-3=7.$
Transposing, $2x=7+3=10.$
By axiom 4, $x=5.$

2. $2x+4=14.$
$2x+4-4=14-4.$
$2x=10.$
$x=5.$

Or
$2x+4=14.$
$2x=14-4=10.$
$x=5.$

4. $2x-3=8+x.$
$2x-3+3=8+3+x.$
$2x=11+x.$
$2x-x=11+x-x.$
$x=11.$

Or
$2x-3=8+x.$
$2x-x=8+3.$
$x=11.$

6. $5x-5=4x-11.$
$5x-5+5-4x=4x-4x-11+5.$
$5x-4x=-11+5.$
$x=-6.$

Or
$5x-5=4x-11.$
$5x-4x=-11+5.$
$x=-6.$

8. $3x+15=x+25.$
$3x+15-15-x=x-x+25-15.$
$3x-x=25-15.$
$2x=10.$
$x=5.$

Or
$3x+15=x+25.$
$3x-x=25-15.$
$2x=10.$
$x=5.$

10. $-x+24=3x+12.$
$-x+24-24-3x=3x-3x+12-24.$
$-x-3x=12-24.$
$-4x=-12.$
$x=3.$

Or
$-x+24=3x+12.$
$-x-3x=-24+12$
$-4x=-12.$
$4x=12.$
$x=3.$

11. $-32x-30x+500=-58.$
By transposing, $-32x-30x=-58-500.$
$32x+30x=58+500.$
$62x=558.$
$x=9.$

13. $5(x-3)-7-2=24+3(8-x)$.

$5x-15-7-2=24+24-3x$.

$5x+3x=24+24+15+7+2$.

$8x=72$.

$x=9$.

15. $.375x-1.875=.12x+1.185$.

$.375x-.12x=1.185+1.875$.

$.255x=3.060$.

$x=12$.

Problems. Page 21.

6. Let x = the brother's number,
then $3x$ = the boy's number.
Since both together have 52,
the equation is $x+3x=52$.
Solving, we obtain $x=13$, and $3x=39$.

7. Let x = the less number,
then $4x$ = the greater number,
and the equation is $x+4x=120$.

8. Let x = Jane's number, then
$3x$ = Ann's number, and
$6x$ = Mary's number.
Since they together have 60 roses,
the equation becomes $x+3x+6x=60$.

Problems. Page 22.

10. Let x = the cost of the calf, then,
$3x$ = the cost of the cow, and
$9x$ = the cost of the horse.
The equation is $x+3x+9x=130$.
$\therefore$ The calf cost \$10, the cow \$30, and the horse \$90.

Problems. Page 23.

16. Let x = what B. paid, then
$x+50$ = what A. paid.
The equation is $2x+50=150$.
Wherefore, B. paid 50, and A. 100.

18. Let x = the least number, then
$x+1$ = the next larger, and
$x+2$ = the largest.
The equation is $3x+3=18$.
Whence $x=5$, $x+1=6$, $x+2=7$.

20. $16x-8=20x-8-5x$.

$16x-20x+5x=-8+8$.

$x=0$.

Problems. Page 24.

22. Let $12x$ = the larger part, then
x = the smaller part.
Equation, $13x = 130$.
Parts = 10 and 120.

23. Let x = the height of the flagstaff, then
$\frac{3}{7}x + \frac{2}{7}x$ = the length of the parts broken off.
The equation is $\frac{3}{7}x + \frac{2}{7}x = 35$.
From which $x = 49$ feet, the height of the flagstaff.

25. Let x = the number.
$2x$ = double the number.
$\therefore\ x + \frac{1}{5}x + \frac{3}{5}x + 9 = 2x$, the equation,
Whence $x = 45$.

26. Let x = the larger number, then
$\frac{3}{4}x$ = the smaller number.
Since the sum of the numbers is 42,
the equation is $x + \frac{3}{4}x = 42$.
The numbers required are 24 and 18.

Exercises. Page 26.

5. $x - (+3x) = x - 3x = -2x$.

7.
$$\begin{array}{r} 4x^2 - 5x \\ 3x^2 + 2x \\ \hline x^2 - 7x. \end{array}$$

9. $3ax - 2ax + 5ax - 3ax =$
$3ax + 5ax - 2ax - 3ax =$
$8ax - 5ax = 3ax$.

10. $6a - (4a - 2) =$
$6a - 4a + 2 = 2a + 2$

11. $3x^3 - 6 - (-x^3 + 2) =$
$3x^3 - 6 + x^3 - 2 =$
$4x^3 - 8$.

Exercises. Page 27.

14.
$$\begin{array}{r} 3 - x + x^2 + x^3 + x^4 \\ 4 + x - x^2 - x^3 - 2x^4 \\ \hline -1 - 2x + 2x^2 + 2x^3 + 3x^4. \end{array}$$

15.
$$\begin{array}{r} x^2 + y^2 - 3z^2 \\ 4x^2 - y^2 + 2z^2 \\ -2x^2 \quad + z^2 \\ x^2 \quad - z^2 \\ \hline 4x^2 \quad - z^2, \end{array}$$

17. $-8m + 5m - 5m = -8m$.

18. $6x - (-3y + 4x) =$
$6x + 3y - 4x =$
$6x - 4x + 3y = 2x + 3y$

19. $0 - (+3x) = -3x$.

21. 1. $8a-10b-2c$
$5a-4b-3c$
$3a+4b-6c$
$16a-10b-11c.$

3. $2x^2-ax+2a^2$
$3x^2+4ax-5a^2$
$x^2+ax+3a^2$
$6x^2+4ax.$

5. $3x^2y^3+x^3y^2$
$-6x^2y^3-3x^3y^2+2x^2y^2$
$-5x^2y^3+5x^3y^2-7x^2y^2$
$-8x^2y^3+3x^3y^2-5x^2y^2.$

24. $a+b$ $\quad$ $a-b$
$a-b$ $\quad$ $a+b$
$2b.$ $\quad$ $-2b.$

25. x^4+x^2+1
$x^4-x^3+x^2-x+1$
x^3+x

Exercises. Page 28.

2. 1. $3x-7y-(x+2y)-(4y-7x).$
$3x-7y-x-2y-4y+7x.$
$3x-x+7x-7y-2y-4y.$
$9x-13y.$

3. $x^2-2xy+y^2-(x^2+xy-y^2)-(-2xy+x^2+y^2).$
$x^2-2xy+y^2-x^2-xy+y^2+2xy-x^2-y^2.$
$x^2-x^2-x^2-2xy-xy+2xy+y^2+y^2-y^2.$
$-x^2-xy+y^2.$

5. $4x-3y-\{(2x+4y)+3x+[y-9x-(2y-x)+(x-y)]\}.$
$4x-3y-\{2x+4y+3x+y-9x-2y+x+x-y\}.$
$4x-3y-2x-4y-3x-y+9x+2y-x-x+y.$
$4x-2x-3x+9x-x-x-3y-4y-y+2y+y.$
$6x-5y.$

6. $m-[n-(m-p)].$
$m-[n-m+p].$
$m-n+m-p.$
$2m-n-p.$

8. $3x-\{2x+(5x-\overline{3x+y})\}.$
$3x-\{2y+(5x-3x-y)\}.$
$3x-\{2y+5x-3x-y\}.$
$3x-2y-5x+3x+y.$
$3x-5x+3x-2y+y.$
$3x+3x-5x-2y+y.$
$x-y.$

11. $3+(a-b)-(5-a^2-\overline{b+8}).$
$3+a-b-(5-a^2-b-8).$
$3+a-b-5+a^2+b+8.$
$3-5+8+b+a^2-b+b.$
$6+a+a^2.$

13. $a-[b+\{a-(b+a)\}].$
$a-[b+\{a-b-a\}].$
$a-[b+a-b-a].$
$a-b-a+b+a.$
$a.$

Exercises. Page 29.

15. $2x-\{a-(2a-[3a-(4a-[5a-(6a-x)])])\}$.
$2x-\{a-(2a-[3a-(4a-[5a-6a+x])])\}$.
$2x-\{a-(2a-[3a-(4a-5a+6a-x)])\}$.
$2x-\{a-(2a-[3a-4a+5a-6a+x])\}$.
$2x-\{a-(2a-3a+4a-5a+6a-x)\}$.
$2x-\{a-2a+3a-4a+5a-6a+x\}$.
$2x-a+2a-3a+4a-5a+6a-x$.
$x+3a$.

4. $4c-[a-(2b-3c)+c]+[a-(2b-5c-a)]$.
$4c-[a-2b+3c+c]+[a-2b+5c+a]$.
$4c-a+2b-3c-c+a-2b+5c+a$.
$4c-3c-c+5c-a+a+a+2b-2b$.
$+5c-a=+15-0=+15$.

6. $a-[-\{c+a=(a-b)-c\}+2a-(b-a)]$.
$a-[-\{c+a-a+b-c\}+2a-b+a]$.
$a-[-c-a+a-b+c+2a-b+a]$.
$a+c+a-a+b-c-2a+b-a$.
$a+a-a-2a-a+b+b+c-c$.
$-2a+2b=0+4=4$.

8. $5a-\{-3a-[3a-(2a-\overline{a-b})-a]+a\}$.
$5a-\{-3a-[3a-(2a-a+b)-a]+a\}$.
$5a-\{-3a-[3a-2a+a-b-a]+a\}$.
$5a-\{-3a-3a+2a-a+b+a+a\}$.
$5a+3a+3a-2a+a-b-a-a$.
$5a+3a+3a-2a+a-a-a-b$.
$8a-b=0-2=-2$.

9. $d-\{d+[b-(d+b+c-\overline{a+b+d})-c]\}$.
$d-\{d+[b-(d+b+c-a-b-d)-c]\}$.
$d-\{d+[b-d-b-c+a+b+d-c]\}$.
$d-\{d+b-d-b-c+a+b+d-c\}$.
$d-d-b+d+b+c-a-b-d+c$.
$d-d+d-d-b+b-b+c+c$.
$-b+2c=-2+6=4$.

10. $d-[5b-\{d-(5c-\overline{2c-b}-4b)+2d-(d-\overline{2b+c})\}]$.
$d-[5b-\{d-(5c-2c+b-4b)+2d-(d-2b-c)\}]$.
$d-[5b-\{d-5c+2c-b+4b+2d-d+2b+c\}]$.
$d-[5b-d+5c-2c+b-4b-2d+d-2b-c]$.
$d-5b+d-5c+2c-b+4b+2d-d+2b+c$.
$d+d+2d-d-5b-b+4b+2b-5c+2c+c$.
$3d-2c=12-6=6$.

Exercises. Page 30.

4. 1. $2a-2b-2c-(-d-e+f)$.

4. $by+ax+bx-(-cx-cz+xy)$.

5. $x+y-z-(-u+4-5a)$.

REVIEW.

3. $\frac{1}{2}x=10$.
$x=20$.

4. $\frac{1}{5}x+5=-9$.
$\frac{1}{5}x=-14$.
$x=-70$.

2. 1.
$$\begin{array}{rrrrr} 3x^3-4a^2- & 4ax^2+ & 5a^2x & & \\ 2x^3 & +3ax^2+ & a^2x+ & 6a^3 & \\ -10x^3 & +18ax^2- & 14a^2x & & \\ \hline 11x^3 & -17ax^2+ & 8a^2x+ & 4a^3 & \\ 6x^3-4a^2 & & +10a^3. & & \end{array}$$

Page 31.

4.
$$\begin{array}{rrrr} a+ & ab- & cd+ & f \\ 3a+ & 5ab- & 6cd- & f \\ -5a- & 6ab+ & 6cd- & 7f \\ \hline -a+ & ab+ & cd+ & 4f \\ -2a+ & ab & -3f. & \end{array}$$

5.
$$\begin{array}{rrrr} 3a+ & b+ & c & \\ 5a+ & 2b & +3ac & \\ a & +c+ & ac & \\ \hline -3a- & 8b & -9ac & \\ 6a- & 5b+ & 2c- & 5ac. \end{array}$$

3. 1.
$$\begin{array}{r} 6ac-5ab+c^2 \\ 3ac+3ab \quad -7c \\ \hline 3ac-8ab+c^2+7c. \end{array}$$

3.
$$\begin{array}{r} 0 \\ a^3-2a^2-a-2 \\ \hline -a^3+2a^2+a+2. \end{array}$$

4. $3x^2-2x+1-(x^2+2x+3)-(2x^2-6x-6)$.
$3x^2-2x+1-x^2-2x-3-2x^2+6x+6$.
$3x^2-x^2-2x^2-2x-2x+6x+1-3+6$.
$2x+4$.

2. $3a-\{a+b-[a+b+c-(a+b+c+d)]\}$.
$3a-\{a+b-[a+b+c-a-b-c-d]\}$.
$3a-\{a+b-a-b-c+a+b+c+d\}$.
$3a-a-b+a+b+c-a-b-c-d$.
$3a-a+a-a-b+b-b+c-c-d$.
$2a-b-d$.

4. $a^2-(b^2-c^2)-b^2+(c^2-a^2)+c^2-(b^2-a^2)$.
$a^2-b^2+c^2-b^2+c^2-a^2+c^2-b^2+a^2$.
$a^2-a^2+a^2-b^2-b^2-b^2+c^2+c^2+c^2$.
$a^2-3b^2+3c^2$.

5. $4x-3y-\{(2x+4y)+3x+[y-9x-(2y-x)+(x-y)]\}$.
$4x-3y-\{2x+4y+3x+[y-9x-2y+x+x-y]\}$.
$4x-3y-\{2x+4y+3x+y-9x-2y+x+x-y\}$.
$4x-3y-2x-4y-3x-y+9x+2y-x-x+y$.
$4x-2x-3x+9x-x-x-3y-4y-y+2y+y$.
$6x-5y$.

5. 1. $\{a-(b-c)\{+\}b-(c-a)\{+\}c-(a-b)\}$.
$\{a-b+c\{+\}b-c+a\{+\}c-a+b\}$.
$a-b+c+b-c+a+c-a+b$.
$a+a-a-b+b+b+c-c+c$.
$a+b+c=10+3+8=21$.

2. $\frac{1}{2}a-\frac{1}{3}b+\frac{1}{4}c-(\frac{1}{5}b-\frac{1}{4}c+\frac{1}{5}a)$.
$\frac{1}{2}a-\frac{1}{3}b+\frac{1}{4}c-\frac{1}{5}b+\frac{1}{4}c-\frac{1}{5}a$.
$\frac{1}{2}a-\frac{1}{5}a-\frac{1}{3}b-\frac{1}{5}b+\frac{1}{4}c+\frac{1}{4}c$.
$5-2-1-1+2+2=5$.

4. $1-[a^3-2-(-2a^3-3)]$.
$1-[a^3-2+2a^3+3]$.
$1-a^3+2-2a^3-3$.
$-3a^3=-3000$.

5. $a-\{a-[b-(a+b+c-\overline{a+b+d})-c]\}$.
$a-\{a-[b-(a+b+c-a-b-d)-c]\}$.
$a-\{a-[b-a-b-c+a+b+d-c]\}$.
$a-\{a-b+a+b+c-a-b-d+c\}$.
$a-a+b-a-b-c+a+b+d-c$.
$a-a-a+a+b-b+b-c-c+d$.
$b-2c+d=3-16+4=-9$.

Problems. Page 32.

1.
$$\begin{array}{r} -10a-4b-15c+12x \\ 8a+4b-5c-x \\ \hline -18a-8b-10c+13x. \end{array}$$

2.
$$\begin{array}{r} 5a+3b-3c \\ 3a-4b+5c \\ \hline 2a+7b-8c. \end{array}$$

3.
$$\begin{array}{r} a^2+2ab+b^2 \\ a^2\qquad -b^2 \\ \hline 2ab+2b^2. \end{array}$$

4.
$$\begin{array}{r} a^2-b^2 \\ a^2+2ab \\ \hline -2ab-b^2. \end{array}$$

5.
$$\begin{array}{r} 0 \\ a^2+2ab+b^2 \\ \hline -a^2-2ab-b^2. \end{array}$$

6.
$$\begin{array}{r} 0 \\ x^2-y^2 \\ \hline -x^2+y^2. \end{array}$$

7.
$$\begin{array}{r} x^2+ax+y^2 \\ x^2-2ax+y^2 \\ \hline +3ax. \end{array}$$

8. $x-p=0$.
Axiom 1, $x-p+p=0+p$.
$x=0+p=p$.

10. Let x = age of youngest.
$x+3$ = age of 3d son.
$x+6$ = age of 2d son.
$x+9$ = age of oldest son.

11. $x-9+y+11=x+y+2$.

12. $\frac{b^c}{d^c}=\frac{2^3}{4^3}=\frac{8}{64}=\frac{1}{8}$.

13. A. $= a^2+3a^2b-5ac+7by$
B. $= -a^2+3a^2b+5ac+7by$
C. $= -3a^2-3a^2b+ac-7by$
$-3a^2+3a^2b+ac+7by$

A. $= a^2+3a^2b-5ac+7by$
B. $= -a^2+3a^2b+5ac+7by$
$6a^2b+14by$
C. $= -3a^2-3a^2b+ac-7by$
$3a^2+9a^2b-ac+21by$

A. $a^2+3a^2b-5ac+7by$
B. $-a^2+3a^2b+5ac+7by$
$2a^2-10ac$
C. $-7by-3a^2+ac-3a^2b$
$2a^2-10ac$
$-7by-a^2-9ac-3a^2b.$

Exercises. Page 35.

1. $9\times10=90.$

2. $-6x\ (-4)=24x.$

4. $70a^2y^3\ 0=0.$

5. $70a^2y^3\ (-1)=-70a^2y^3.$

8. $-12ax^2\ (-9ax^2)=108a^2x^4.$

9. $14xy\ (-4ay)=-56axy^2.$

11. $8fg\ (-f^2g^3)=-8f^3g^4.$

14. $-8a^n\cdot 6a^n=-48a^{2n}.$

15. $10x^ny^m\cdot(-3x^ny^m)=-30x^{2n}y^{2m}.$

16. $6y^{n+1}\cdot(-6y^{n-1})=-36y^{2n}.$

18. $ay^{2n+1}\cdot ay^{2n-1}=a^2y^{4n}.$

20. 1. $3x-2y$
$-3abxy$
$-9abx^2y+6abxy^2.$

Page 36.

20. 2. $7ax-4by$
$2xy$
$14ax^2y-8byxy^2.$

4. $4xc^2-4m^2n$
$4an$
$16ac^2nx-16am^2n^2.$

5. $m+m^2-m^3$
$-9m^2n$
$-9m^3n-9m^4n+9m^5n.$

6. $3x^n-4x^{n-1}$
x^2
$3x^{n+2}-4x^{n+1}.$

8. $7x^{1-n}-5x^{2-n}$
$5x^n$
$35x-25x^2.$

21. 3. $-abc^2\ a^2b^2c\ (-a^2bc)=a^5b^4c^4.$

4. $-3m^2np^3\ (mnp^2)\ (-m^2np)=3m^5n^3p^6.$

5. $-\frac{1}{2}xy\ (-\frac{2}{3}cx)\ (-\frac{2}{5}a^2y)=-\frac{4}{30}a^2cx^2y^2=-\frac{2}{15}a^2cx^2y^2.$

Exercises. Page 37.

2. 1. $2x - 5$
$3x + 2$
$6x^2 - 15x$
$4x - 10$
$6x^2 - 11x - 10.$

4. $4x - 5y$
$5x - 4y$
$20x^2 - 25xy$
$-16xy + 20y^2$
$20x^2 - 41xy + 20y^2.$

5. $8x^2 - 6y^2$
$5x^2 + 4y^2$
$40x^4 - 30x^2y^2$
$+32x^2y^2 - 24y^4$
$40x^4 + 2x^2y^2 - 24y^4.$

8. $12x^4y - 8x^2y^3$
$4x^2 + 3y^2$
$48x^6y - 32x^4y^3$
$36x^4y^3 - 24x^2y^5$
$48x^6y + 4x^4y^3 - 24x^2y^5.$

9. $a^2 + 2ab$
$x + y$
$a^2x + 2abx + a^2y + 2aby.$

10. $x^3 + 2x^2y$
$a - b$
$ax^3 + 2ax^2y - bx^3 - 2bx^2y.$

1. $m^2 + mn + n^2$
$m - n$
$m^3 + m^2n + mn^2$
$-m^2n - mn^2 - n^3$
$m^3 - n^3.$

Page 38.

2. 3. $x^5 + 7ax^5 + a^2x^5$
$x + ax$
$x^6 + 7ax^6 + a^2x^6$
$ax^6 + 7a^2x^6 + a^3x^6$
$x^6 + 8ax^6 + 8a^2x^6 + a^3x^6.$

5. $x^3 + x^2y + xy^2 + y^3$
$x - y$
$x^4 + x^3y + x^2y^2 + xy^3$
$-x^3y - x^2y^2 - xy^3 - y^4$
$x^4 - y^4.$

7. $x^3 + x^2 + 1 + x$
$x - 1$
$x^4 + x^3 + x + x^2$
$-x^3 - x^2 - 1 - x$
$x^4 - 1.$

9. $x^3 - 5x^2 + 1$
$2x^3 + 5x + 1$
$2x^6 - 10x^5 + 2x^3$
$5x^4 - 25x^3 + 5x$
$x^3 - 5x^2 + 1$
$2x^6 - 10x^5 + 5x^4 - 22x^3 - 5x^2 + 5x + 1.$

10. $x^2 - x + 1$
$x^2 + x + 1$
$x^4 - x^3 + x^2$
$x^3 - x^2 + x$
$x^2 - x + 1$
$x^4 + x^2 + 1$
$x^4 + x^2 + 1$
$x^8 + x^6 + x^4$
$x^6 + x^4 + x^2$
$x^4 + x^2 + 1$
$x^8 + 2x^6 + 3x^4 + 2x^2 + 1$

Exercises. Page 39.

2. $(a+b)\ (a-b)-b^2=a^2-b^2-b^2=a^2-2b^2$.

4. $(a^2+ab+b^2)\ (a-b)-(a^3-b^3)=a^3-b^3-(a^3-b^3)=0$.

5. $(a^2-ab+b^2)\ (a+b)+(a^3+b^3)=a^3+b^3+a^3+b^3=2a^3+2b^3$.

1. $(x+y+z)\ (x+y-z)-(2xy-z^2)=(x+y)^2-z^2-2xy+z^2=x^2+2xy+y^2-2xy=x^2+y^2$.

3. $[xz-(x-y)\ (y+z)]-y\ [y-(x-z)]=xz-xy+y^2-xz+yz-y^2+xy-yz=0$.

4. $(a-1)\ (a-2)-3a\ (a+3)+2\ [(a+2)\ (a+1)-3]=a^2-3a+2-3a^2-9a+2a^2+6a+4-6=-6a$.

5. $(x+y)\ (x^3-y^3)\ [x^2-y\ (x-y)]=(x^4-xy^3+x^3y-y^4)\ (x^2-xy+y^2)=x^6-y^6$.

REVIEW. Page 39.

1.
$$\begin{array}{r} -11x^m \\ -\ 8x^m \\ \hline 88x^{m+m}=88x^{2m}. \end{array}$$

2.
$$\begin{array}{r} 4x^my^n \\ -\ x^n\,y^nz^2 \\ \hline -\ 4x^{m+n}y^{2n}z^2 \\ \quad 16y^2z^2 \\ \hline -64x^{m+n}y^{2n+2}z^4. \end{array}$$

Page 40.

6.
$$\begin{array}{r} 3x^{\,m+2}y\ -\ 3x\,y^{n+1} \\ 4x^{\,m+5}y^3-\ 4x^4y^{n+2} \\ \hline 12x^{2m+7}y^3-12x^{m+6}y^{n+3} \\ \quad -12x^{m+6}y^{n+3}+12x^5y^{2n+3} \\ \hline 12x^{2m+7}y^3-24x^{m+6}y^{n+3}+12x^5y^{2n+3}. \end{array}$$

7.
$$\begin{array}{r} a\ +\ 1 \\ 3a\ -\ 2 \\ \hline 3a^2+\ 3a \\ \quad -\ 2a\ -2 \\ \hline 3a^2+\ \ a\ -2 \\ 3a^2-\ \ a\ -2 \\ \hline 9a^4+\ 3a^3-6a^2 \\ \quad -\ 3a^3-\ a^2\ +2a \\ \hline \quad\quad -6a^2-2a+4 \\ \hline 9a^4-13a^2\ +4. \end{array}$$

9.
$$\begin{array}{r} 6\ (x+y)^{\,n+1}+4\ (x+y)^n\ \ -2\ (x+y)^{\,n-1} \\ \quad (x+y)^{\,n+2} \\ \hline 6\ (x+y)^{2n+3}+4\ (x+y)^{2n+2}-2\ (x+y)^{2n+1}. \end{array}$$

11. x^2+x+1
x^2-x+1
$x^4+x^3+x^2$
$-x^3-x^2-x$
x^2+x+1
x^4+x^2+1
x^2+x-1
$x^6+x^4+x^2$
x^5+x^3+x
$-x^4-x^2-1$
$x^6+x^5+x^3+x-1.$

13. $a^2+b^2+c^2-bc-ac-ab$
$a+b+c$
$a^3+ab^2+ac^2-abc-a^2c-a^2b$
$a^2b+b^3+bc^2-b^2c-abc-ab^2$
$a^2c+b^2c+c^3-bc^2-ac^2-abc$
$a^3+b^3+c^3-3abc.$

14. $a^m+b^p-2c^n$
$2a^m-3b$
$2a^{2m}+2a^mb^p-4a^mc^n$
$-3a^mb-3b^{p+1}+6bc^n$
$2a^{2m}+2a^mb^p-4a^mc^n-3a^mb-3b^{p+1}+6bc^n.$

2. 1. $a+b$
$a+b$
a^2+ab
$ab+b^2$
$a^2+2ab+b^2.$

2. $a-b$
$a-b$
a^2-ab
$-ab+b^2$
$a^2-2ab+b^2.$

3. $a+b$
$a-b$
a^2+ab
$-ab-b^2$
$a^2-b^2.$

8. $a+1$
$a+1$
a^2+2a+1
$a+1$
a^3+2a^2+a
a^2+2a+1
$a^3+3a^2+3a+1.$

9. $a-1$
$a-1$
a^2-2a+1
$a-1$
a^3-2a^2+a
$-a^2+2a-1$
$a^3-3a^2+3a-1.$

12. $a+b+c$
$a+b+c$
$a^2+ab+ac$
$ab+b^2+bc$
$ac+bc+c^2$
$a^2+2ab+2ac+b^2+2bc+c^2=a^2+b^2+c^2+2ab+2ac+2bc.$

13. $a - b - c$
$a - b - c$
$a^2 - ab - ac$
$- ab + b^2 + bc$
$- ac + bc + c^2$
$a^2 - 2ab + b^2 - 2ac + 2bc + c^2 = a^2 + b^2 + c^2 - 2ab - 2ac + 2bc.$

3. 1. $(x+y)(x-y) = x^2 - y^2.$
2. $(x+y)^2 = x^2 + 2xy + y^2.$
3. $(x-y)^2 = x^2 - 2xy + y^2.$
4. $(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3.$
5. $(x-y)^3 = x^3 - 3x^2y + 3xy^2 - y^3.$

Page 41.

4. 1. $a - 2(b+3a) - 3\{b+2(a-b)\}$
$a - 2b - 6a - 3\{b+2a-2b\}$
$a - 2b - 6a - 3b - 6a + 6b$
$-11a + b = -0 + 1 = 1.$

3. $4a - [2a - \{2b(a+c) - 2b(b+c)\}]$
$4a - [2a - \{2ab + 2bc - 2b^2 - 2bc\}]$
$4a - [2a - 2ab - 2bc + 2b^2 + 2bc]$
$4a - 2a + 2ab + 2bc - 2b^2 - 2bc$
$2a + 2ab - 2b^2 = 0 + 0 - 2 = -2.$

5. $5a - 7(b-c) - [6a - (3b+2c) + 4c]$
$5a - 7b + 7c - [6a - 3b - 2c + 4c]$
$5a - 7b + 7c - 6a + 3b + 2c - 4c$
$-a - 4b + 5c = -0 - 4 - 5 = -9.$

5. 3. $\frac{a+b+c}{2} - a + \frac{a+b+c}{2} - b + \frac{a+b+c}{2} - c = \frac{b+b+2b}{2} - b + \frac{b+b+2b}{2} -$
$-b + \frac{b+b+2b}{2} - 2b = \frac{2b+2b+0}{2} = \frac{4b}{2} = 2b.$

4. $(m+1)^c (m+1)^c = (m+1)^{2c} = (m+1)^{4b}.$
5. $(x^2 - (a+b)x + ab)(x-c) = (x^2 - 2bx + b^2)(x-2b) =$
$x^3 - 4bx^2 + 5b^2x - 2b^3.$

Page 42.

1. 1. $4a^6 \div a^4 = 4a^2.$
3. $-70y^9 \div 7y^7 = -10y^2.$
5. $40x^9y^4 \div -5x^2y^3 = -8x^7y.$
7. $56a^8b^6 \div -14a^2b^4 = -4a^6b^2.$
9. $-48m^8n^6 \div -8m^4n^4 = 6m^4n^2.$

2. 1. $-16a^3) -16a^3 - 32a^5$
$1 + 2a^2.$
3. $-5a^5) -15a^5 + 10a^7$
$3 - 2a^2.$
5. $-6x^3z^3) -6x^5z^5 - 12x^3z^3$
$x^2z^2 + 2.$
7. $-3a) 6a^3 - 12a^2b - 18ab^2$
$-2a^2 + 4ab + 6b^2.$
9. $-4x) 16x^3 - 8x^2y - 12xy^2$
$-4x^2 + 2xy + 3y^2.$

3. 1. $-m) -m + m^2 + m^3 - m^4$
$+ 1 - m - m^2 + m^3.$
2. $x) 0 + x + x^2 - x^3$
$0 + 1 + x - x^2.$

Page 43.

3. $a^m)a^m-a^{m+1}+a^{m+2}$
$1-a+a^2.$

6. $1)1^5-2^4+3^3-4^2+5$
$1-16+27-16+5=1.$

10. $5x^my^nz)15x^my^nz^r-35x^{m+2}y^{2n}z$
$3z^{r-1}-7x^2y^n.$

7. $x^{n+4})x^{n+3}+x^{n+2}+x^{n+1}+x^n$
$x^{-1}+x^{-2}+x^{-3}+x^{-4}.$

8. $x^2)13x^{m+3}+7x^{n+2}-x^n$
$13x^{m+1}+7x^n-x^{n-2}.$

Page 45.

1.
$$\begin{array}{l|l} 4x^2+12x+9 & 2x+3 \\ \underline{4x^2+6x} & \underline{2x+3} \\ 6x+9 & \\ \underline{6x+9} & \\ \end{array}$$

3.
$$\begin{array}{l|l} x^2-9y^2 & x+3y \\ \underline{x^2+3xy} & \underline{x-3y} \\ -3xy-9y^2 & \\ \underline{-3xy-9y^2} & \\ \end{array}$$

5.
$$\begin{array}{l|l} a^3-3a^2y+3ay^2-y^3 & a-y \\ \underline{a^3-a^2y} & \underline{a^2-2ay+y^2} \\ -2a^2y+3ay^2 & \\ \underline{-2a^2y+2ay^2} & \\ ay^2-y^3 & \\ \underline{ay^2-y^3} & \\ \end{array}$$

6.
$$\begin{array}{l|l} 6x^4-96 & 3x-6 \\ \underline{6x^4-12x^3} & \underline{2x^3+4x^2+8x+16} \\ 12x^3-96 & \\ \underline{12x^3-24x^2} & \\ 24x^2-96 & \\ \underline{24x^2-48x} & \\ 48x-96 & \\ \underline{48x-96} & \\ \end{array}$$

9.
$$\begin{array}{l|l} 144x^4-145x^2y^2+36y^4 & 4x+3y \\ \underline{144x^4+108x^3y} & \underline{36x^3-27x^2y-16xy^2+12y^3} \\ -108x^3y-145x^2y^2 & \\ \underline{-108x^3y-81x^2y^2} & \\ -64x^2y^2+36y^4 & \\ \underline{-64x^2y^2-48xy^3} & \\ 48xy^3+36y^4 & \\ \underline{48xy^3+36y^4} & \\ \end{array}$$

12.
$$\begin{array}{l|l} a^3-4ab^2-a-4ab & a-2b-1 \\ \underline{a^3-2a^2b-a^2} & \underline{a^2+2ab+a} \\ 2a^2b-4ab^2-4ab & \\ \underline{2a^2b-4ab^2-2ab} & \\ a^2-2ab-a & \\ \underline{a^2-2ab-a} & \\ \end{array}$$

14. $x^3+y^3+z^3-3xyz \mid x+y+z$

$x^3+x^2y+x^2z \mid x^2-xy+y^2-xz-yz+z^2$

$-x^2y-x^2z+y^3+z^3-3xyz$

$-x^2y-xy^2 \quad -xyz$

$xy^2-x^2z-2xyz+y^3+z^3$

$xy^2+y^3+y^2z$

$-x^2z-2xyz-y^2z+z^3$

$-x^2z-xyz-xz^2$

$-xyz+xz^2-y^2z+z^3$

$-xyz \quad -y^2z-yz^2$

$xz^2+yz^2+z^3$

$xz^2+yz^2+z^3.$

16. $x^4-y^4 \mid x-y$

$x^4-x^3y \mid x^3+x^2y+xy^2+y^3$

x^3y-y^4

$x^3y-x^2y^2$

$x^2y^2-y^4$

$x^2y^2-xy^3$

xy^3-y^4

$xy^3-y^4.$

17. $x^{2n}-y^{2n} \mid x^n+y^n$

$x^{2n}+x^ny^n \mid x^n-y^n$

$-x^ny^n-y^{2n}$

$-x^ny^n-y^{2n}.$

18. $x^4-x^2-x^{-2}+x^{-4} \mid x-x^{-1}$

$x^4-x^2 \mid x^3-x^{-3}$

$-x^{-2}+x^{-4}$

$-x^{-2}+x^{-4}.$

20. $27a^3-8b^3-27c^3-54abc \mid 3a-2b-3c$

$27a^3-19a^2b-27a^2c \mid 9a^2+9ac+9c^2+6ab+4b^2-6bc$

$27a^2c+18a^2b-8b^3-27c^3-54abc$

$27a^2c-27ac^2 \quad -18abc$

$27ac^2+18a^2b-8b^3-27c^3-36abc$

$27ac^2-18bc^2-27c^3$

$18a^2b+18bc^2-8b^3-36abc$

$18a^2b-12ab^2-18abc$

$12ab^2+18bc^2-8b^3-18abc$

$12ab^2-12b^2c-8b^3$

$-18abc+18bc^2+12b^2c$

$-18abc+18bc^2+12b^2c.$

REVIEW. Page 45.

2.

$2a+b-4c$

$7a \quad -3c+m$

$-9a \quad +6c \quad +3ab+7am$

$b-c+m+3ab+7am.$

3.

$5\,(a+x)$

$-2\,(a+x)$

$-3\,(a+x)$

$0\,(a+x)=0.$

4.
$$\begin{array}{r} 4\sqrt{x}+2a^2x \\ -\ \sqrt{x}-\ a^2x \\ \hline 4\sqrt{x}-\ a^2x \\ -3\sqrt{x} \\ \hline 4\sqrt{x}. \end{array}$$

5.
$$\begin{array}{l} (\ b-a)\sqrt{x} \\ (2a-b)\sqrt{x} \\ \underline{(\ a+c)\sqrt{x}} \\ (2a+c)\sqrt{x}. \end{array}$$

Page 46.

2. 2. $51a^3b^2c-18abc-14a^2y-41a^3b^2c+27abc+14a^2y=$
$10a^3b^2c+9abc.$

4. $\frac{3}{2}a+36+\frac{1}{4}c+\frac{1}{2}bc-\frac{5}{2}a+76+3bc-\frac{1}{2}c=-a+112-\frac{1}{4}c+\frac{7}{2}bc.$

5. $\frac{1}{2}y-\frac{5}{2}a-\frac{3}{4}x+\frac{1}{3}a-3y-\frac{11}{4}a+\frac{2}{3}x=-\frac{5}{2}y-1\frac{5}{12}a-1\frac{1}{2}x.$

3. 3. $[(1+x)-(1+2x)]+[(1-x)+(1-2x)]-[(1-x)-(1-2x)]=$
$[1+x-1-2x]+[1-x+1-2x]-[1-x-1+2x]=$
$1+x-1-2x+1-x+1-2x-1+x+1-2x=$
$1-1+1+1-1+1+x-2x-x-2x+x-2x=$
$2-5x.$

5. $4x-3y-\{(2x+4y)+3x+[y-9x-(2y-x)+(x-y)]\}=$
$4x-3y-\{2x+4y+3x+[y-9x-2y+x+x-y]\}=$
$4x-3y-\{2x+4y+3x+y-9x-2y+x+x-y\}=$
$4x-3y-2x-4y-3x-y+9x+2y-x-x+y=$
$4x-2x-3x+9x-x-x-3y-4y-y+2y+y=$
$6x-5y.$

4. 1.
$$\begin{array}{r|l} x^2+2ax+a^2 & x+a \\ \underline{x^2+\ ax} & \underline{x+a} \\ ax+a^2 & \\ ax+a^2. & \end{array}$$

3.
$$\begin{array}{r|l} x^2-a^2 & x+a \\ \underline{x^2+ax} & \underline{x-a} \\ -ax-a^2 & \\ -ax-a^2. & \end{array}$$

5.
$$\begin{array}{r|l} x^3+a^3 & x+a \\ \underline{x^3+ax^2} & \underline{x^2-ax+a^2} \\ -ax^2+a^3 & \\ \underline{-ax^2-a^2x} & \\ a^2x+a^3 & \\ a^2x+a^3. & \end{array}$$

7.
$$\begin{array}{r|l} x^4-a^4 & x+a \\ \underline{x^4+ax^3} & \underline{x^3-ax^2+a^2x-a^3} \\ -ax^3-a^4 & \\ \underline{-ax^3-a^2x^2} & \\ +-a^2x^2-a^4 & \\ \underline{+-a^2x^2+a^3x} & \\ -a^3x-a^4 & \\ -a^3x-a^4. & \end{array}$$

9.
$$\begin{array}{r|l} x^4+a^2x^2+a^4 & x^2+ax+a^2 \\ \underline{x^4+ax^3+a^2x^2} & \underline{x^2-ax+a^2} \\ -ax^3+a^4 & \\ \underline{-ax^3-a^2x^2-a^3x} & \\ a^2x^2+a^3x+a^4 & \\ a^2x^2+a^3x+a^4. & \end{array}$$

11. $x^3+y^3+z^3-3xyz \mid x+y+z$

$x^3+x^2y+x^2z \mid x^2-xy+y^2-xz+z^2-yz$

$-x^2y-x^2z+y^3+z^3-3xyz$

$-x^2y-xy^2 \quad -xyz$

$xy^2-x^2z+y^3+z^3-2xyz$

$xy^2 \quad +y^3+x^2z$

$-x^2z+z^3-y^2z-2xyz$

$-x^2z-xz^2 \quad -xyz$

$xz^2+z^3-y^2z-xyz$

$xz^2+z^3+yz^2$

$-xyz-y^2z-yz^2$

$-xyz-y^2z-yz^2.$

5. 1. $x-3$

$x-5$

x^2-3x

$-5x+15$

$x^2-8x+15$

$x-7$

x^3-8x^2+15x

$-7x^2+56x-105$

$x^3-15x^2+71x-105.$

Page 47.

5. $(x-y)^2\,(x+y)^2\,(x^2+y^2)^2=(x^2-y^2)\,(x^2-y^2)\,(x^2+y^2)^2=$
$(x^2-y^2)^2\,(x^2+y^2)^2=(x^4-y^4)\,(x^4-y^4)=(x^4-y^4)^2=$
$x^8-2x^4y^4+y^8.$

6. $(x^2+1)^3\,(x+1)^3\,(x-1)^3=(x^6+3x^4+3x^2+1)\,(x^6-3x^4+3x^2-1)=$
$x^{12}-3x^8+3x^4-1.$

9. $a^m+b^p-2c^n$

$2a^m-3b$

$2a^{2m}+2a^mb^p-4a^mc^n$

$-3a^mb \quad -3b^{p+1}+6bc^n$

$2a^{2m}+2a^mb^p-4a^mc^n-3a^mb-3b^{p+1}+6bc^n.$

10. $a^{2n}+x^{2n}$

$2a^{2n}-2x^{2n}$

$2a^{4n}+2a^{2n}x^{2n}$

$-2a^{2n}x^{2n}-2x^{4n}$

$2a^{4n}-2x^{4n}.$

11. $\frac{2}{3}x^2+\frac{1}{4}y^2$

$\frac{2}{3}x^2-\frac{1}{4}y^2$

$\frac{4}{9}x^4+\frac{1}{6}x^2y^2$

$-\frac{1}{6}x^2y^2-\frac{1}{16}y^4$

$\frac{4}{9}x^4 \quad -\frac{1}{16}y^4.$

12. $(1-x)\,(1+x)\,(1+x^2)\,(1+x^4)=(1-x^2)\,(1+x^2)\,(1+x^4)=$
$(1-x^4)\,(1+x^4)=1-x^8.$

6. 1. $8.4x^4-1.6x^3-10.3x^2+10.2x-3.9 \mid 2.4x^2+1.6x-2.6$

$8.4x^4+5.6x^3-9.1x^2 \mid 3.5x^2-3x+1.5$

$-7.2x^3-1.2x^2+10.2x-3.9$

$-7.2x^3-4.8x^2+7.8x$

$3.6x^2+2.4x-3.9$

$3.6x^2+2.4x-3.9.$

2. $2x^{10}+8.5x^9-19.25x^8+9.65x^7-1.05x^6-.075x^5 \mid .5x^7+2.5x^6-3x^5-.15x^4$

$2x^{10}+10x^9-12x^8-.60x^7 \mid 4x^3-3x^2+.5x$

$-1.5x^9-7.25x^8+10.25x^7-1.05x^6-.075x^5$

$-1.5x^9-7.5x^8+9x^7+.45x^6$

$.25x^8+1.25x^7-1.50x^6-.075x^5$

$.25x^8+1.25x^7-1.50x^6-.075x^5.$

3. $-\frac{1}{8}d^5+d^2-\frac{4}{24}d^3+\frac{5}{6}d^4 \mid -\frac{3}{4}d^2+2d$

$-\frac{1}{8}d^5+\frac{2}{6}d^4 \mid \frac{1}{6}d^3-\frac{2}{3}d^2+\frac{1}{2}d$

$\frac{3}{6}d^4-\frac{4}{24}d^3+d^2$

$\frac{3}{6}d^4-\frac{4}{3}d^3$

$-\frac{9}{24}d^3+d^2$

$-\frac{9}{24}d^3+d^2.$

4. $x^4-\frac{19x^2}{6}+\frac{x}{3}+\frac{1}{6} \mid x^2-2x+\frac{1}{2}$

$x^4-2x^3+\frac{1}{2}x^2 \mid x^2+2x+\frac{1}{3}$

$2x^3-\frac{22}{6}x^2+\frac{x}{3}+\frac{1}{6}$

$2x^3-4x^2+x$

$\frac{1}{3}x^2-\frac{2}{3}x+\frac{1}{6}$

$\frac{1}{3}x^2-\frac{2}{3}x+\frac{1}{6}.$

5. $1.2a^4x-5.494a^3x^2+4.8a^2x^3+0.9ax^4-x^5 \mid 0.6ax-2x^2$

$1.2a^4x-4a^3x^2 \mid 2a^3-2.49a^2x-.3ax^2+.5x^3$

$-1.494a^3x^2+4.8a^2x^3+0.9ax^4-x^5$

$-1.494a^3x^2+4.98a^2x^3$

$-.18a^2x^3+0.9ax^4-x^5$

$-.18a^2x^3+.6ax^4$

$.3ax^4-x^5$

$.3ax^4-x^5.$

7. 2. $3x^2-\{5x-[4-(x-1)(2x-3)-7x=x^2-7x+1$

$3x^2-5x+[4-(x-1)(2x-3)-7x=x^2-7x+1$

$3x^2-5x+4-(x-1)(2x-3)-7x=x^2-7x+1$

$3x^2-5x+4-2x^2+5x-3-7x=x^2-7x+1$

$x^2-7x+1=x^2-7x+1.$

4. $3x+\left(\frac{x+bcx+4ax}{x}\right)-\left(\frac{4bc}{4}\right)-\left(\frac{8a^2+2a}{2a}\right)=3x$

$3x+1+bc+4a-bc-4a-1=3x$

$3x+1-1+bc-bc+4a-4a=3x$

$3x=3x.$

5. $-a\,[a\,\{(a-b)-(a+b)\}-\overline{a-2ab}]+a^3=2a^2$

$-a\,[a\,\{a-b-a-b\}-a+2ab]+a^2=2a^2$

$-a\,[a\,\{-2b\}-a+2ab]+a^2=2a^2$

$-a\,[-2ab-a+2ab]+a^2=2a^2$

$2a^2b+a^2-2a^2b+a^2=2a^2$

$2a^2=2a^2.$

Page 49.

Squares:

1. $(b+c)\ (b+c)=(b+c)^2=b^2+2bc+c^2.$

3. $(a+x)\ (a+x)=(a+x)^2=a^2+2ax+x^2.$

5. $(3x-2d)\ (3x-2d)=(3x-2d)^2=9x^2-12dx+4d^2.$

7. $(2a^2+3x^2)\ (2a^2+3x^2)=4a^4+12a^2x^2+9x^4.$

9. $(3a+5)\ (3a+5)=(3a+5)^2=9a^2+30a+25.$

10. $(7-2x^2)\ (7-2x^2)=(7-2x^2)^2=49-28x^2+4x^4.$

Expansions:

11. $(b+d)\ (b-d)=b^2-d^2.$

13. $(2a^2+2b^2)\ (2a^2-2b^2)=4a^4-4b^4.$

15. $(a+7b^2)\ (a-7b^2)=a^2-49b^4.$

Page 50.

2. $a^2+6ab+9b^2=(a+3b)^2=(a+3b)\ (a+3b).$

3. $x^2-2x^2y+x^2y^2=(x-xy)^2=(x-xy)\ (x-xy)=x\ (1-y).$
$x\ (1-y)=x^2\ (1-y)\ (1-y).$

4. $1-x^2=(1+x)\ (1-x).$

5. $25-16a^2=(5+4a)\ (5-4a).$

9. $36m^2-36mn+9n^2=(6m-3n)^2=(6m-3n)\ (6m-3n)=$
$3\ (2m-n)\cdot 3\ (2m-n)=9\ (2m-n)^2.$

10. $x^6+8x^5+16x^4=(x^3+4x^2)=(x^3+4x^2)\ (x^3+4x^2)=x^2\ (x+4).$
$x^2\ (x+4)=x^4\ (x+4)\ (x+4).$

14. $144a^2b^4-225c^6=(12ab^2+15c^3)\ (12ab^2-15c^3)=3\ (4ab^2+5c^3).$
$3\ (4ab^2-5c^3)=9\ (4ab^2+5c^3)\ (4ab^2-5c^3).$

15. $4a^4b^2+52a^3b^3+169a^2b^4=(2a^2b+13ab^2)^2=(2a^2b+13ab^2)\ (2a^2b+$
$13ab^2)=ab\ (2a+13b).ab\ (2a+13b)=a^2b^2\ (2a+13b)^2.$

Factors:

1. $(x-y)^2-z^2=(x-y+z)\ (x-y-z).$

Factors:

3. $(3a^2-b^2)^2-(a^2-3b^2)^2=(3a^2-b^2+a^2-3b^2)\ (3a^2-b^2-a^2+3b^2)$
$=(4a^2-4b^2)\ (2a^2+2b^2)=4\ (a^2-b^2)\cdot 2\ (a^2+b^2)=$
$8\ (a+b)\ (a-b)\ (a^2+b^2)$.

5. $m^2-(x-y)^2=(m+x-y)\ (m-x+y)$.

7. $(3a+5)^2-(2a-3)^2=(3a+5+2a-3)\ (3a+5-2a+3)=$
$(5a+2)\ (a+8)$.

9. $(a-b)^2-(c-d)^2=(a-b+c-d)\ (a-b-c+d)$.

10. $(7x+4y)^2-(2x+3y)^2=(7x+4y+2x+3y)\ \ (7x+4y-2x-3y)=$
$(9x+7y)\ (5x+y)$.

Page 51.

Factors:

1. $a^2+2a+1-d^2=(a+1)^2-d^2=(a+1+d)\ (a+1-d)=$
$(a+d+1)\ (a-d+1)$.

5. $c^2+2cd+d^2-1=(c+d)^2-1=(c+d+1)\ (c+d-1)$.

6. $(a^2-2am+m^2)-(b^2+2bn+n^2)=(a-m)^2-(b+n)^2=$
$(a-m+b+n)\ (a-m-b-n)$.

7. $(a^2+2am+m^2)-(b^2-2bn+n^2)=(a+m)^2-(b-n)^2=$
$(a+m+b-n)\ (a+m-b+n)$.

9. $(1-2x+x^2)-(4-4a+a^2)=(1-x)^2-(2-a)^2=(1-x+2-a)$
$(1-x-2+a)=(3-x-a)\ (-1-x+a)$.

10. $(a^2+2ax+x^2)-(4a^2b^4+8ab^4x+4b^4x^2)=(a^2+2ax+x^2)-4b^4$
$(a^2+2ax+x^2)=(a+x)^2-4b^4\ (a+x)^2=(a+x)^2\ (1-4b^4)=$
$(a+x)\ (a+x)\ (1+2b^2)\ (1-2b^2)$.

Page 52.

Factors:

1. $x^3+2x^2y+xy^2-4x=x\ (x^2+2xy+y^2-4)=x\ [(x+y)^2-4]=$
$x\ (x+y+2)\ (x+y-2)$.

3. $4bm-m-4b^2m+4m^5=m\ (4b-1-4b^2+4m^4)=m\ (4m^4-$
$[4b^2-4b+1])=m\ (2m^2+2b-1)\ (2m^2-2b+1)$.

4. $5a^2-10am+5m^2-5b^2-10bn-5n^2=5\ (a^2-2am+m^2-b^2-$
$2bn-n^2)=5\ [(a-m)^2-(n+b)^2]=5\ (a-m+n+b)$
$(a-m-n-b)$.

5. $q^2a^2-q^2b^2+q^2c^2-q^2d^2+2acq^2-2bdq^2=q^2\ (a^2-b^2+c^2-d^2+$
$2ac-2bd)=q^2\ [(a^2+2ac+c^2)-(b^2+2bd+d^2)]=q^2\ [(a+c)^2$
$-(b+d)^2]=q^2\ (a+c+b+d)\ (a+c-b-d)$.

1. $6x^2+3xy-2ax-ay=3x\ (2x+y)-a\ (2x-y)=(2x-y)\ (3x-a)$.

4. $y^3-y^2+y-1=y^2\ (y-1)+(y-1)=(y-1)\ (y^2+1)$.

Factors:

Page 53.

5. $amx^2+bmxy-anxy-bny^2=mx\ (ax+by)-ny\ (ax+by)=$
$(ax+by)\ (mx-ny)$.

Factors:

7. $4x^3-2x^2-2ax+a=2x^2\ (2x-1)-a\ (2x-1)=(2x-1)\ (2x^2-a)$.

9. $x+y-x^2-2xy-y^2=(x+y)-(x^2+2xy+y^2)=(x+y)-(x+y)\ (x+y)=(x+y)\ (1-x-y)$.

10. $c^2-x^2-(-c^2-x^2-2xc)=(c^2-x^2)+(c^2+x^2+2xc)=(c^2-x^2)+(c+x)^2=(c+x)\ (c-x)+(c+x)^2=(c+x)\ (c-x+c+x)=(c+x)\ 2c$.

1. $x^4+x^2+1=x^4+2x^2+1-x^2=(x^2+1)^2-x^2=(x^2+1+x)\ (x^2+1-x)=(x^2+x+1)\ (x^2-x+1)$.

3. $4a^4-13a^2+1=4a^4-4a^2+1-9a^2=(2a^2-1)^2-9a^2=(2a^2-1+3a)\ (2a^2-1-3a)=(2a^2+3a-1)\ (2a^2-3a-1)$.

5. $49b^4-11b^2d^2+25d^4=49b^4+70b^2d^2+25d^4-81b^2d^2=(7b^2+5d^2)^2-81b^2d^2=(7b^2+5d^2+9bd)\ (7b^2+5d^2-9bd)$.

7. $25x^4-36x^2y^2+4y^4=25x^4-20x^2y^2+4y^4-16x^2y^2=(5x^2-2y^2)^2-16x^2y^2=(5x^2-2y^2+4xy)\ (5x^2-2y^2-4xy)$.

9. $225a^4-46a^2b^2+49b^4=225a^4-210a^2b^2+49b^4-256a^2b^2=(15a^2-7b^2)^2-256a^2b^2=(15a^2+7b^2+16ab)(15a^2-7b^2-16ab)$.

10. $256c^4-36c^2+25^4=256c^4-160c^2+25d^4-196c^2d^2=(16c^2-5d^2)^2-196c^2d^2=(16c^2-5d^2+14cd)\ (16c^2-5d^2-14cd)$.

Page 55.

Factors:

1. $x^2+6x+9=(x+3)\ (x+3)$.

3. $x^2+17x+70=(x+10)\ (x+7)$.

6. $x^4-7x^2-18=(x^2-9)\ (x^2+2)=(x+3)\ (x-3)\ (x^2+2)$.

8. $a^6-12a^5+35a^4=(a^3-7a^2)\ (a^3-5a^2)=a^4\ (a-7)\ (a-5)$.

10. $3a^3b+18a^2b+27ab=3ab\ (a^2+6a+9)=3ab\ (a+3)\ (a+3)$.

Page 57.

Factors:

1. $12x^2+24x+12=12\ (x^2+2x+1)=12\ (x+1)\ (x+1)$.

3. $6x^2-x-12=(2x-3)\ (3x+4)$.

5. $15x^2+32x-7=(5x-1)\ (3x+7)$.

7. $-6x^2+7x+20=(5-2x)\ (3x+4)$.

9. $axy+(acx+by)+bc=(ax+b)\ (y+c)$.

10. $a^4b^5+(10a^4-9b^3)-90=a^4b^3-9b^3+10a^4-90=b^3\ (a^4-9)+10\ (a^4-9)=(a^4-9)\ (b^3+10)=(a^2+3)\ (a^2-3)\ (b^3+10)$.

Page 58.

Factors:

1. $m^3+n^3=(m+n)\ (m^2-mn+n^2)$.

2. $a^3+64=a^3+4^3=(a+4)\ (a^2-4a+16)$.

3. $x^3y^3-64=x^3y^3-4^3=(xy-4)\ (x^2y^2+4xy+16)$.

4. $343-x^3=7^3-x^3=(7-x)\ (49+7x+x^2)$.

Factors:

5. $8a^3-b^3=2^3a^3-b^3=(2a-b)\ (4a^2+2ab+b^2)$.

6. $x^6-y^6=(x^3+y^3)\ (x^3-y^3)=(x+y)\ (x-y)\ (x^2-xy+y^2)$ (x^2+xy+y^2).

7. $1-x^8=(1+x^4)\ (1-x^4)=(1+x^4)\ (1+x^2)\ (1-x^2)=(1+x^4)\ (1+x^2)$ $(1+x)\ (1-x)$.

8. $a^5-b^5=(a-b)\ (a^4+a^3b+a^2b^2+ab^3+b^4)$.

9. $x^8-y^8=(x^4+y^4)\ (x^4-y^4)=(x^4+y^4)\ (x^2+y^2)\ (x^2-y^2)=(x^4+y^4)$ $(x^2+y^2)\ (x+y)\ (x-y)$.

10. $(m+n)^3+8=(m+n+2)\ (m+n)^2-2\ (m+n)+4=(m+n+2)$ $(m^2+2mn+n^2-2m-2n+4)$.

Page 59.

Factors:

1. $x^4-y^4+x^2+y^2=(x^4-y^4)+(x^2+y^2)=$
$(x^2+y^2)\ (x^2-y^2)+(x^2+y^2)=(x^2+y^2)\ (x^2-y^2+1)$.

2. $x^3-a^3+x-a=(x-a)\ (x^2+ax+a^2)+(x-a)=$
$(x^2+ax+a^2+1)\ (x-a)$.

3. $8x^3-y^3-2x+y=(8x^3-y^3)-(2x-y)=$
$(2x-y)\ (4x^2+2xy+y^2-1)$.

4. $a^3+8b^3+a^2+5ab+6b^2=(a^3+8b^3)+(a^2+5ab+6b^2)=$
$(a+2b)\ (a^2-2ab+4b^2)+(a+3b)\ (a+2b)=$
$(a+2b)\ (a^2-2ab+4b^2+a+3b)$.

5. $216-a^3+36-a^2=(6^3-a^3)+(6^2-a^2)=$
$(6-a)\ (6^2+6a+a^2+6+a)=(6-a)\ (42+7a+a^2)$.

6. $m^4-n^4+2m^3n-2mn^3=(m^4-n^4)+2mn\ (m^2-n^2)=$
$(m^2-n^2)\ (m^2+n^2+2mn)=(m+n)\ (m-n)\ (m+n)^2$.

7. $a^8-b^8-a^4+b^4=(a^8-b^8)-(a^4-b^4)=$
$(a^4-b^4)\ (a^4+b^4-1)=(a^4+b^4-1)\ (a^2+b^2)\ (a+b)\ (a-b)$.

8. $27a^3-27y^3-(9a^2-18ay+9y^2)=$
$(3a-3y)\ (9a^2+9ay+9y^2)-(9a^2-18ay+9y^2)=$
$(3a-3y)\ (9a^2+9ay+9y^2-3a+3y)=$
$3\ (a-y)\cdot 3(3a^2+3ay+3y^2-a+y)=$
$9\ (a-y)\ (3a^2+3ay+3y^2-a+y)$.

9. $x^3p^2-8y^3p^2-4x^3q^2+32y^3q^2=$
$p^2\ (x^3-8y^3)-4q^2\ (x^3-8y^3)=$
$(x^3-8y^3)\ (p^2-4q^2)=$
$(x-2y)\ (p-2q)\ (x^2+2xy+4y^2)\ (p+2q)$.

10. $a^9-64a^3-a^6+64=(a^9-a^6)-(64a^3-64)=$
$a^6\ (a^3-1)-64\ (a^3-1)=(a^3-1)\ (a^6-64)=(a^3-1)\ (a^6-2^6)=$
$(a-1)\ (a^2+a+1)\ (a^3+2^3)\ (a^3-2^3)=$
$(a-1)\ (a^2+a+1)\ (a+2)\ (a^2-2a+4)\ (a-2)\ (a^2+2a+4)$.

Page 60.

Factors:

1. $4a^2-11ab+17ac-3b^2+bc+4c^2=$
$4a^2+17ac-4c^2-11ab+bc-3b^2=$
$(4a+c)\ (a+4c)-3b^2$, etc. $=$
$(4a+c+b)\ (a+4c-3b)$.

2. $2x^2+ax+cx-a^2+10ac-21c^2=$
$2x^2+ax-a^2+cx+10ac-21c^2=$
$(2x-a)\ (x+a)-21c^2$, etc. $=$
$(2x-a+7c)\ (x+a-3c)$.

3. $3x^2+5xy-7cx-2y^2+7cy-6c^2=$
$3x^2+5xy-2y^2-7cx+7cy-6c^2=$
$(3x-y)\ (x+2y)-6c^2$, etc. $=$
$(3x-y+2c)\ (x+2y-3c)$.

4. $4a^2+10ab+3ac-6b^2-5bc-c^2=$
$4a^2+10ab-6b^2+3ac-5bc-c^2=$
$(4a-2b)\ (a+3b)-c^2$, etc. $=$
$(4a-2b-c)\ (a+3b+c)$.

5. $8x^2+6xy+10xz-2y^2+2z^2=$
$8x^2+6xy-2y^2+10xz+2z^2=$
$(4x-y)\ (2x+2y)+2z^2$, etc. $=$
$(4x-y+z)\ (2x+2y+2z)$.

6. $2a^2-4ab-4ac+2b^2+4bc+2c^2=$
$2a^2-4ab+2b^2-4ac+4bc+2c^2=$
$(2a-2b)\ (a-b)+2c^2$, etc. $=$
$(2a-2b-2c)\ (a-b-c)$.

7. $2a^2+6ax-18a+4x^2-30x+36=$
$2a^2+6ax+4x^2-18a-30x+36=$
$(2a+4x)\ (a+x)+36$, etc. $=$
$(2a+4x-6)\ (a+x-6)$.

Factors:

8. $4a^2-11ab+17ac-3b^2+bc+4c^2=$
$4a^2-11ab-3b^2+17ac+bc+4c^2=$
$(a-3b)\ (4a+b)+4c^2$, etc. =
$(a-3b+4c)\ (4a+b+c)$.

9. $3x^2+5xy-7cx-2y^2+7cy-6c^2=$
$3x^2+5xy-2y^2-7cx+7cy-6c^2=$
$(x+2y)\ (3x-y)-6c^2$, etc. =
$(x+2y-3c)\ (3x-y+2c)$.

Page 61.

Factors:

1. $x^3-3x-2=x^3+1-3x-3=$
$(x^3+1)-(3x+3)=(x+1)\ (x^2-x+1)-3\ (x+1)=$
$(x+1)\ (x^2-x+1-3)=(x+1)\ (x^2-x-2)$.

2. $x^4-3x^2-2x=x\ (x^3-3x-2)=x\ (x^3+1)-(3x+3)=$
$x\ (x+1)\ (x^2-x+1)-3\ (x+1)=x\ (x+1)\ (x^2-x+1-3)=$
$x\ (x+1)\ (x^2-x-2)$.

3. $a^5+3a^2-4=a^3-1+3a^2-3=(a^3-1)+3\ (a^2-1)=$
$(a-1)\ (a^2+a+1)+3\ (a^2-1)=(a-1)\ (a^2+a+1+3a+3)=$
$(a-1)\ (a^2+4a+4)=(a-1)\ (a+2)\ (a+2)$.

4. $x^3-7x-6=x^3+1-7x-7=(x^3+1)-7\ (x+1)=$
$(x+1)\ (x^2-x+1)-7\ (x+1)=(x+1)\ (x^2-x+1-7)=$
$(x+1)\ (x^2-x-6)=(x+1)\ (x+2)\ (x-3)$.

5. $3x^3+7x^2-4=3x^3+7x^2-4x^2-4+4x^2=$
$3x^3+3x^2-4+4x^2=3x^2\ (x+1)-4\ (1-x^2)=$
$3x^2\ (1+x)-4\ (1-x^2)=(1+x)\ (3x^2-4\ (1+x))=$
$(x+1)\ (3x^2-4+4x)=(x+1)\ (x+2)\ (3x-2)$.

6. $x^3-3x^2-10x+24=x^3-2x^2-x^2-10x+24=$
$x^2\ (x-2)-(x^2+10x-24)=x^2\ (x-2)-(x+12)\ (x-2)=$
$(x-2)\ (x^2-x-12)=(x-2)\ (x-4)\ (x+3)$.

7. $x^3-8x^2+17x-10=x^3-2x^2-6x^2+17x-10=$
$x^2\ (-2)-(6x^2-17x+10)=x^2\ (x-2)-(6x-5)\ (x-2)=$
$(x-2)\ (x^2-6x+5)=(x-2)\ (x-1)\ (x-5)$.

8. $6x^5-23x^2+16x-3=6x^3-3x^2-20x^2+16x-3=$
$3x^2\ (2x-1)-(20x^2-16x+3)=$
$3x^2\ (2x-1)-(10x-3)\ (2x-1)=$
$(2x-1)\ (3x^2-10x+3)=$
$(2x-1)\ (3x-1)\ (x-3)$.

Factors:

9. $4x^3-19x+15=4x^3-4x-15x+15=$
$(4x^3-4x)-(15x-15)=4x\,(x^2-1)-15\,(x-1)=$
$(x-1)\,(4x^2+4x-15)=(x-1)\,(2x+5)\,(2x-3).$

10. $2x^3+6x^2+3x^2+4x-15=2x^2\,(x+3)+(3x^2-4x-15)=$
$2x^2\,(x+3)+(3x-5)\,(x+3)=$
$(x+3)\,(2x^2+3x-5)=$
$(x+3)\,(2x+5)\,(x-1).$

Page 62.

Factors:

1. $6a^3b^2c=2.3.a.a.a.b.b.c.$

3. $3a^2-6ab=3a\,(a-2b).$

5. $a^2+2ab+b^2=(a+b)\,(a+b).$

8. $a^3-9a=a\,(a^2-9)=a\,(a+3)\,(a-3).$

9. $a^4-16b^4=(a^2+4b^2)\,(a^2-4b^2)=(a^2+4b^2)\,(a+2b)\,(a-2b).$

13. $a^2-2a-8=(a+2)\,(a-4).$

14. $m^2+2mn+n^2-1=(m+n)^2-1=(m+n+1)\,(m+n-1).$

15. $x^4+x^2y^2+y^4=x^4+2x^2y^2+y^4-x^2y^2=(x^2+y^2)^2-x^2y^2=$
$(x^2+y^2+xy)\,(x^2+y^2-xy).$

16. $x^6-64=(x^3+8)\,(x^3-8)=(x+2)\,(x^2-2x+4)\,(x-2)\,(x^2+2x+4).$

17. $x^3+8y^3z^3=x^3+2^3y^3z^3=(x+2yz)\,(x^2-2xyz+4y^2z^2).$

18. $a^2-b^2+a-b=(a+b)\,(a-b)+(a-b)=(a-b)\,(a+b+1).$

19. $x^4-2x^3-1+2x=(x^4-2x^3+x^2)-(x^2-2x+1)=$
$(x^2-x)^2-(x-1)^2=x^2\,(x-1)^2-(x-1)^2=(x-1)^2\,(x^2-1)=$
$(x-1)\,(x-1)\,(x-1)\,(x+1).$

20. $x^2+13xy-30y^2=(x-2y)\,(x+15y).$

22. $27x^3+64y^3=3^3x^3+4^3y^3=(3x+4y)\,(9x^2-12y^2+16y^2).$

23. $x^4-(x-2)^2=(x^2+x-2)\,(x^2-x+2)=$
$(x+2)\,(x-1)\,(x^2-x+2).$

24. $x^3-9x^2+11x+21=x^3-2x^2-3x-7x^2+14x+21=$
$(x^3-2x^2-3x)-(7x^2-14x-21)=$
$x\,(x^2-2x-3)-7\,(x^2-2x-3)=$
$(x^2-2x-3)\,(x-7)=$
$(x+1)\,(x-3)\,(x-7).$

25. $x^4y-x^2y^3-x^3y^2+xy^4=xy\,(x^3-xy^2-x^2y+y^3)=$
$xy\,[x\,(x^2-y^2)-y\,(x^2-y^2)]=$
$xy\,(x^2-y^2)\,(x-y)=xy\,(x+y)\,(x-y)\,(x-y).$

27. $a^4-7a^2+6=(a^2-6)\,(a^2-1)=(a^2-6)\,(a+1)\,(a-1).$

28. $(x+y)^2-x^2+y^2=(x+y)^2-(x^2-y^2)=$
$(x+y)\,(x+y)-(x+y)\,(x-y)=$
$(x+y)\,(x+y-x+y)=(x+y)\,2y.$

Factors:

30. $ax^2+bx^2-5bx-5ax+6b+6a=$
$x^2\ (a+b)-5x\ (a+b)+6\ (a+b)=$
$(a+b)\ (x^2-5x+6)=(a+b)\ (x-2)\ (x-3).$

31. $5x^4-15x^3-90x^2=5x^2\ (x^2-3x-18)=5x^2\ (x+3)\ (x-6).$

32. $14a^2x^3-35a^3x^2+14a^4x=7a^2x\ (2x^2-5ax+2a^2)=$
$7a^2x\ (2x-a)\ (x-2a).$

34. $9a^6-10a^4b^2+a^2b^4=a^2\ (9a^4-10a^2b^2+b^4)=$
$a^2\ (9a^4-6a^2b^2+b^4)-4a^2b^2=a^2\ (3a^2-b^2)^2-4a^2b^2=$
$a^2\ (3a^2-b^2+2ab)\ (3a^2-b^2-2ab)=$
$a^2\ (3a-b)\ (a+b)\ (3a+b)\ (a-b).$

35. $9x^4+21x^2y^2+25y^4=9x^4+30x^2y^2+25y^4-9x^2y^2=$
$(3x^2+5y^2)^2-9x^2y^2=(3x^2+5y^2+3xy)\ (3x^2+5y^2-3xy).$

36. $(ax+by)^2-(ax-by)^2=(ax+by+ax-by)\ (ax+by-ax+by)=$
$2ax.2by=4abxy.$

37. $a^2+b^2-2ab-c^2=a^2-2ab+b^2-c^2=(a-b)^2-c^2=$
$(a-b+c)\ (a-b-c).$

38. $80x^2y^5-5x^6y=5x^2y\ (16y^4-x^4)=5x^2y\ (4y^2+x^2)\ (4y^2-x^2)=$
$5x^2y\ (4y^2+x^2)\ (2y+x)\ (2y-x).$

39. $x^{16}+x^8+1=x^{16}+2x^8+1-x^8=$
$(x^8+1)^2-x^8=(x^8+1+x^4)\ (x^8+1-x^4)=$
$(x^8+x^4+1)\ (x^8-x^4+1)=(x^8+2x^4+1-x^4)\ (x^8-x^4+1)=$
$[(x^4+1)^2-x^4]\ [x^8-x^4+1]=(x^4+1+x^2)\ (x^4+1-x^2)$
$[x^8-x^4+1]=(x^2+1+x)\ (x^2+1-x)\ (x^4-x^2+1)\ (x^8-x^4+1).$

41. $(a^2-b^2-c^2)^2-4b^2c^2=(a^2-b^2-c^2+2bc)\ (a^2-b^2-c^2-2bc)=$
$[a^2-(b^2-2bc+c^2)]\ [a^2-(b^2+2bc+c^2)]=$
$[a^2-(b-c)^2]\ [a^2-(b+c)^2]=$
$(a+b-c)\ (a-b+c)\ (a+b+c)\ (a-b-c).$

43. $x^3-3x^2+3x-2=x^3-x^2+x-2x^2+2x-2=$
$x\ (x^2-x+1)-2\ (x^2-x+1)=(x^2-x-1)\ (x-2).$

Factors:

Page 63.

44. $a^3+a^2c+abc+b^2c-b^3=(a^3-b^3)+c\ (a^2+ab+b^2)=$
$(a-b)\ (a^2+ab+b^2)+c\ (a^2+ab+b^2)=$
$(a^2+ab+b^2)\ (a-b+c).$

45. $m^7+n^7=(m+n)\ (m^6-m^5n+m^4n^2-m^3n^3+m^2n^4-mn^5+n^6).$

47. $1-x-x^3+x^4=(1-x)-x^3\ (1-x)=(1-x)\ (1-x^3)=$
$(1-x)\ (1-x)\ (1+x+x^2).$

49. $x^2-2bx-a^2+b^2=x^2-2bx+b^2-a^2=(x-b)^2-a^2=$
$(x-b+a)\ (x-b-a).$

50. $a^2-b^2+2bc-c^2=a^2-(b^2-2bc+c^2)=a^2-(b-c)^2=$
$(a+b-c)\ (a-b+c).$

Factors:

52. $a^6-1=(a^3+1)\ (a^3-1)=(a+1)\ (a-1)\ (a^2-a+1)\ (a^2+a+1)$.

53. $a^3+b^3+a+b=(a^3+b^3)+(a+b)=(a+b)\ (a^2-ab+b^2+1)$.

54. $a^2-9b^2+a+3b=(a^2-9b^2)+(a+3b)=(a+3b)\ (a-3b+1)$.

55. $x^5-23x^4+132x^3=x^3\ (x^2-23x+132)=x^3\ (x-12)\ (x-11)$.

56. $x^2+2xy-a^2-2ay=(x^2-a^2)+2y\ (x-a)=(x-a)\ (x+a+2y)$.

57. $a^6-64y^6=(a^3+2^3y^3)\ (a^3-2^3y^3)=$
$(a+2y)\ (a^2-2ay+4y^2)\ (a-2y)\ (a^2+2ay+4y^2)$.

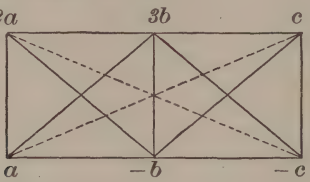
58. $abx^3+x+ab+1=x+1+abx^3+ab=(x+1)+ab\ (x^3+1)=$
$[x+1]\ [1+ab\ (x^2-x+1)]$.

59. $36a^4-21a^2+1=(36a^4-12a^2+1)-9a^2=(6a^2-1)^2-9a^2=$
$(6a^2-1+3a)\ (6a^2-1-3a)$.

60. $x^2+ax+bx-ab-2a^2=x^2+2ax+bx-ax-2a^2-ab=$
$x\ (x+2a+b)-a\ (x+2a+b)=(x+2a+b)\ (x-a)$.

61. $a^2-b^2+a-b=(a^2-b^2)+(a-b)=(a-b)\ (a+b+1)$.

62. $2a^2+ab-3b^2-4bc-ac-c^2=$



$=(2a+3b+c)\ (a-b-c)$.

63. $a^4-a^2-b^4+b^2=(a^4-b^4)-(a^2-b^2)=$
$(a^2-b^2)\ (a^2+b^2-1)=(a+b)\ (a-b)\ (a^2-b^2-1)$.

64. $(c+d)^3+(c-d)^3=c^3+3c^2d+3cd^2+d^3+c^3-3c^2d+3cd^2-d^3=$
$2c^3+6cd^2=2c\ (c^2+3d^2)$.

65. $4(x-y)^3-(x-y)=(x-y)\ [(4\ (x-y)^2-1)]=$
$(x-y)\ [2\ (x-y)+1]\ [2\ (x-y)-1]=$
$(x-y)\ [2x-2y+1]\ [2x-2y-1]$.

66. $x^4y-x^2y^3-x^3y^2+xy^4=xy\ (x^3-xy^2-x^2y+y^3)=$
$xy\ [x\ (x^2-y^2)-y\ (x^2-y^2)]=xy\ (x^2-y^2)\ (x-y)$.

67. $ax^2+bx^2-5bx-5ax+6b+6a=$
$x^2\ (a+b)-5x\ (a+b)+6\ (a+b)=$
$(a+b)\ (x^2-5x+6)=(a+b)\ (x-3)\ (x-2)$.

68. $4(xy-ab)^2-(x^2+y^2-a^2-b^2)^2=$
$[2(xy-ab)+(x^2+y^2-a^2-b^2)]\ [2\ (xy-ab)-(x^2+y^2-a^2-b^2)]=$
$[2xy-2ab+x^2+y^2-a^2-b^2]\ [2xy-2ab-x^2-y^2+a^2+b^2]=$
$[x^2+2xy+y^2-(a^2+2ab+b^2)]\ [a^2-2ab+b^2-(x^2-2xy+y^2)]=$
$[(x+y)^2-(a+b)^2]\ [(a-b)^2-(x-y)^2]=$
$(x+y+a+b)\ (x+y-a-b)\ (a-b+x-y)\ (a-b-x+y)$.

Factors:

69. $a^3+3a^2b+3ab^2+b^3=(a^3+b^3)+3ab\ (a+b)=$
$(a+b)\ (a^2-ab+b^2+3ab)=$
$(a+b)\ (a^2+2ab+b^2)=(a+b)\ (a+b)\ (a+b).$

70. $a^3-3a^2b+3ab^2-b^3=(a-b)\ (a-b)\ (a-b).$

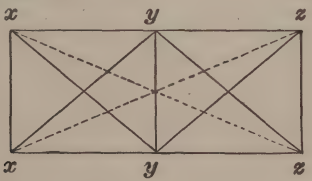
71. $a^3+3a^2+3a+1=(a+1)\ (a+1)\ (a+1).$

72. $a^3-3a^2+3a-1=(a-1)\ (a-1)\ (a-1).$

73. $x^3+6x^2+12x+8=x^3+2x^2+4x^2+12x+8=$
$x^2\ (x+2)+4(x^2+3x+2)=x^2\ (x+2)+4(x+2)\ (x+1)=$
$(x+2)\ (x^2+4(x+1))=(x+2)\ (x^2+4x+4)=$
$(x+2)\ (x+2)\ (x+2).$

74. $a^6-3a^4b^2+3a^2b^4-b^6=(a^6-b^6)-(3a^4b^2-3a^2b^4)=$
$(a^3+b^3)\ (a^3-b^3)-3a^2b^2\ (a^2-b^2)=$
$(a+b)\ (a^2-ab+b^2)\ (a-b)\ (a^2+ab+b^2)-3a^2b^2\ (a^2-b^2)=$
$(a^2-b^2)\ [(a^2-ab+b^2)\ (a^2+ab+b^2)-3a^2b^2]=$
$(a^2-b^2)\ (a^4-2a^2b^2+b^4)=(a^2-b^2)\ (a^2-b^2)\ (a^2-b^2).$

75. $x^2+y^2+z^2+2xy+2xz+2yz=$



$=(x+y+z)\ (x+y+z).$

76. $x^2+y^2+z^2-2xy+2xz-2yz=$
$(x-y+z)\ (x-y+z).$

77. $x^8y^4+7x^4y^2+12=(x^4y^2+4)\ (x^4y^2+3).$

78. $a^3+b^3+a^2-ab+b^2=(a+b)\ (a^2-ab+b^2)+(a^2-ab+b^2)=$
$(a^2-ab+b^2)\ (a+b+1).$

79. $x^3-y^3-3xy\ (x-y)=(x-y)\ (x^2+xy+y^2)-3xy\ (x-y)=$
$(x-y)\ (x^2+xy+y^2-3xy)=$
$(x-y)\ (x^2-2xy+y^2)=(x-y)\ (x-y)\ (x-y).$

80. $4a^4-9a^2+6a-1=4a^4-(9a^2-6a+1)=$
$4a^4-(3a-1)^2=(2a^2+3a-1)\ (2a-3a+1).$

81. $(m-n)^2-12(m-n)+27=(m-n-9)\ (m-n-3).$

82. $(a+b)^4+4a^2\ (a+b)^2+4a^4=(3a^2+2ab+b^2)\ (3a^2+2ab+b^2).$

Page 64.

Factors:

83. $(x^2+4x+8)^2+3x\ (x^2+4x+8)+2x^2=$
$(x^2+4x+8)\ (x^2+4x+8+3x)+2x^2=$
$(x^2+4x+8)\ (x^2+7x+8)+2x^2=$
$x^4+11x^3+46x^2+88x+64=$
$(x^2+6x+8)\ (x^2+5x+8)=$
$(x+2)\ (x+4)\ (x^2+5x+8).$

84. $x^8-1=(x^4+1)\ (x^4-1)=(x^4+1)\ (x^2+1)\ (x^2-1)=$
$(x^4+1)\ (x^2+1)\ (x+1)\ (x-1).$

85. $15x^3-5x^2+33x-11=5x^2\ (3x-1)+11(3x-1)=$
$(3x-1)\ (5x^2+11).$

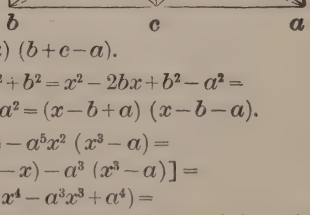
86. $x^3-6x^2+11x-6=(x^3-6x^2+8x)+(3x-6)=$
$x(x^2-6x+8)+3(x-2)=$
$x(x-4)\ (x-2)+3(x-2)=$
$(x-2)\ (x^2-4x+3)=$
$(x-2)\ (x-3)\ (x-1).$

87. $x^8+x^6y^2+x^2y+y^3=x^6\ (x^2+y^2)+y(x^2+y^2)=$
$(x^2+y^2)\ (x^6+y).$

88. $a^2-9+9b^2-6ab=(a^2-6ab+9b^2)-9=$
$(a-3b)^2-9=(a-3b+3)\ (a-3b-3).$

89. $8(x+y)^3-(2x-y)^3=$
$8x^3+24x^2y+24xy^2+8y^3-(8x^3-12x^2y+6xy^2-y^3)=$
$36x^2y+18xy^2+9y^3=9y(4x^2+2xy+y^2).$

90. $(b+c)^2-6a\ (b+c)+5a^2=(b+c)\ (b+c-6a)+5a^2=$
$b^2+2bc-6ab+c^2-6ac+5a^2=$



$=(b+c-5a)\ (b+c-a).$

91. $x^2-2bx-a^2+b^2=x^2-2bx+b^2-a^2=$
$(x-b)^2-a^2=(x-b+a)\ (x-b-a).$

92. $a^2x^5\ (a^3-x)-a^5x^2\ (x^3-a)=$
$a^2x^2\ [x^3\ (a^3-x)-a^3\ (x^3-a)]=$
$a^2x^2\ (a^3x^3-x^4-a^3x^3+a^4)=$
$a^2x^2\ (a^4-x^4)=a^2x^2\ (a^2+x^2)\ (a+x)\ (a-x).$

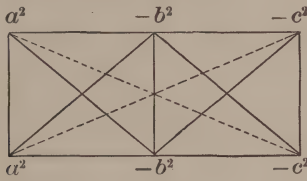
Factors:

93. $(a+b)^2-1-ab\ (a+b+1)=[(a+b)^2-1]-ab\ (a+b+1)=$
$(a+b+1)(a+b-1)-ab(a+b+1)=(a+b+1)\ (a+b-1-ab)=$
$(a+b+1)\ [(a-1)-b\ (a-1)]=(a+b+1)\ (a-1)\ (1-b).$

94. $8x^3-6xy\ (2x+3y)+27y^3=(8x^3+27y^3)-6xy\ (2x+3y)=$
$(2^3x^3+3^3y^3)-6xy\ (2x+3y)=(2x+3y)\ (4x^2-6xy+9y^2-6xy)=$
$(2x+3y)\ (4x^2-12xy+9y^2)=(2x+3y)\ (2x-3y)\ (2x-3y).$

95. $(x^2+4)^2-16x^2=(x^2+4+4x)\ (x^2+4-4x)=$
$(x+2)\ (x+2)\ (x-2)\ (x-2).$

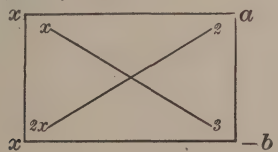
96. $a^4+b^4-c^4-2a^2b^2-2a^2c^2+2b^2c^2=$



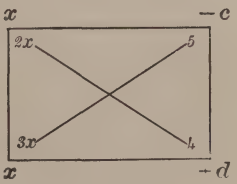
$=(a^2-b^2-c^2)\ (a^2-b^2-c^2).$

97. $x^3+3x^2+3x+1=(x^3+1)+3x\ (x+1)=$
$(x+1)\ (x^2-x+1+3x)=(x+1)\ (x^2+2x+1)=$
$(x+1)\ (x+1)\ (x+1).$

98. $x^2+(a-b)x-ab=$

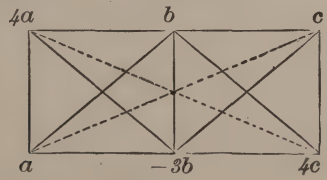


$=(x+a)\ (x-b).$



99. $x^2-(c-d)x-cd=$
$(x-c)\ (x+d).$

100. $4a^2-11ab+17ac-3b^2+bc+4c^2=$



$=(4a+b+c)\ (a-3b+4c).$

Page 65.

1. $12a^3b^4c^4 = 2.2.3aaabbbbcccc.$
$9a^2b^3c = 3.3aabbbc.$
$6a^5b^2c = 2.3aaaaabbc.$
H. C. F. $= 3aabbc = 3a^2b^2c.$

3. $42a^2x^3 = 2.3.7aaxxx.$
$60a^2x^2 = 2.2.3.5aaxx.$
H. C. F. $= 2.3aaxx = 6a^2x^2.$

5. $a^3b^4c^4 = aaabbbcccc.$
$a^2b^3 = aabbb.$
$a^5bc^2 = aaaaabcc.$
H. C. F. $= aab = a^2b.$

7. $60a^2x^2 = 2.2.3.5aaxx.$
$45ax^3 = 3.3.5axxx.$
$90a^9x^5y = 2.3.3.5aaxxxxxy.$
H. C. F. $= 3.5axx = 15ax^2.$

9. $64x^{11} = 2.2.2.2.2.2xxxxxxxxxx.$
$8x^9 = 2.2.2xxxxxxxxx.$
$32x^7 = 2.2.2.2.2xxxxxxx.$
$4x^5 = 2.2xxxxx.$
H. C. F. $= 2.2xxxxx = 4x^5.$

Page 66

1. $5x^5y - 15x^2y^2 = 5x^2y\ (x^3 - 3y).$
$25x^3y^3 - 45xy^5 = 5xy^3\ (5x^2 - 9y^2).$
H. C. F. $= 5xy.$

2. $x^2 - y^2 = (x+y)\ (x-y).$
$x^3 + y^3 = (x+y)\ (x^2 - xy + y^2).$
H. C. F. $= x+y.$

3. $x^2 - 1 = (x+1)\ (x-1).$
$x^3 - 1 = (x-1)\ (x^2 + x + 1).$
H. C. F. $= x-1.$

4. $x^3 + 1 = (x^2 - x + 1)\ (x+1).$
$x^3 + mx^2 + mx + 1 = (x+1)\ (x^2 - x + 1 + mx).$
H. C. F. $= x+1.$

5. $x^2 + x = x(x+1).$
$x^2 - 1 = (x+1)\ (x-1).$
$x^2 - x - 2 = (x+1)\ (x-2).$
H. C. F. $= x+1.$

6. $a^3b - ab^3 = ab\ (a^2 - b^2) = ab(a+b)\ (a-b).$
$a^5b^2 - a^2b^5 = a^2b^2(a^3 - b^3) = a^2b^2(a-b)\ (a^2 + ab + b^2).$
H. C. F. $= ab(a-b).$

7. $a^2 - 5ab + 4b^2 = (a-b)\ (a-4b).$
$a^3 - 5a^2b + 4b^3 = (a-b)\ (a - 4ab - 4b^2).$
H. C. F. $= a-b.$

8. $a^2 + 2a + 1 = (a+1)\ (a+1).$
$a^3 + 2a^2 + 2a + 1 = (a+1)\ (a^2 + a + 1).$
H. C. F. $= a+1.$

9. $x^3-1 = (x-1)(x^2+a+1)$.
$x^2-1 = (x-1)(x-1)$.
$(x-1)^2 = (x+1)(x-1)$.
H. C. F. $= x-1$.

10. $10x^3y+9x^2y^2-9xy^3 = xy(5x-3y)(2x+3y)$.
$4xy^2+15y^3-4x^2y = y(5y-2x)(3y+2x)$.
H. C. F. $= (2x+3y)y$.

11. $x^2-2xy+y^2 = (x-y)(x-y)$.
$(x-y)^3 = (x-y)(x-y)(x-y)$.
H. C. F. $= (x-y)^2$.

12. $x^3+8y^3 = (x+2y)(x^2-2xy+4y^2)$.
$x^2+xy-2y^2 = (x+2y)(x-y)$.
H. C. F. $= x+2y$.

13. $15x^3-19x^2y+6xy^2 = x(5x-3y)(3x-2y)$.
$10x^4-x^3y-3x^2y^2 = x^2(5x-3y)(2x+y)$.
H. C. F. $= x(5x-3y)$.

14. $x^6-y^6 = (x^3+y^3)(x^3-y^3) =$
$(x+y)(x^2-xy+y^2)(x-y)(x^2+xy+y^2)$.
$x^4+xy^3 = x(x^3+y^3) = x(x+y)(x^2-xy+y^2)$.
$x^6+2x^3y^3+y^6 = (x^3+y^3)^2 =$
$(x+y)(x^2-xy+y^2)(x+y)(x^2-xy+y^2)$.
H. C. F. $= (x+y)(x^2-xy+y^2)$.

15. $20x^4+x^2-1 = (5x^2-1)(4x^2+1)$.
$75x^4+15x^3-3x-3 = (5x^2-1)(15x^2+3x+3)$.
H. C. F. $= 5x^2-1$.

Page 67.

1.
$$\begin{array}{r|l} 6x^2-x-12)6x^3-13x^2-6x+18 & (x-2 \\ \underline{6x^3-x^2-12x} & \\ -12x^2+6x+18 & \\ \underline{-12x^2+2x+24} & \\ 2)4x-6 & \\ \underline{2x-3} & \end{array}$$

H. C. F. $= 2x-3)6x^2-x-12(3x+2$

$$\begin{array}{r} 6x^2-9x \\ \hline 2)8x-12 \\ \underline{4x-6} \\ 4x-6 \\ \hline 0. \end{array}$$

2. $x^4-2ax^3+a^4)x^4+\ a^3x-2a^4(1$

$x^4-2ax^3+\ a^4$

$a)2ax^3+\ a^3x-3a^4$

$2x^3\ +\ a^2x-3a^3$

$2x^3+a^2x-3a^3)\ x^4-2ax^3\ +\ a^4(x-2a$

2

$2x^4-4ax^3\ +2a^4$

$2x^4+\ a^2x^2-3a^3x$

$-4ax^3\ -\ a^2x^2+3a^3x+2a^4$

$-4ax^3\ -2a^3x\ +6a^4$

$-a^2)-\ a^2x^2+5a^3x-4a^4$

$x^2-5ax\ +4a^2$

$x^2-5ax+4a^2)2x^3+\ \ a^2x-\ 3a^3\ (2x+10$

$2x^3-10ax^2+\ 8a^2x$

$a)10ax^2-\ 7a^2x-\ 3a^3$

$10x^2\ -\ 7ax\ -\ 3a^2$

$10x^2\ -50ax\ +40a^2$

$43a)43ax\ -43a^2$

$x-\ \ a$

H. C. F. $=x-a)x^2-5ax+4a^2(x-4a$

$x^2-\ ax$

$-4ax+4a^2$

$-4ax+4a^2$

$0.$

3. $a)a^4-4a^2+3a$

$a^3-4a\ +3\ \ )a^3+4a^2-5(1$

$a^3-4a\ +3$

$4)4a^2+4a-8$

$a^2+\ a-2)a^3-4a\ +3\ (a$

$a^3+\ a^2-2a$

$-1)-\ a^2-2a+3$

$a^2+2a-3(1$

$a^2+\ a-2$

H. C. F. $=a-1)a^2+a-2(a+2$

a^2-a

$2a-2$

$2a-2$

$0.$

4. $x^3-2x-1)x^3+2x^2+2x+1$

x^3-2x-1

$2)2x^2+4x+2$

$x^2+2x+1)x^3-2x-1(x-2$

x^3+2x^2+x

$-2x^2-3x-1$

$-2x^2-4x-2$

H. C. F. $= x+1)x^2+2x+1(x+1$

x^2+x

$x+1$

$x+1$

$0.$

Page 68.

5. x^4+2x^2+9

7

$7x^3-11x^2+15x+9)7x^4+14x^2+63\quad(x+11$

$7x^4-11x^3+15x^2+9x$

$11x^3-x^2-9x+63$

7

$77x^3-7x^2-63x+441$

$77x^3-121x^2+165x+99$

$114)114x^2-228x+342$

x^2-2x+3

H. C. F. $= x^2-2x+3)7x^3-11x^2+15x+9(7x+3$

$7x^3-14x^2+21x$

$3x^2-6x+9$

$3x^2-6x+9$

$0.$

6. $3x)18x^4-6x^3-39x^2-18x$

$6x^3-2x^2-13x-6$

$6x^3-2x^2-13x-6)12x^3-x^2-30x-16(2$

$12x^3-4x^2-26x-12$

$3x^2-4x-4)6x^3-2x^2-13x-6(2x+2$

$6x^3-8x^2-8x$

$6x^2-5x-6$

$6x^2-8x-8$

$3x+2$

$2x)24x^4-2x^3-60x^2-32x$

$12x^3-x^2-30x-16$

$3x+2)3x^2-4x-4(x+1$

$3x^2+2x$

$-2)-6x-4$

$3x+2$

$3x+2$

$0.$

H. C. F. $= x(3x+2)$.

7. $x^3+x^2-10x+8)x^3+2x^2-13x+10(1$

$\underline{x^3+\ x^2-10x+\ 8}$

$x^2-\ 3x+\ 2$

$x^2-3x+2)x^3+\ x^2-10x+8(x$

$\underline{x^2-3x^2+\ 2x}$

$4)4x^2-12x+8$

H. C. F. $=\ x^2-\ 3x+2)x^2-3x+2(1$

$\underline{x^2-3x+2}$

$0.$

8. $x^3-5x^2-99x+40)x^3-6x^2-86x+35(1$

$\underline{x^3-5x^2-99x+40}$

$-1)-\ x^2+13x-\ 5$

$x^2-13x+\ 5$

$x^2-13x+5)x^3-\ 5x^2-\ 99x+40(x$

$\underline{x^3-13x^2+\ \ 5x}$

$8)8x^2-104x+40$

H. C. F. $=x^2-\ 13x+\ 5)x^2-13x+5(1$

$\underline{x^2-13x+5}$

$0.$

9. $6x^3-6x^2y+2xy^2-2y^3)12x^2-15xy\ +3\ y^2(2$

x

$\underline{12x^3-15x^2y+3xy^2}$

$\underline{12x^3-12x^2y+4xy^2-4y^3}$

$-y)-\ 3x^2y-\ xy^2+4y^3$

$3x^2\ +\ xy\ -4y^2$

$3x^2+xy-4y^2)6x^3-6x^2y+\ 2xy^2-2y^3(2x$

$\underline{6x^3+2x^2y-\ 8xy^2}$

$-2y)-8x^2y+10xy^2-2y^3$

$4x^2\ -\ 5xy\ +\ y^2)\ 3x^2+\ \ xy-\ 4y^2$

4

$\underline{12x^2+\ 4xy-16y^2(3}$

$\underline{12x^2-15xy+\ 3y^2}$

$19xy-19y^2$

$x-y)4x^2-5xy+y^2(4x$

$\underline{4x^2-4xy}$

$-y)-\ xy+y^2$

H. C. F. $=x-y)x-y(1$

$\underline{x-y}$

$0.$

10. $3x^3+5x^2-x+2)12x^4+27x^3+6x^2-6x-12(4x+7$

$12x^4+20x^3-4x^2+8x$

$7x^3+10x^2-14x-12$

3

$21x^3+30x^2-42x-36$

$21x^3+35x^2-7x+14$

$-5)-5x^2-35x-50$

$x^2+7x+10$

$x^2+7x+10)3x^3+5x^2-x+2(3x+16$

$3x^3+21x^2+30x$

$-1)-16x^2-31x+2$

$16x^2+31x-2$

$16x^2+112x+160$

$-81)-81x-162$

H. C. F. $=x+2)x^2+7x+10(x+5$

$x+2x$

$5x+10$

$5x+10$

$0.$

11. $5x)10x^3+35x^2+30x$

$2x^2+7x+6$

$2x)4x^4+26x^3+58x^2+42x$

$2x^3+13x^2+29x+21$

$2x^2+7x+6)2x^3+13x^2+29x+21(x+3$

$2x^3+7x^2+6x$

$6x^2+23x+21$

$6x^2+21x+18$

$2x+3)2x^2+7x+6(x+2$

$2x^2+3x$

$4x+6$

$4x+6$

$0.$ H. C. F. $=x(2x+3)$.

12. $2)2x^2-14x+20$

$x^2-7x+10)4x^3-25x^2+20x+25(4x+3$

$4x^3-28x^2+40x$

$3x^2-20x+25$

$3x^2-21x+30$

$x-5)x^2-7x+10(x$

x^2-5x

$-2)-2x+10$

H. C. F. $=x-5)x-5(1$

$x-5$

$0.$

13. $x^3-7x^2+11x-5)x^4-10x^3+35x^2-50x+24(x-3$

$$\underline{x^4-7x^3+11x^2-5x}$$
$$\underline{-3x^3+24x^2-45x+24}$$
$$\underline{-3x^3+21x^2-33x+15}$$
$$3)\underline{3x^2-12x+9}$$
$$x^2-4x+3$$

$x^2-4x+3)x^3-7x^2+11x-5(x-3$

$$\underline{x^3-4x^2+3x}$$
$$-3x^2+8x-5$$
$$\underline{-3x^2+12x-9}$$
$$-4)-4x+4$$

H. C. F. $=x-1)x^2-4x+3(x-3$

$$\underline{x^2-x}$$
$$-3x+3$$
$$\underline{-3x+3}$$
$$0.$$

14. $x^3+2x^2+2x+1)x^3-4x^2-4x-5(1$

$$x^3+2x^2+2x+1$$
$$-6)-6x^2-6x-6$$

H. C. F. $=x^2+x+1)x^3+2x^2+2x+1(x+1$

$$\underline{x^3+x^2+x}$$
$$x^2+x+1$$
$$\underline{x^2+x+1}$$
$$0.$$

15. $8x^2+2x-3)24x^3+20x^2-8\ (3x+14$

$$\underline{24x^3+6x^2-9x}$$
$$14x^2+9x-8$$
$$\underline{8}$$
$$112x^2+72x-64$$
$$\underline{112x^2+28x-42}$$
$$22)\underline{44x-22}$$
$$2x-1)8x^2+2x-3(4x$$
$$\underline{8x^2-4x}$$
$$3)6x-3$$

H. C. F. $=2x-1)2x-1(1$

$$\underline{2x-1}$$
$$0.$$

16. $2x^3-3x^2-16x+24)4x^5+2x^4-28x^3-16x^2-32x(x+2$

Dividing by $2x$, . . . $2x^4+x^3-14x^2-8x-16$

$2x^4-3x^3-16x^2+24x$

$4x^3+2x^2-32x-16$

$4x^3-6x^2-32x+48$

$8)8x^2-64$

$x^2-8)2x^3-3x^2-16x+24(2x$

$2x^3-16x$

$-3)-3x^2+24$

H. C. F. $=x^2-8)x^2-8(1$

x^2-8

$0.$

17. $8x^2)64x^{11}-8x^2y6$

$8x^9-y^6$

$4x^6-4x^3y^2+y^4)8x^9-y^6(2x^3+2y^2$

$8x^9-8x^6y^2+2x^3y^4$

$8x^6y^2-2x^3y^4-y^6$

$8x^6y^2-8x^3y^4+2y^6$

$3y^4)6x^3y^4-3y^6$

$2x^3-y^2)4x^6-4x^3y^2+y^4(2x^3-y^2$

$4x^6-2x^3y^2$

$-2x^3y^2+y^4$

$-2x^3y^2+y^4$

H. C. F. $=x^2(2x^3-y^2)$. $0.$

$3x^2)12x^8+3x^2y^4-12x^5y^2$

$4x^6+y^4-4x^3y^2$

18. $3a^4-2a^3b+2a^2b^2-5ab^3-2b^4)6a^4-a^3b+2a^2b^2-2ab^3-b^4(2$

$6a^4-4a^3b+4a^2b^2-10ab^3-4b^4$

$3a^3b-2a^2b^2+8ab^3+3b^4$

$3a^3b-2a^2b^2+8ab^3+3b^4)3a^4b-2a^3b^2+2a^2b^3-5ab^4-2b^5(a+1$

$3a^4b-2a^3b^2+8a^2b^3+3ab^4$

$-2b^3)-6a^2b^3-8ab^4-2b^5$

$3a^2+4ab+b^2$

$3a^2+4ab+b^2)3a^3b-2a^2b^2+8ab^3+3b^4(ab-2b^2$

$3a^3b+4a^2b^2+ab^3$

$-6a^2b^2+7ab^3+3b^4$

$-6a^2b^2-8ab^3-2b^4$

$5b^3)15ab^3+5b^4$

H. C. F. $=3a+b)3a^2+4ab+b^2(a+b$

$3a^2+ab$

$3ab+b^2$

$3ab+b^2$

$0.$

1. $x^3-9x^2+26x-24)x^3-10x^2+31x-30(1$

$x^3-9x^2+26x-24$

$-1)-x^2+5x-6$

$x^2-5x+6)x^3-9x^2+26x-24(x$

x^3-5x^2+6x

$-4)-4x^2+20x-24$

H. C. F. $= x^2-5x+6$

$x^2-5x+6)x^3-11x^2+38x-40(x+3$

x^3-5x^2+6x

$-2)-6x^2+32x-40$

$3x^2-16x+20$

$3x^2-15x-18$

Final H. C. F. $= x-2)x^2-5x+6(x-3$

x^2-2x

$-3x+6$

$-3x+6$

0.

Page 69.

2. $2x^2-5x-42)4x^2+8x-21(2$

$4x^2-10x-84$

$9)18x+63$

$2x+7)2x^2-5x-42(x$

$2x^2+7x$

$-6)-12x-42$

H. C. F. $= 2x+7)2x+7(1$

$2x+7$

0.

Final H. C. F. $= 2x+7)6x^2+23x+7(3x+1$

$6x^2+21x$

$2x+7$

$2x+7$

0.

3. $x^3-x^2-2x)x^3-2x^2+3x-6(1$

x^3-x^2-2x

$-1)-x^2+5x-6$

$x^2-5x+6)x^3-x^2-2x(x$

x^3-5x^2+6x

$4x)4x^2-8x$

H. C. F. $= x-2$

Final H. C. F. $= x-2)2x^3-3x^2-\ x-2(2x^2+x+1$

$2x^3-4x^2$

$x^2-\ x-2$

x^2-2x

$x-2$

$x-2$

0.

4. $x^4-10x^2+9)x^4+10x^3+20x^2-10x-21(1$

$x^4-10x^2+\ 9$

$10)10x^3+30x^2-10x-30$

H. C. F. $= x^3+\ 3x^2-\ x-\ 3)x^4-10x^2+9 \quad (x-3$

$x^4+\ 3x^3-\ x^2-3x$

$-\ 3x^3-9x^2+3x+9$

$-\ 3x^3-9x^2+3x+9$

H. C. F. $= x^3+3x^2-x-3)x^4+4x^3-22x^2-4x+21(x-7$

$x^4+3x^3-\ x^2-3x$

$-7x^3-21x^2+7x+21$

$-7x^3-21x^2+7x+21$

0

$x^3+3x^2-x-\ 3)x^4+4x^3-22x^2-4x+21(x+1$

$x^4+3x^3-\ x^2-3x$

$x^3-21x^2-\ x+21$

$x^3+\ 3x^2-\ x-\ 3$

$-24) \quad -24x^2 \quad +24$

$x^2 \quad -\ 1)x^3+3x^2-x-3(x$

$x^3-\ x$

$3)3x^2-3$

Final H. C. F. $= x^2-1)x^2-1(1$

x^2-1

0.

Page 70.

1. 2 | $6ab,\ 10abc,\ 14ac$

a | $3ab,\ 5bc,\ 7ac$

b | $3b,\ 5bc,\ 7c$

c | $3,\ 5c,\ 7c$

$3,\ 5,\ 7.$ L. C. Dd. $= 2.3.5.7abc = 210abc.$

2. 2 | $8,\ 24,\ 18$

3 | $4,\ 12,\ 9$

4 | $4,\ 4,\ 3$

$1,\ 1,\ 3$

Factors of literal part $= a^4b^2c^3.$

L. C. Dd. $= 2.3.3.4a^4b^2c^3 = 72a^4b^2c^3.$

Page 71.

6. 2 | 7, 4, 6, 21
3 | 7, 2, 3, 21
7 | 7, 2, 1, 7
| 1, 2, 1, 1.

Factors of literal part $= a^2bc^2$.

L. C. Dd. $= 2.2.3.7a^2bc^2 = 84a^2bc^2$.

8. 11 | 66, 44, 24
2 | 6, 4, 24
2 | 3, 2, 12
3 | 3, 1, 6
| 1, 1, 2.

Factors of literal part $= a^4b^4c^4$.

L. C. Dd. $= 2.2.2.3.11a^4b^4c^4 = 264a^4b^4c^4$.

10. 2.2 | 3, 8, 6, 12, 21, 7
3.7 | 3, 2, 3, 3, 21, 7
| 1, 2, 1, 1, 1, 1.

Factors of literal part $= x^3y^2$.

L. C. Dd. $= 2.2.2.3.7x^3y^2 = 168x^3y^2$.

1. $x^2 = x^2$

$x^2 - 3x = x(x-3)$.

L. C. Dd. $= x^2(x-3)$.

2. $x^2-1 = (x+1)(x-1)$

$x^2+x = x(x+1)$.

L. C. Dd. $= (x+1)(x-1)x$.

4. $x^3 - x = x(x^2-1) = x(x+1)(x-1)$

$x^3 - 1 = (x-1)(x^2+x+1)$.

L. C. Dd. $= x(x+1)(x-1)(x^2+x+1)$.

6. $x^3 - 3x + 2 = (x^3-x^2)+(x^2-3x+2) = x^2(x-1)+(x-2)(x-1) =$
$(x-1)(x^2+x-2)$

$x^3 + 2x^2 - x - 2 = (x+2)(x+1)(x-1)$.

L. C. Dd. $= (x-1)(x+1)(x+2)(x^2+x=2)$.

8. $2axy(x-y)^2 = 2axy(x-y)(x-y)$

$3ax^2(x^2-y^2) = 3ax^2(x+y)(x-y)$

$4y^2(x+y)^2 = 4y^2(x+y)(x+y)$.

L. C. Dd. $= 2.2.3ax^2y^2(x+y)^2(x-y)^2 = 12ax^2y^2(x^2-y^2)^2$.

10. $a^2 - ax + x^2 = a^2 - ax + x^2$

$a^2 + ax + x^2 = a^2 + ax + x^2$

$a^3 + x^3 = (a+x)(a^2 - ax + x^2)$.

L. C. Dd. $= (a^2 - ax + x^2)(a^2 + ax + x^2)(a+x)$.

Page 72.

12. $x^2 - y^2 = (x+y)(x-y)$

$x^2 - 3xy + 2y^2 = (x-2y)(x-y)$.

L. C. Dd. $= (x+y)(x-y)(x-2y) = (x^2-y^2)(x-2y)$.

15. $x^2 - y^2 - z^2 + 2yz = x^2 - (y^2 - 2yz + z^2) = x^2 - (y-z)^2 =$
$(x+y-z)(x-y+z)$

$x^2 - y^2 + z^2 + 2xz = (x^2 + 2xz + z^2) - y^2 = (x+z)^2 - y^2 =$
$(x+z+y)(x+z-y)$.

L. C. Dd. $= (x+y+z)(x+y-z)(x-y+z)$.

16. $a^2-x^2-a-x=(a^2-x^2)-(a+x)=(a+x)\ (a-x-1)$

$5a^2b-10abx+5bx^2=5b(a^2-2ax+x^2)=5b(a-x)\ (a-x)$

$a^2b-abx-ab=ab(a-x-1)$.

L. C. Dd. $=5ab\ (a-x-1)\ (a+x)\ (a-x)^2$.

18. $2x^2+xy-6y^2=(2x-3y)\ (x+2y)$

$3x^2-8xy+4y^2=(3x-2y)\ (x-2y)$.

L. C. Dd. $=(3x-2y)\ (2x-3y)\ (x^2-4y^2)$.

Page 73.

1. H. C. F. $=3x-4$.

$3x-4)\overline{6x^2+13x-28,\ 12x^2-31x+20}$.

$2x+7,\quad 4x-5$

L. C. Dd. $=(3x-4)\ (2x+7)\ (4x-5)$.

4. H. C. F. $=4x+1$.

$4x+1)\overline{8x^2+30x+7,\ 12x^2-29x-8}$.

$2x+7,\quad 3x-8$

L. C. Dd. $=(4x+1)\ (2x+7)\ (3x-8)$.

5. H. C. F. $=7x-4$.

$7x-4)\overline{21x^2-26x+8,\ 7x^3-4x^2-21x+12}$.

$3x-2,\qquad x^2-3$

L. C. Dd. $=(7x-4)\ (3x-2)\ (x^2-3)$.

7. H. C. F. $=x^2-5x+6$.

$x^2-5x+6)\overline{x^3-6x^2+11x-6,\ x^3-9x^2+26x-24}$.

$x-1\qquad x-4$

L. C. Dd. $=(x^2-5x+6)\ (x-1)\ (x-4)$.

9. H. C. F. $=x^2+2x-3$.

$x^2+2x-3)\overline{2x^4+x^3-20x^2-7x+24,\ 2x^4+3x^3-13x^2+7x+15}$.

$2x^2-3x-8\qquad 2x^2-x-5$

L. C. Dd. $=(x^2+2x-3)\ (2x^2-3x-8)\ (2x^2-x-5)$.

12. H. C. F. $=x^2-ax+a^2$.

$x^2-ax+a^2)\overline{x^4+ax^3+a^3x+a^4,\ x^4+a^2x^2+a^4}$.

$x^2+2ax+a^2\qquad x^2+ax+a^2$

L. C. Dd. $=(x^2-ax+a^2)\ (x^2+2ax+a^2)\ (x^2+ax+a^2)=$

$(x+a)^2\ (x^2-ax+a^2)\ (x^2+ax+a^2)$.

13. H. C. F. $=x^2-2x+3$.

$x^2-2x+3)\overline{x^3-5x^2+9x-9,\ x^4-4x^2+12x-9}$.

$x-3\qquad x^2+2x-3$

L. C. Dd. $=(x^2-2x+3)\ (x-3)\ (x^2+2x-3)$.

15. H. C. F. $=x^2-2x+4$.

$x^2-2x+4)\overline{x^4-x^3+8x-8,\ x^4+4x^3-8x^2+24x}$.

$x^2+x-2\qquad x^2+6x$

L. C. Dd. $=(x^2-2x+4)\ (x^2+x-2)\ (x^2+6x)$.

Page 74.

1. H. C. F. $= x^2 - 5x + 6$.

$$x^2-5x+6)\underset{x-1}{\underset{(1)}{x^3-6x^2+11x-6}},\quad \underset{x-4}{\underset{(2)}{x^3-9x^2+26x-24}}.$$

First
L. C. Dd. $= (x^2-5x+6)\,(x-1)\,(x-4) = (x-2)\,(x-3)\,(x-1)\,(x-4)$.
(3) $= (x-1)\,(x-3)\,(x-4)$.
Second
L. C. Dd. $= (x-1)\,(x-2)\,(x-3)\,(x-4)$.

2. H. C. F. $= 3a+1$.

$$3a+1)\underset{a-1}{\underset{(1)}{3a^2-2a-1}},\quad \underset{2a-1}{\underset{(2)}{6a^2-a-1}}.$$

First
L. C. Dd. $= (3a+1)\,(a-1)\,(2a-1)$.
(3) $= (3a-2)\,(3a+1)$.
Second
L. C. Dd. $= (3a+1)\,(3a-2)\,(a-1)\,(2a-1)$.

3. H. C. F. $= 7x^2 - 5$.

$$7x^2-5)\underset{3x+4}{\underset{(1)}{21x^3+28x^2-15x-20}},\quad \underset{3x-4}{\underset{(2)}{21x^3-28x^2-15x+20}}.$$

First
L. C. Dd. $= (7x^2-5)\,(3x+4)\,(3x-4)$.
(3) $= (2x-3)\,(7x^2-5)$.
Second
L. C. Dd. $= (7x^2-5)\,(2x-3)\,(3x+4)\,(3x-4)$.

5. H. C. F. $= x-3$.

$$x-3)\underset{x^2-2x+3}{\underset{(1)}{x^3-5x^2+9x-9}},\quad \underset{x^2+2x-3}{\underset{(2)}{x^3-x^2-9x+9}}.$$

First
L. C. Dd. $= (x-3)\,(x^2-2x+3)\,(x^2+2x-3)$.
(3) $= (x-3)\,(x^3+3x^2+5x+3)$.
Second
L. C. Dd. $= (x-3)\,(x^2-2x+3)\,(x^2+2x-3)\,(x^3+3x^2+5x+3)$.

Page 76.

3. $+\frac{-m}{-n}=+d$; $+\frac{+m}{-n}=-d$; $-\frac{-m}{+n}=+d$;

$+\frac{+m}{-n}=-d$; $-\frac{-m}{-n}=-d$; $+\frac{+m}{+n}=+d$.

7. $\frac{-a}{a-b}+\frac{b}{a-b}+\frac{c}{a-b}$; $\frac{1}{a(a-b)(a-c)}+\frac{1}{a(a-b)(a-c)}$.

9. $\frac{y-x}{(b-a)(x-y)(a-b)}=\frac{x-y}{(a-b)(x-y)(a-b)}=\frac{1}{(a-b)^2}$

10. $\frac{a-b}{(a+b)(c-b)(b-a)}=\frac{a-b}{(a+b)(b-c)(a-b)}$.

Page 78.

1. $\frac{58ab^2c^2}{87a^4b^3c^2}=\frac{2.29ab^2c^2}{3.29a^4b^3c^2}=\frac{2}{3a^3b}$. **2.** $\frac{12x^5y^2z^3}{30xy^3z^3}=\frac{2.2.3x^5y^2z^3}{2.3.5xy^3z^3}=\frac{2x^4}{5y}$.

4. $\frac{2a^2cd+2abcd}{6a^2xy+6abxy}=\frac{acd+bcd}{3axy+3bxy}=\frac{cd(a+b)}{3xy(a+b)}=\frac{cd}{3xy}$.

6. $\frac{20(x^3-y^3)}{5x^2+5xy+5y^2}=\frac{20(x-y)(x^2+xy+y^2)}{5(x^2+xy+y^2)}=\frac{4(x-y)}{1}$.

7. $\frac{(xy-3y^2)^2}{x^3y^2-27y^5}=\frac{y^2(x^2-6xy+9y^2)}{y^2(x^3-27y^3)}=\frac{(x-3y)(x-3y)}{(x-3y)(x^2+3xy+9y^2)}=$

$\frac{x-3y}{x^2+3xy+9y^2}$.

9. $\frac{x^2-ax}{a^2-x^2}=\frac{x(x-a)}{(a+x)(a-x)}=\frac{-x(a-x)}{(a+x)(a-x)}=\frac{-x}{a+x}=-\frac{x}{a+x}$.

1. $\frac{x^4-9x^2}{x^4-x^3-6x^2}=\frac{(x^2+3x)(x^2-3x)}{x^2(x+2)(x-3)}=\frac{x^2(x+3)(x-3)}{x^2(x+2)(x-3)}=\frac{x+3}{x+2}$.

3. $\frac{a^4-14a^2-51}{a^4-2a^2-15}=\frac{(a^2+3)(a^2-17)}{(a^2+3)(a^2-5)}=\frac{a^2-17}{a^2-5}$.

Page 79.

5. $\frac{2x^2+17x+21}{3x^2+26x+35}=\frac{(2x+3)(x+7)}{(3x+5)(x+7)}=\frac{2x+3}{3x+5}$.

7. $\frac{a^2-(b-c)^2}{(a+c)^2-b^2}=\frac{(a+b-c)(a-b+c)}{(a+c+b)(a+c-b)}=\frac{a+b-c}{a+b+c}$.

10. $\frac{x^{3m}+x^{2m}-2}{x^{2m}+x^m-2}=\frac{(x^m-1)(x^{2m}+2x^m+2)}{(x^m-1)(x^m+2)}=\frac{x^{2m}+2x^m+2}{x^m+2}$.

1. $\frac{x^2-3x+4}{x}=x-3+\frac{4}{x}$. **4.** $\frac{x^3+x^2-1}{x^2}=x+1-\frac{1}{x^2}$.

Page 80.

5. $\dfrac{x^2+6x+12y-5y^2}{x-2y+6} = x+2y-\dfrac{y^2}{x-2y+6}$.

7. $\dfrac{4ax-2x^2-a^2}{2a-x} = 2x-\dfrac{a^2}{2a-x}$.

9. $\dfrac{x^5-x^3-2x^2-2x-1}{x^2-x-1} = x^3+x^2+x-\dfrac{x+1}{x^2-x-1}$.

10. $\dfrac{x^4+1}{x-1} = x^3+x^2+x+1+\dfrac{2}{x-1}$.

Page 81.

1. $y+\dfrac{a^2-y^2-c}{y} = \dfrac{y^2+a^2-y^2-c}{y} = \dfrac{a^2-c}{y}$.

3. $x+1+\dfrac{1}{x-1} = \dfrac{x^2-1+1}{x-1} = \dfrac{x^2}{x-1}$.

5. $\dfrac{3x-2}{x+1}-x-2 = \dfrac{3x-2-(x+2)\,(x+1)}{x+1} = \dfrac{3x-2-x^2-3x-2}{x+1} =$

$\dfrac{-4-x^2}{x+1} = -\dfrac{4+x^2}{x+1}$.

7. $x-1-\dfrac{x-2}{x^2+2x-1} = \dfrac{x^3+x^2-3x+1-(x-2)}{x^2+2x-1} =$

$\dfrac{x^3+x^2-3x+1-x+2}{x^2+2x-1} = \dfrac{x^3+x^2-4x+3}{x^2+2x-1}$.

8. $a+b-\dfrac{a^2+b^2}{a-b} = \dfrac{a^2-b^2-(a^2+b^2)}{a-b} = \dfrac{a^2-b^2-a^2-b^2}{a-b} = \dfrac{-2b^2}{a-b}$.

10. $\dfrac{x^4}{x^2+x-1}-x^2+x-1 = \dfrac{x^4-(x^2-x+1)\,(x^2+x-1)}{x^2+x-1} =$

$\dfrac{x^4-x^4+x^2-2x+1}{x^2+x-1} = \dfrac{x^2-2x+1}{x^2+x-1}$.

Page 83.

1. $\dfrac{n\,.\,(n-3)}{1\,.\,(n-3)} = \dfrac{n\,(n-3)}{n-3} = \dfrac{n^2-3n}{n-3}$; $\dfrac{1+3n}{n-3} = \dfrac{1+3n}{n-3}$.

3. L. C. Dd. $= a^3x^3$.

$a^3x^3 \div a^3x^2$, ax^3, $a^2x = x$, a^2, ax^2.

$\dfrac{2}{a^3x^2}\,.\,x = \dfrac{2x}{a^3x^3}$; $\dfrac{3}{ax^3}\,.\,a^2 = \dfrac{3a^2}{a^3x^3}$; $\dfrac{4}{a^2x}\,.\,ax^2 = \dfrac{4ax^2}{a^3x^3}$.

5. L. C. Dd. $= a^2-x^2$.

$(a^2-x^2) \div (a-x) = a+x$.

$\dfrac{1}{(a-x)\,.\,(a+x)} = \dfrac{a+x}{a^2-x^2}$; $\dfrac{1}{a^2-x^2} = \dfrac{1}{a^2-x^2}$.

6. L. C. Dd. $= (a+1)\,(a-1)^2$.

L. C. Dd. $\div (a-1)$, (a^2-1), $(a^2-2a+1) = a^2-1$, $a-1$, $a+1$.

$\frac{x}{(a-1)} \cdot (a^2-1) = \frac{a^2x-x}{(a+1)\,(a-1)^2}$; $\frac{y}{(a^2-1)} \cdot (a-1) = \frac{ay-y}{(a^2-1)\cdot(a-1) = (a+1)\,(a-1)^2}$.

$\frac{z}{(a^2-2a+1)\cdot(a+1) = (a+1)\,(a-1)^2} \cdot (a+1) = \frac{az+z}{}$.

7. L. C. Dd. $= x^3y^3\,(x^2-y^2)$.

L. C. Dd. $\div x^3y(x+y)$, $xy^3(x-y)$, $x^2y^2(x^2-y^2) = y^2(x-y)$, $x^2(x+y)$, xy.

Multiplying severally by these three, we obtain

$$\frac{ay^2(x-y),\ bx^2(x+y),\ cxy}{x^3y^3(x^2-y^2)}.$$

8. L. C. Dd. $= (a+3)\,(a-5)\,(a-2)$.

L. C. Dd. $\div (a^2-2a-15)$, (a^2+a-6), $(a^2-7a+10) =$ $a-2$, $a-5$, $a+3$.

Multiplying severally by these three, we obtain

$$\frac{(a-1)\,(a-2),\ (a+2)\,(a-5),\ (a+3)\,(a+3)}{(a+3)\,(a-5)\,(a-2)}.$$

9. L. C. Dd. $= (a-b)\,(b-c)$.

L. C. Dd. $\div (a-b)\,(b-c)$, $(a-b)\,(b-c) = a,\ -b$.

Multiplying severally by these two, we obtain

$$\frac{a}{(a-b)\,(b-c)},\quad \frac{-b}{(a-b)\,(b-c)}.$$

10. L. C. Dd. $= (x-1)\,(x^2+x+1)$.

L. C. Dd. $\div x-1$, x^2+x+1, $x^3-1 = x^2+x+1$, $x-1$, x^3.

Multiplying severally by these three, we obtain

$$\frac{(x+1)\,(x^2+x+1),\ (x^2+1)\,(x-1),\ x^3}{(x-1)\,(x^2+x+1)}.$$

Page 84.

1. $\frac{2x}{5} + \frac{x}{3} - \frac{2x}{3} = \frac{6x+5x-10x}{15} = \frac{x}{15}$.

3. $\frac{7x-5}{8} - \frac{3x+2}{3} + \frac{x+1}{4} = \frac{21x-15-(24x+16)+6x+6}{24} = \frac{3x-25}{24}$.

5. $\frac{1-a}{1+a} + \frac{1}{1-a} - \frac{a^2}{1-a^2} = \frac{1-2a+a^2+1+a-a^2}{1-a^2} = \frac{2-a}{1-a^2}$.

8. $\frac{x-1}{x-2} - \frac{x-3}{x-1} = \frac{x^2-2x+1-(x^2-5x+6)}{x^2-3x+2} = \frac{3x-5}{x^2-3x+2}$.

9. $\frac{a}{a-1} - a + \frac{1}{a^2-a} + \frac{1}{a} = \frac{a^2-a(a^2-a)+1+a-1}{a^2-a} =$

$\frac{a^2-a^3+a^2+1+a-1}{a^2-a} = \frac{2a^2-a^3+a}{a^2-a} = \frac{a(2a-a^2+1)}{a(a-1)} =$

$\frac{2a-a^2+1}{a-1}.$

10. $\frac{bc}{(a-b)(a-c)} + \frac{-bc}{(a-b)(b-c)} + \frac{ab}{(a-c)(b-c)} =$

$\frac{bc(b-c)-bc(a-c)+ab(a-b)}{(a-b)(a-c)(b-c)} =$

$\frac{b^2c-bc^2-abc+bc^2+a^2b-ab^2}{\text{L. C. D.}} = \frac{b^2c-abc+a^2b-ab^2}{\text{L. C. D.}} =$

$\frac{(b^2-ab)(c-a)}{\text{L. C. D.}} = \frac{b(b-a)(c-a)}{(a-b)(a-c)(b-c)} = \frac{b}{b-c}.$

Page 85.

1. $\frac{18a^2c^2-3acf}{27ac^2-6ac^3} = \frac{3ac(6ac-f)}{3ac(9c-2c^2)} = \frac{6ac-f}{9c-2c^2}.$

3. $\frac{x^2+x-2}{2x^2-3x+1} = \frac{(x+2)(x-1)}{(x-1)(2x-1)} = \frac{x+2}{2x-1}.$

5. $\frac{x^{3m}+x^{2m}-2}{x^{2m}+x^m-2} - \frac{(x^{2m}+2x^m+2)(x^m-1)}{(x^m+2)(x^m-1)} = \frac{x^{2m}+2x^m+2}{x^m+2}.$

6. $\frac{x^6+a^2x^3y}{x^6-a^4y^2} = \frac{x^3(x^3+a^2y)}{(x^3+a^2y)(x^3-a^2y)} = \frac{x^3}{x^3-a^2y}.$

8. $\frac{x^3-8x-3}{x^4-7x^2+1} = \frac{(x-3)(x^2+3x+1)}{(x^2-3x+1)(x^2+3x+1)} = \frac{x-3}{x^2-3x+1}.$

9. $\frac{x^4-1}{x^6-1} = \frac{(x^2+1)(x^2-1)}{(x^3+1)(x^3-1)} = \frac{(x^2+1)(x+1)(x-1)}{(x+1)(x-1)(x^2-x+1)(x^2+x+1)}$

$\frac{x^2+1}{x^4+x^2+1}.$

10. $\frac{3x^4-14x^3-9x+2}{2x^4-9x^3-14x+3} = \frac{(3x^2+x+2)(x^2-5x+1)}{(2x^2+x+3)(x^2-5x+1)} = \frac{3x^2+x+2}{2x^2+x+3}.$

Page 86.

7. 1. $x - \frac{a^2-x^2}{x} = \frac{x^2-a^2+x^2}{x} = \frac{2x^2-a^2}{x}.$

3. $1 - \frac{x-a-1}{a} = \frac{a-x+a+1}{a} = \frac{2a-x+1}{a}.$

5. $3x-1 - \frac{x+a}{3a-2} = \frac{9ax-6x-3a+2-x-a}{3a-2} = \frac{9ax-7x-4a+2}{3a-2}.$

7. $3+\frac{3}{x^2-1}=\frac{3x^2-3+3}{x^2-1}=\frac{3x^2}{x^2-1}$.

8. $a^2-ax+x^2+\frac{2x^3}{a+x}=\frac{a^3+x^3+2x^3}{a+x}=\frac{a^3+3x^3}{a+x}$.

9. $x+1-\frac{1-x^2}{x-1}=\frac{x^2-1-1+x^2}{x-1}=\frac{2x^2-2}{x-1}=\frac{2(x^2-1)}{x-1}=2(x+1)$.

10. $x+1+\frac{1-x^2}{x-1}=\frac{x^2-1+1-x^2}{x-1}=\frac{0}{x-1}=0$.

8. 2. $1-\frac{a}{a+b}+\frac{a}{a-b}=\frac{a^2-b^2-a^2+ab+a^2+ab}{a^2-b^2}=\frac{a^2+2ab-b^2}{a^2-b^2}$.

3. $a-y-\frac{a^2-y^2}{a+y}=\frac{a^2-y^2-a^2+y^2}{a+y}=\frac{0}{a+y}=0$.

Page 87.

5. $\frac{a}{b^2}+\frac{a^2+b}{ab^2}+\frac{1}{ab}=\frac{a^2+a^2+b+b}{ab^2}=\frac{2a^2+2b}{ab^2}=\frac{2(a^2+b)}{ab^2}$.

6. $\frac{a^4x^3-a^3x^5}{x^4-ax^2}+a^3x-a^4=\frac{a^4x^3-a^3x^5+a^3x^5-a^4x^4-a^4x^3+a^5x^2}{x^4-ax^2}=$
$\frac{a^5-a^4x^2}{x^2-a}=\frac{a^4(a-x^2)}{x^2-a}=-a^4$.

7. $\frac{1}{(x+y)^2}-\frac{1}{(x-y)^2}=\frac{x^2-2xy+y^2-x^2-2xy-y^2}{(x^2-y^2)^2}=\frac{-4xy}{(x^2-y^2)^2}$.

8. $1+\frac{a-b}{a+b}-\frac{a^2-b^2}{a^2-b^2}=1+\frac{a-b}{a+b}-1=\frac{a-b}{a+b}$.

9. $\frac{a^2}{b^3-a^3}+\frac{b^3a^2}{a^6-b^6}=\frac{-a^2}{a^3-b^3}+\frac{b^3a^2}{a^6-b^6}=\frac{-a^2(a^3+b^3)+b^3a^2}{a^6-b^6}=$
$\frac{-a^5-a^2b^3+a^2b^3}{a^6-b^6}=\frac{-a^5}{a^6-b^6}$.

10. $\frac{x^2-yz}{(x+y)(x+z)}+\frac{y^2-xz}{(y+z)(x+y)}+\frac{z^2-xy}{(y+z)(x+z)}=$
$\frac{(y+z)(x^2-yz)+(x+z)(y^2-xz)+(x+y)(z^2-xy)}{(x+y)(x+z)(y+z)}=$
$\frac{x^2y-y^2z+x^2z-yz^2+xy^2-x^2z+y^2z-xz^2+xz^2-x^2y+z^2y-xy^2}{(x+y)(x+z)(y+z)}=0$.

Page 89.

2. $\left(b+\frac{bx}{a}\right)\times\frac{a}{x}=\frac{ab+bx}{a}\times\frac{a}{x}=\frac{ab+bx}{x}=b+\frac{ab}{x}$.

3. $\frac{x^2-b^2}{bc}\times\frac{x^2+b^2}{b+c}=\frac{x^4-b^4}{b^2c+bc^2}=\frac{x^4-b^4}{bc(b+c)}$.

4. $\frac{x-b}{8cd}\times\frac{4d}{3cx}=\frac{x-b}{6c^2x}$.

5. $\frac{x^4-b^4}{x^2-2bx+b^2} \times \frac{x^2-b^2}{-a} = \frac{(x^2+b^2)\,(x^2-b^2)\,(x^2-b^2)}{-a\,(x-b)\,(x-b)} =$
$\frac{(x^2+b^2)\,(x+b)\,(x+b)}{-a}$.

6. $\frac{x^2-1}{x} \div \frac{x+1}{x} = \frac{x^2-1}{x} \times \frac{x}{x+1} = x-1$.

7. $\frac{1+a}{a} \times \frac{a^3}{1+a} = \frac{a^3}{a} = a^2$.

Page 90.

8. $\left(\frac{a}{b} - \frac{b}{c} + \frac{c}{d}\right) \times \frac{b^2c^2d^2}{a-b+c} = \frac{adc-b^2d+bc^2}{bcd} \times \frac{b^2c^2d^2}{a-b+c} =$
$\frac{(acd-b^2d+bc^2)\,bcd}{(a-b+c)}$.

9. $\frac{a^3-x^3}{x^3} \times \frac{a-x}{x^2+ax+a^2} = \frac{(a-x)^2}{x^3}$.

10. $\frac{(a+1)^2}{2x} \times \frac{x^2}{a+1} = \frac{(a+1)\,x}{2}$.

14. $\frac{3\,(a-b)^2}{4\,(a+b)^2} \times \frac{7\,(a^2-b^2)}{9\,(a-b)^3} \times \frac{8\,(a+b)}{14ab} = \frac{(a+b).}{(a+b).3.ab} = \frac{1}{3ab}$.

15. $\frac{x^4-y^4}{x^4} \times \frac{xy}{x^2+y^2} = \frac{(x^2-y^2)\,y}{x^3}$.

16. $\frac{4x^2-y^2}{y^2} \cdot \frac{2x-2x+y}{2x-y} = \frac{(2x+y)\,y}{y^2} = \frac{2x+y}{y} = 1+\frac{2x}{y}$.

17. $\frac{a^3-a^2y+ay^2}{x-3} \times \frac{x\,(a+y)-3\,(a+y)}{a^3+y^3} =$
$\frac{a\,(a^2-ay+y^2)}{x-3} \times \frac{(a+y)\,(x-3)}{a^3+y^3} = \frac{a\,(a+y)\,(a^2-ay+y^2)}{a^3-y^3} = a$.

18. $\left(\frac{m^2-mn+mn}{m-n}\right)\left(\frac{mn+n^2-mn}{m+n}\right) = \frac{m^2}{m-n} \times \frac{n^2}{m+n} = \frac{m^2n^2}{m^2-n^2}$.

19. $\frac{(x+2)\,(x+1)}{(x+4)\,(x+5)} \times \frac{(x+4)\,(x+3)}{(x+2)\,(x+3)} = \frac{x+1}{x+5}$.

20. $\frac{(a^3+x^3)\,(a^3-x^3)}{a^2-2ax+x^2} \times \frac{a-x}{(a^2+ax+x^2)\,(a^2-ax+x^2)} = 1$.

Page 92.

1. $\left(a+\frac{m}{n}\right) \div \left(b-\frac{c}{d}\right) = \frac{an+m}{n} \times \frac{d}{bd-c} = \frac{adn+md}{bdn-nc} = \frac{d\,(an+m)}{n\,(bd-c)}$.

2. $\left(\frac{a}{4}+c\right) \div x+\frac{z}{2} = \left(\frac{a+4c}{4}\right)\left(\frac{2}{2x+z}\right) = \frac{a+4c}{4x+2z}$.

3. $\left(1-\frac{3}{a}\right) \div \left(1-\frac{1}{x}+\frac{1}{x^2}\right) = \frac{a-3}{a} \times \frac{x^2}{x^2-x+1} = \frac{x^2\,(a-3)}{a\,(x^2-x+1)}$.

5. $\dfrac{a}{b+\dfrac{c}{d+\dfrac{e}{f}}} = \dfrac{a}{b+\dfrac{c}{\dfrac{fd+e}{f}}} = \dfrac{a}{b+\dfrac{fc}{fd+e}} = \dfrac{a}{\dfrac{bfd+be+fc}{fd+e}} = \dfrac{afd+ae}{bfd+be+fc}.$

6. $\dfrac{1-a-1-a}{1-a^2} \div \dfrac{1+a+1-a}{1-a^2} = \dfrac{-2a}{1-a^2} \times \dfrac{1-a^2}{2} = -a.$

7. $\dfrac{x^3+y^3}{x^2-y^2} \times \dfrac{x-y}{x^2-xy+y^2} = \dfrac{x+y}{x+y} = 1.$

8. $\left(\dfrac{1}{a}+\dfrac{1}{b}+\dfrac{1}{c}\right) \div \left(\dfrac{a}{b}+\dfrac{b}{c}+\dfrac{a}{c}\right) = \dfrac{bc+ac+ab}{abc} \times \dfrac{bc}{ac+b^2+ab} = \dfrac{bc+ac+ab}{a(ac+b^2+ab)}.$

9. $\dfrac{a^2-ab+ab+b^2}{a^2-b^2} \times \dfrac{a^2-b^2}{a^2+ab-ab+b^2} = 1.$

Page 93.

10. $\dfrac{(1+x^2)^2-(1-x^2)^2}{1-x^4} \times \dfrac{1-x^2}{1-x-1-x} = \dfrac{4x^2}{1-x^4} \times \dfrac{1-x^2}{-2x} = \dfrac{-2x}{1+x^2}.$

2. $\dfrac{a^3-x^3}{a^2-x^2} = \dfrac{(a-x)(a^2+ax+x^2)}{(a+x)(a-x)} = \dfrac{a^2+ax+x^2}{a+x}.$

4. $\dfrac{1-a}{1-a} = \dfrac{1}{1} = 1.$

6. $\dfrac{a^4-m^4}{a^3-a^2m-am^2+m^3} = \dfrac{(a^2+m^2)(a^2-m^2)}{a^2(a-m)-m^2(a-m)} = \dfrac{(a^2+m^2)(a^2-m^2)}{(a-m)(a^2-m^2)} = \dfrac{a^2+m^2}{a-m}.$

1. $\dfrac{a^4-b^4}{a^2-2ab+b^2} = a^2+2ab+3b^2+\dfrac{4b^3(a-b)}{a^2-2ab+b^2} = \dfrac{4b^3}{a-b}+a^2+2ab+3b^2.$

4. $\dfrac{a^3-x^3+1}{a^2+ax-1} = a-x+\dfrac{ax^2-x+a-x^3+1}{a^2+ax-1}.$

5. $\dfrac{x^3+ax^2-3a^2x-3a^3}{x-2a} = x^2+3ax+3a^2+\dfrac{3a^3}{x-2a}.$

3. 1. $a+b-\dfrac{a^3-b^3}{a^2-ab+b^2} = \dfrac{a^3+b^3-a^3+b^3}{a^2-ab+b^2} = \dfrac{2b^3}{a^2-ab+b^2}.$

2. $\dfrac{a^2+b^2+c^2}{2bc}-1 = \dfrac{a^2+b^2+c^2-2bc}{2bc} = \dfrac{a^2+(b-c)^2}{2bc}.$

4. $(a-1)^4-\dfrac{(a-1)^4}{a} = \dfrac{a(a-1)^4-(a-1)^4}{a} = \dfrac{(a-1)^4(a-1)}{a}.$

6. $x-1-\dfrac{x-1}{x^2+x+1} = \dfrac{x^3-1-x+1}{x^2+x+1} = \dfrac{x^3-x}{x^2+x+1} = \dfrac{x(x^2-1)}{x^2+x+1}.$

7. $x^2-\left[1-\left(1-\frac{2}{x+1}\right)\right]=x^2-\left[1-1+\frac{2}{x+1}\right]=x^2-1+1-\frac{2}{x+1}=$
$x^2-\frac{2}{x+1}=\frac{x^3+x^2-2}{x+1}$.

4. 1. L. C. D. $=ab\,(x^2-y^2)$.
L. C. D. $\div a\,(x-y)$, $b\,(x-y)$, $x^2-y^2=b\,(x+y)$, $a\,(x+y)$, ab.
Multiplying severally by these, we have
$$\frac{b\,(x+y),\ a\,(x+y),\ a^2b^2}{ab\,(x^2-y^2)}.$$

Page 94.

2. $-\frac{1}{a-c}=\frac{-1}{a-c}$.
$-\frac{1}{c-a}=\frac{-1}{c-a}=\frac{1}{a-c}$.

3. $\frac{a}{x-y}=\frac{a}{x-y}$.
$\frac{-a}{y-x}=\frac{a}{x-y}$.

4. $\frac{1}{1-x+x^2}=\frac{1}{1-x+x^2}$.
$\frac{-1}{x-1-x^2}=\frac{1}{1-x+x^2}$.

5. $-\frac{1-a}{a^3-1}=\frac{a-1}{a^3-1}$.
$\frac{-(1-a)}{1-a^3}=\frac{a-1}{1-a^3}=\frac{1-a}{a^3-1}$.

6. L. C. D. $=x\,(a-x)$.
L. C. D. $\div 1$, x, $a-x=x\,(a-x)$, $(a-x)$, x.
Multiplying severally by these, we obtain
$$\frac{(1-a)\,(ax-x^2),\ (a-b)\,(a-x),\ bx}{x\,(a-x)}.$$

7. L. C. D. $=x\,(x-2)\,(x+1)\,(x+2)$.
L. C. D. $\div x\,(x-2)$, $x\,(x+2)$, $x^2-x-2=$
$(x+1)\,(x+2)$, $(x-2)\,(x+1)$, $x\,(x+2)$.
Hence, we have
$$\frac{(x+1)^2\,(x+2),\ (3x+2)\,(x-2)\,(x+1),\ (2x-1)\,(x+2)\,x}{x\,(x^2-4)\,(x+1)}.$$

5. 1. $\frac{a-b}{c-d}\times\frac{a+b}{c+d}\times\frac{c^2-d^2}{a^2-b^2}=1$.

2. $-\frac{a}{b}\div\frac{a}{b}=\frac{-a}{b}\times\frac{b}{a}=-1$.

3. $\frac{a-c}{a-b}\times\frac{a+c}{a+b}=\frac{a^2-c^2}{a^2-b^2}$.

Page 95.

4. $\frac{a}{b}\times\frac{b}{c}=\frac{a}{c}$.

5. $\frac{a+b}{a-b}\times\frac{a+c}{a-c}=\frac{a^2+a\,(b-c)+bc}{a^2-a\,(b+c)+bc}$.

6. $\frac{b+a}{b}\times\frac{b}{b-a}=\frac{a+b}{b-a}$.

7. $\frac{y-x}{y}\times\frac{x}{y}=\frac{xy-x^2}{y^2}$.

8. $\frac{a-b-c}{b} \times \frac{b}{a+1} = \frac{a-b-c}{a+1}$.

9. $\frac{1-a}{1} \times \frac{a}{1-a} = a$.

10. $\left(\frac{1}{a} + \frac{1}{ab^3}\right) \div \left(b + \frac{1}{b} - 1\right) = \frac{b^3+1}{ab^3} \times \frac{b}{b^2-b+1} = \frac{b+1}{ab^2}$.

11. $\frac{1}{2a-1+\frac{1}{1+\frac{a}{1-a}}} \times \frac{a}{2} = \frac{1}{2a-1+\frac{1}{\frac{1-a+a}{1-a}}} \times \frac{a}{2} =$

$\frac{1}{2a-1+\frac{1-a}{1}} \times \frac{a}{2} = \frac{1}{\frac{2a-1+1-a}{1}} \times \frac{a}{2} =$

$\frac{1}{2a-1+1-a} \times \frac{a}{2} = \frac{a}{2a} = \frac{1}{2}$.

12. $\frac{yz+xz+xy}{xyz} \times \frac{xyz}{x^2z+xy^2+yz^2} = \frac{yz+xz+xy}{x^2z+xy^2+yz^2}$.

6. 1. $(y+3)^3+2(y+3)^2-(y+3)-10=$
$y^3+9y^2+27y+27+2y^2+12y+18-y-3-10=$
$y^3+11y^2+38y+32$.

4. $(a+1)^3+(a-1)(a+1)^2+(a-1)^2(a+1)+(a-1)^3=$
$a^3+3a^2+3a+1+a^3+a^2-a-1+a^3-a^2-a+1+a^3-3a^2+3a-1=$
$4a^3+4a=4a(a^2+1)$.

5. $\frac{\frac{a+1}{ab+1}+\frac{ab+a}{ab+1}-1}{\frac{a+1}{ab+1}-\frac{ab+a}{ab+1}-1} = \frac{a+1+ab+a-ab-1}{a+1-ab-a-ab-1} = \frac{2a}{-2ab} = -\frac{1}{b}$.

Page 96.

7. 1.
$$\begin{array}{l|l} 1+x)x & (x-x^2+x^3-x^4+\frac{x^5}{1+x} \\ \underline{x+x^2} & \\ -x^2 & \\ \underline{-x^2-x^3} & \\ x^3 & \\ \underline{x^3+x^4} & \\ -x^4 & \\ \underline{-x^4-x^5} & \\ x^5. & \end{array}$$

2. $a-b)a \quad (1+\frac{b}{a}+\frac{b^2}{a^2}+\frac{b^3}{a^3}+\frac{b^4}{a^4-a^3b}$

$$\begin{array}{l} \underline{a-b} \\ b \\ \quad b-\frac{b^2}{a} \\ \quad \underline{\quad\quad} \\ \quad\quad \frac{b^2}{a} \\ \quad\quad \underline{\frac{b^2}{a}-\frac{b^3}{a^2}} \\ \quad\quad\quad\quad \frac{b^3}{a^2}-\frac{b^4}{a^3} \\ \quad\quad\quad\quad \underline{\quad\quad\quad} \\ \quad\quad\quad\quad\quad \frac{b^4}{a^3} \end{array} \qquad \frac{b^4}{a-b} = \frac{b^4}{a^4-a^3b}.$$

4. $1-x+x^2)1 \quad (1+x-x^3-x^4+\frac{x^6}{1-x+x^2}$

$$\begin{array}{l} \underline{1-x+x^2} \\ \quad x-x^2 \\ \quad \underline{x-x^2+x^3} \\ \quad\quad -x^3 \\ \quad\quad \underline{x^3+x^4-x^5} \\ \quad\quad\quad -x^4+x^5 \\ \quad\quad\quad \underline{-x^4+x^5-x^6} \\ \quad\quad\quad\quad x^6. \end{array}$$

Page 101.

1. 1. $\begin{array}{l|l} x^2+3x+2 & x+1 \\ \underline{x^2+\ x\ \ } & x+2 \\ \quad 2x+2 & \\ \quad \underline{2x+2} & \\ \quad\quad 0. & \end{array}$

2. $\begin{array}{l|l} 8x^3-\ \ y^3 & 4x^2+2xy+y^2 \\ \underline{8x^3+4x^2y+2xy^2} & 2x\ -y \\ \quad -4x^2y-2xy^2-y^3 & \\ \quad \underline{-4x^2y-2xy^2-y^3} & \\ \quad\quad 0. & \end{array}$

3. $(a-x)\ (x-a)\ (1-a)\ (a-1)=(a^2+x^2-2ax)\ (a-1)^2$
$(a-x)\ (x-a)\ (1-a)\ (a-1)=(a-x)\ (a-x)\ (a-1)\ (a-1)$
$+1.+1=-1.-1$
$1=1.$

4. $(a^0\times a^{-1}\times b^0\times b^{-1}\times c^0\times c^{-1})\ abc=1$
$(1\times a^{-1}\times 1\times b^{-1}\times 1\times c^{-1})\ abc=1$
$a^{-1}b^{-1}c^{-1}\times abc=1$
$a^0b^0c^0=1$
$1\times 1\times 1=1.$

5. $\frac{a^2}{a^2b^0} \times \frac{n+1}{n-1} \times \frac{b^0}{n+1} \times \frac{1}{(n-1)^{-1}} = 1$

$\frac{a^2}{a^2} \times \left(\frac{n+1}{n-1} \times \frac{1}{n+1} \times \frac{n-1}{1}\right) = 1$

$1 \times 1 = 1.$

Page 102.

2. 9. $6(23-x)-3x=3(4x-27)$

$138-6x-3x=12x-81$

$-21x=-219$

$x=10\frac{3}{7}.$

10. $x-[3+\{x-(3+x)\}]=5$

$x-[3+\{x-3-x\}]=5$

$x-[3+x-3-x]=5$

$x-3-x+3+x=5$

$x=5.$

3. 1. $2x-\frac{1}{2}x+1=5x-2$

$4x-x+2=10x-4$

$-7x=-6$

$x=\frac{6}{7}.$

6. $\frac{x}{2}+\frac{x}{3}+\frac{x}{5}=\frac{x}{4}+\frac{47}{6}$

$30x+20x+12x=15x+470$

$47x=470$

$x=10.$

4. $x-\frac{x}{4}+25=\frac{x}{3}+\frac{x}{2}+21$

$12x-3x+300=4x+6x+252$

$-x=-48$

$x=48.$

10. $\frac{3x}{5}+2\frac{1}{2}+11=\frac{x}{4}+17$

$12x+50+220=5x+340$

$7x=70$

$x=10.$

Page 103.

4. 1. $.2x-3.5=.6x+1.5$

$-.4x=+5$

$x=\frac{5}{-.4}=-12\frac{1}{2}.$

4. $.6x-1.5x=.2-.15x$

$-.75x=.2$

$x=\frac{.2}{-.75}=-2\frac{2}{3}.$

2. $.4x-.075x=12-.275x$

$.600x=12$

$x=\frac{12}{.6}=20.$

5. $.125x-.0625x=.375$

$.0625x=.375$

$x=\frac{.3750}{.0625}=6.$

3. $.375x-1.875=.12x+1.185$

$.255x=3.060$

$x=\frac{3.060}{.255}=12.$

5. 1. $ay+2b=3by+4a$

$ay-3by=4a-2b$

$y(a-3b)=4a-2b$

$y=\frac{4a-2b}{a-3b}.$

2. $ab-3cx-2ax-c=0$

$-3cx-2ax=c-ab$

$3cx+2ax=ab-c$

$x(3c+2a)=ab-c$

$x=\frac{ab-c}{3c+2a}.$

5. 4. $am-ax=-am+2ax$
$-ax-2ax=-am-am$
$3ax=2am$
$x=\frac{2am}{3a}=\frac{2m}{3}.$

6. $am-5x=bc-ax$
$ax-5x=bc-am$
$x(a-5)=bc-am$
$x=\frac{bc-am}{a-5}.$

7. $abc-a^2x=ax-a^2b$
$-a^2x-ax=-a^2b-abc$
$a^2x+ax=a^2b+abc$
$x(a^2+a)=a^2b+abc$
$x=\frac{a^2b+abc}{a^2+a}=\frac{b(a^2+ac)}{a^2+a}=\frac{b(a+c)}{a+1}.$

8. $3acx-6bcd=12cdx+abc$
$3acx-12cdx=6bcd+abc$
$x(3ac-12cd)=bc(6d+a)$
$x=\frac{bc(6d+a)}{3c(a-4d)}=\frac{b(6d+a)}{3(a-4d)}.$

9. $x^2-a^2=x-a$
$(x+a)(x-a)=x-a$
$x+a=1$
$x=1-a.$

10. $(a+x)(b+x)=x(x-c)$
$ab+ax+bx+x^2=x^2-cx$
$ax+bx+cx=-ab$
$x(a+b+c)=-ab$
$x=\frac{-ab}{a+b+c}.$

Page 103. (Special Cases.)

1. $x^2+a^2=(b-x)^2$
$x^2+a^2=b^2-2bx+x^2$
$2bx=b^2-a^2$
$x=\frac{b^2-a^2}{2b}.$

3. $(x+1)^2+(x-5)^2=2(x+5)^2$
$x^2+2x+1+x^2-10x+25=2x^2+20x+50$
$2x-10x-20x=-1-25+50=24$
$-28x=24$
$x=-\frac{24}{28}=-\frac{6}{7}.$

5. $(x-1)^3+x^3+(x+1)^3=3x(x^2-1)$
$x^3-3x^2+3x-1+x^3+x^3+3x^2+3x+1=3x^3-3x$
$9x=0$
$x=0.$

Page 104.

1. $\frac{4(x+2)}{5} = 7-5x$

$4x+8 = 35-25x$

$29x = 27$

$x = \frac{27}{29}$.

4. $\frac{ax}{c} = b-x$

$ax = bc-cx$

$ax+cx = bc$

$x(a+c) = bc$

$x = \frac{bc}{a+c}$.

5. $x-(3x-4)-\frac{5-2x}{4} = 2$

$4x-12x+16-5+2x = 8$

$-6x = -3$

$x = \frac{3}{6} = \frac{1}{2}$.

3. 1. $4-\frac{x-9}{8} = \frac{x}{22}-\frac{1}{2}$

$352-11x+99 = 4x-44$

$15x = -352-99-44 = -495$

$x = \frac{495}{15} = 33$.

Page 105.

2. $\frac{8-5x}{12}+\frac{5x-6}{4}-\frac{7x+5}{6} = 0$

$8-5x+15x-18-14x-10 = 0$

$-4x = +20$

$x = -5$.

4. $\frac{ax-b}{c}-\frac{bx+c}{a} = abc$

$a^2x-ab-bcx-c^2 = a^2bc^2$

$a^2x-bcx = a^2bc^2+ab+c^2$

$x(a^2-bc) = a^2bc^2+ab+c^2$

$x = \frac{a^2bc^2+ab+c^2}{a^2-bc}$.

5. $\frac{1}{3}(x-3)-\frac{1}{4}(x-8)+\frac{1}{5}(x-5) = 0$

$20x-60-15x+120+12-60 = 0$

$20x = 15x = 60+60-120 = 0$

$x = 0$.

1. $\frac{3}{1-x}-\frac{1}{1+x}-\frac{1}{1-x^2} = 0$

$3+3x-2+2x-1 = 0$

$5x = 3-3 = 0$

$x = 0$.

3. $\frac{a}{x+b}-\frac{b}{x-b} = \frac{a^2-b^2}{x^2-b^2}$

$ax-ab-bx-b^2 = a^2-b^2$

$ax-bx = ab+a^2$

$x(a-b) = ab+a^2$

$x = \frac{ab+a^2}{a-b} = \frac{a(a+b)}{a-b}$.

4. $\frac{6}{x-1} - \frac{4x}{x^2-1} = \frac{3}{x+1}$

$6x+6-4x=3x-3$

$-x=-9$

$x=9.$

5. $b+\frac{x}{1+\frac{1}{b}} = 1+\frac{x}{1-\frac{1}{b^2}}$

$b+\frac{bx}{b+1} = 1+\frac{b^2x}{b^2-1}$

$b^3-b+bx\,(b-1) = b^2-1+b^2x$

$b^3-b+b^2x-bx = b^2-1+b^2x$

$-bx = b^2-1-b^3+b$

$x = -b+\frac{1}{b}+b^2-1 =$

$b^2-b-1+\frac{1}{b}.$

Page 106.

1. $\frac{8x+5}{14} + \frac{7x-3}{6x+2} = \frac{4x+6}{7}$

$\frac{8x+5}{14} - \frac{8x+12}{14} + \frac{7x-3}{6x+2} = 0$

$\frac{8x+5-8x-12}{14} + \frac{7x-3}{6x+2} = 0$

$\frac{7x-3}{6x+2} = \frac{1}{2}$

$14x-6 = 6x+2$

$8x = 8$

$x = 1.$

4. $2\frac{1}{2} - \frac{3}{2x+4} = \frac{2x-1}{4} - \frac{x}{2}$

$\frac{10+1}{4} = \frac{3}{2x+4}$

$\frac{11}{4} = \frac{3}{2x+4}$

$22x+44 = 12$

$22x = -32$

$x = -\frac{32}{22} = -\frac{16}{11}.$

5. $\frac{10x+17}{18} - \frac{12x+2}{11x-8} = \frac{5x-4}{9}$

$\frac{10x+17-10x+8}{18} = \frac{12x+2}{11x-8}$

$\frac{25}{18} = \frac{12x+2}{11x-8}$

$275x-200 = 216x+36$

$59x = 236$

$x = 4.$

Page 107.

1. $\frac{x}{x-2} - \frac{x+1}{x-1} = \frac{x-8}{x-6} - \frac{x-9}{x-7}$

$\frac{x^2-x-x^2+x+2=2}{(x-2)\,(x-1)} = \frac{x^2-15x+56-x^2+15x-54=2}{(x-6)\,(x-7)}$

$(x-2)\,(x-1) = (x-6)\,(x-7)$

$x = 4.$

3. $\frac{x-7}{x-9}-\frac{x-9}{x-11}=\frac{x-13}{x-15}-\frac{x-15}{x-17}$

$\frac{x^2-18x+77-x^2+18x-81=4}{(x-9)\ (x-11)}=\frac{x^2-30x+221-x^2+30x-225=4}{(x-15)\ (x-17)}$

$(x-9)\ (x-11)=(x-15)\ (x-7)$

$x=13.$

5. $\frac{x+2}{x}+\frac{x-7}{x-5}-\frac{x+3}{x+1}=\frac{x-6}{x-4}$

$\frac{x+2}{x}-\frac{x+3}{x+1}=\frac{x-6}{x-4}-\frac{x-7}{x-5}$

$x^2+x=x^2-9x+20$

$x=2.$

1. $\frac{4x-17}{x-4}+\frac{10x-13}{2x-3}=\frac{8x-30}{2x-7}+\frac{5x-4}{x-1}$

$\frac{4x-17}{x-4}-\frac{8x-30}{2x-7}=\frac{5x-4}{x-1}-\frac{10x-13}{2x-3}$

$\frac{1}{(x-4)\ (2x-7)}=\frac{1}{(x-1)\ (2x-3)}$

$x=2\frac{1}{2}.$

Page 108.

4. $\frac{x}{x-2}+\frac{x-9}{x-7}=\frac{x+1}{x-1}+\frac{x-8}{x-6}$

$\frac{x}{x-2}-\frac{x+1}{x-1}=\frac{x-8}{x-6}-\frac{x-9}{x-7}$

$\frac{2}{(x-2)\ (x-7)}=\frac{2}{(x-6)\ (x-7)}$

$x=7.$

5. $\frac{x^2-5}{x^2-6}+\frac{x^2-11}{x^2-12}=\frac{x^2-7}{x^2-8}+\frac{x^2-9}{x^2-10}$

$\frac{x^2-5}{x^2-6}-\frac{x^2-7}{x^2-8}=\frac{x^2-9}{x^2-10}-\frac{x^2-11}{x^2-12}$

$(x^2-6)\ (x^2-8)=(x^2-10)\ (x^2-12)$

$x^2=9$

$x=3.$

Page 109.

4. $ab+bx+ax+x^2-(cd-dx-cx+x^2)=0$

$ab+bx+ax+x^2-cd+dx+cx-x^2=0$

$ax+bx+cx+dx=cd-ab$

$x\ (a+b+c+d)=cd-ab$

$x=\frac{cd-ab}{a+b+c+d}.$

5. $x^2+2x+1+x^2-10x+25=2x^2+20x+50$

$$28x=-24$$

$$x=-\tfrac{24}{28}=-\tfrac{6}{7}.$$

6. $2bx-ab-a^2=2ax+2bx-3ab$

$$-2ax=ab-3ab+a^2$$

$$x=\frac{2ab+a^2}{2a}=b+\frac{a}{2}.$$

8. $ax-bx-a^2+ab=ax+bc-ab-cx$

$$-bx+cx=a^2-ab+bc-ab$$

$$bx-cx=2ab-a^2-bc$$

$$x(b-c)=2ab-a^2-bc$$

$$x=\frac{2ab-a^2-bc}{b-c}.$$

10. $x^2+a^2+b^2-2ax-2bx+2ab-(x^2-ax-bx+ab)+ab=0$

$$x^2+a^2+b^2-2ax-2bx+2ab-x^2+ax+bx-ab+ab=0$$

$$-2ax-2bx+ax+bx=-a^2-b^2-2ab$$

$$ax+bx=a^2+2ab+b^2$$

$$x(a+b)=(a+b)^2$$

$$x=a+b.$$

13. $\dfrac{a^2-x}{c}-\dfrac{b^2-x}{a}-\dfrac{c^2-x}{b}=\dfrac{a^2}{c}-\dfrac{b^2}{a}$

$$\frac{-x}{c}+\frac{x}{a}-\frac{c^2-x}{b}=0$$

$$-abx+bcx-ac^3+acx=0$$

$$x(ac-ab+bc)=ac^3$$

$$x=\frac{ac^3}{ac-ab+bc}$$

14. $\dfrac{x^2+2x+1}{x^2+4x+4}=\dfrac{x-4}{x-2}$

$$x^3-3x-2=x^3-12x-16$$

$$9x=-14$$

$$x=-\tfrac{14}{9}.$$

15. $(ax+b)(ax+b)-3b(ax-b)=a^2x^2+b^2$

$$a^2x^2+2abx+b^2-3abx+3b^2=a^2x^2+b^2$$

$$+abx=3b^2$$

$$x=\frac{3b^2}{ab}=\frac{3b}{a}.$$

Page 110.

16. $\frac{1}{4}x^2-\frac{1}{4}ax-\frac{x^2+2ax+a^2}{4}=\frac{2ax}{3}-\frac{2a^2}{6}$

$3x^2-3ax-3x^2-6ax-3a^2=8ax-4a^2$

$17ax=a^2$

$17x=a$

$x=\frac{a}{17}.$

18. $\frac{4+x}{4}-\frac{1-x}{7}-\frac{1}{5}(8-x)=\frac{x-23}{5}+7$

$140+35x-20+20x-224+28x=28x-644+980$

$35x+20x=980+20+224-644-140$

$55x=440$

$x=8.$

20. $\frac{ax+b}{c}+\frac{ax+b}{cx+b}=\frac{2ax+d}{2c}+\frac{b}{c}$

$\frac{2ax+2b}{2c}+\frac{ax+b}{cx+b}=\frac{2ax+d}{2c}+\frac{2b}{2c}$

$\frac{ax+b}{cx+b}=\frac{2ax+d+2b-2ax-2b}{2c}=\frac{d}{2c}$

$2acx+2bc=cdx+bd$

$2acx-cdx=bd-2bc$

$x(2ac-cd)=bd-2bc$

$x=\frac{bd-2bc}{2ac-cd}.$

Page 110. REVIEW.

4. $4a^{2m}-4a^mb^n+b^{2n}=(2a^m-b^n)(2a^m-b^n).$

5. $a^4+a^2b^2+b^4=a^4+2a^2b^2+b^4-a^2b^2=(a^2+b^2)^2-a^2b^2=$
$(a^2+ab+b^2)(a^2-ab+b^2).$

7. $a^2x^2-4a^2x+4a^2=a^2(x^2-4x+4)=a^2(x-2)(x-2).$

9. $x^2-3x^2y+3xy^2-y^2=(x^2-y^2)-(3x^2y-3xy^2)=$
$(x^2-y^2)-3xy(x-y)=(x-y)(x+y-3xy).$

10. $3a^2+a-2=(3a+2)(a-1).$

2. $\left(\frac{a^2-ax}{a^2-x^2}\times\frac{a^2-x^2}{a^2-2ax+x^2}\right)\left(\frac{a^2-x^2}{a^2}\right)=$

$\left(\frac{a(a-x)}{(a-x)(a-x)}\right)\left(\frac{a^2-x^2}{a^2}\right)=\frac{a(a+x)}{a^2}=\frac{a+x}{a}.$

3. $\frac{a^2+b^2-ab}{b}\times\frac{ab}{b-a}\times\frac{a^2-b^2}{a^3+b^3}=\frac{a}{b-a}\times\frac{a^2-b^2}{a+b}=$

$\frac{-a(a-b)}{a-b}=-a.$

5. $x-[5y-\{x-(5z-\overline{2z-y}+2x-(x-2y-z)\}]$
$x-[5y-\{x-(5z-2z+y)+2x-x+2y+z\}]$
$x-[5x-\{x-5z\times 2z-y+2x-x+2y+z\}]$
$x-[5y-x+5z-2z+y-2x+x-2y-z]$
$x-5y+x-5z+2z-y+2x-x+2y+z$
$3x-3y-2z.$

Page 111.

3. $a^2c+bx^2=b(a+x)(b+x)-ab(b+c)$
$a^2c+bx^2=ab^2+b^2x+abx+bx^2-ab^2-abc$
$b^2x+abx=abc+a^2c$
$bx(b+a)=ac(b+a)$
$bx=ac$
$x=\frac{ac}{b}.$

5. $a^3b+a^2x-b^3+bx=b^2x-b^3-a^3b+a^2x$
$x(1-b)=-2a^3$
$x=-\frac{2a^3}{1-b}=\frac{2a^3}{b-1}.$

8. $6a^2x-36a^3+3x=4ax-48a^4+72a^5$
$6a^2x+3x-4ax=36a^3-48a^4+72a^5$
$x(6a^2+3-4a)=36a^3-48a^4+72a^5$
$x=12a^3.$

9. $x^2+ax+cx+ac+x^2+2bx+b^2=2x^2+2cx+2bx+2bc$
$ax-cx=2bc-ac-b^2$
$x(a-c)=2bc-ac-b^2$
$x=\frac{2bc-ac-b^2}{a-c}.$

10. $\frac{3a^2m-2b^2m}{a+b}=\frac{2a^2-b^2}{ax+bx}+\frac{a-b}{x}$
$3a^2mx-2b^2mx=2a^2-b^2+a^2-b^2$
$x(3a^2m-2b^2m)=3a^2-2b^2$
$x=\frac{3a^2-2b^2}{m(3a^2-2b^2)}=\frac{1}{m}.$

Page 112.

1. x = the number
$\frac{x}{3}-\frac{x}{4}=16$
$x=192.$

2. x = C.'s share
$3x$ = B.'s share
$x+3x=1000.$

Page 113.

3. x = brother's number
$x+8$ = Carl's number
$2x+8=24$.

4. x = the number
$2x$ = its double
$\frac{x}{2}$ = its half
$2x-\frac{x}{2}=150$.

5. x = the number
$\frac{x}{2}+\frac{x}{5}=56$.

6. x = son's age
$3x$ = father's age
$3x-x=24$.

7. $x-\left(\frac{x}{3}+\frac{x}{10}+\frac{x}{12}\right)=87$.

8. x = cost of horse
$35+\frac{x}{2}=x+10$.

9. $2x$ = A.'s share
$3x$ = B.'s share
$2x+3x=2000$.

10. x = value of the bridle
$10x$ = value of the horse
$x+10x=132$.

11. x = the smaller
$70-x$ = the larger
$3x=2(70-x)$.

12. x = Raymond's age now
$2x$ = Ross' age now
$x-8$ = Raymond's age 8 yrs. ago
$2x-8$ = Ross' age 8 yrs. ago
$2x-8=6(x-8)$.

13. x = son's age
$2x$ = man's age
$(x-15)3=2x-15$.

Page 114.

14. x = one part
$42-x$ = other part
$\frac{x}{5}+\frac{42-x}{6}=8$.

15. x = investment of one
$\frac{4}{5}x$ = investment of the other
$x+\frac{4}{5}x=2600$.

16. x = the first number
$2x$ = the second number
$3x$ = the third number
$x+2x+3x=72$.

17. x = B.'s age
$x+16$ = A.'s age
$x+16-50=40-x$
$x=37$, $x+16=43$.

18. x = length, $\frac{2}{3}x$ = width } of 1st room
$x-3$ = length, $\frac{2}{3}x+3$ = width } of 2d room
$x-3=\frac{2}{3}x+3$.

19. x = years required
$48+x$ = father's age then
$12+x$ = son's age then
$48+x=2(12+x)$.

20. x = length of post
$\frac{x}{4}+\frac{x}{3}+10=x$.

21. x = one number
$x+6$ = other number
$2x+6=20$.

22. x = one number
$x+1$ = other number
$(x+1)^2 - x^2 = 41$
$x^2 + 2x + 1 - x^2 = 41$.

23. x = a woman's share
$x+1$ = a man's share
$\frac{x}{2}$ = a child's share
$6(x+1) + 10x + \frac{40x}{2} = 150$.

24. x = days required
A. can do $\frac{1}{6}$ work in 1 day
B. can do $\frac{1}{8}$ work in 1 day
Both can do $\frac{1}{x}$ work in 1 day
$\frac{1}{6} + \frac{1}{8} = \frac{1}{x}$.

Page 115.

25. x = the money of each
$x - 40$ = Mary's money after giving
$x + 40$ = Martha's money after receiving
$x - 40 = \frac{1}{3}(x+40)$.

26. x = sum owed to A.
$2x$ = sum owed to B.
$6x$ = sum owed to C.
$x + 2x + 6x = 900$.

27. x = first part
$\frac{x}{2}$ = second part
$\frac{x}{6}$ = third part
$\frac{x}{24}$ = fourth part
$x + \frac{x}{2} + \frac{x}{6} + \frac{x}{24} = 410$.

28. x = B.'s share
$\frac{5}{6}x + 25$ = A.'s share
$\frac{4}{15}x$ = C.'s share
$x + \frac{5}{6}x + 25 + \frac{4}{15}x = 1285$.

29. x = distance wagon moved
$\frac{x}{2} - \frac{x}{5} = 10$.

30. x = weight of body
$\frac{x}{2} + 9$ = weight of head
$\frac{x}{2} + 18$ = weight of body
$\frac{x}{2} + 18 = x$
$x = 36$, body
$\frac{x}{2} + 9 = 27$, head
9, tail
72, fish.

31. x = days he worked
$48 - x$ = days he was idle
$24x - 12(48 - x) = 504$.

32. x = leaps required
$\frac{3}{2}x$ = leaps of fox
$\frac{7}{3}x$ = hound's distance
$\frac{3}{2}x$ = fox's distance
$\frac{7}{3}x - \frac{3}{2}x = 60$.

34. x = time required

To gain 1 space requires $5\frac{5}{11}$ min.

$x = 6 \times 5\frac{5}{11} = 32\frac{8}{11}$ min.

35. x = time required

To gain 1 space requires $5\frac{5}{11}$ min.

$x = 9 \times 5\frac{5}{11} = 49\frac{1}{11}$ min. (after 9).

36. x = time required

At 7 o'clock the hands are 7 spaces apart

The required position holds the hands 6 spaces apart

∴ The min. hand has but 1 space to gain

∴ $x = 5\frac{5}{11}$ min. (after 7).

37. x = L.'s share

$x - 387$ = M.'s share

$x - 387 + 2223$ = N.'s share

$3x + 1449 = 9000$.

38. x = distance required

$\frac{x}{9}$ = hrs. spent in going

$\frac{x}{3}$ = hrs. spent in returning

$\frac{x}{9} + \frac{x}{3} = 8$.

Page 117.

39. x = time required

The pipes respectively fill in $\frac{1}{3}$ and $\frac{1}{5}$ in 1 hour

Hence, $\frac{x}{3} + \frac{x}{5} = 1$ (cistern).

40. x = the number

$\frac{x}{7} + \frac{x}{8} = 150$.

41. x = the less part

$49 - x$ = the greater part

$\frac{x}{6} + \frac{49 - x}{5} = 9$.

42. x^2 = area

$x^2 = (x+6)(x-4)$.

43. x = one number

nx = other number

$x + nx = b$.

44. x = idle days

$a - x$ = days he worked

$(a - x)c$ = receipt

dx = forfeit

$(a + x)c - dx = m$

$x = \frac{ac - m}{c + d}$.

45. $x = \frac{12 \times 100 - 900}{100 + 50} = 2$ days (idle)

x = idle days

$12 - x$ = days he worked

$(12 - x)100$ = receipt

$50x$ = forfeit

$(12 - x)100 - 50x = 900$

$x = 2$ days (idle).

46. x = distance required

$\frac{x}{a}$ = revolutions of fore wheel

$\frac{b}{a}$ = revolutions of hind wheel

$\frac{x}{a} - \frac{x}{b} = c$.

47. x = A.'s money

$\frac{2}{3}x$ = B.'s money

$\frac{3}{4}$ of $\frac{2}{3}x$ = C.'s money

$x + \frac{2}{3}x + \frac{1}{2}x = 650$.

Page 118.

48. x = the distance

$\frac{x}{b}$ = hours he rode

$\frac{x}{c}$ = hours he walked

$\frac{x}{b} + \frac{x}{c} = a.$

49. x = the days required

A. can plow $\frac{1}{a}$ in 1 day

B. can plow $\frac{1}{b}$ in 1 day

Together $\frac{1}{x}$ in 1 day

$\frac{1}{a} + \frac{1}{b} = \frac{1}{x}.$

50. x = pieces of 1st kind

$p - x$ = pieces of 2d kind

$\frac{1}{n}$ = value of each piece of 1st kind

$\frac{1}{m}$ = value of each piece of 2d kind

$\frac{x}{n}$ = value of x pieces

$\frac{p-x}{m}$ = value of $p-x$ pieces

$$\frac{x}{n} + \frac{p-x}{m} = 1$$

$$x = \frac{n(m-p)}{m-n}.$$

Page 119.

1.
$$\begin{array}{r} 2x - 3y = 4 \\ 18x - 3y = 84 \\ \hline 16x = 80 \\ x = 5 \\ 6x - 9y = 12 \\ 6x - y = 28 \\ \hline 8y = 16 \\ y = 2. \end{array}$$

3.
$$\begin{array}{r} 10x + 15y = 35 \\ 10x + 14y = 38 \\ \hline y = -3 \\ 3y = -9 \\ 2x - 9 = 7 \\ x = 8. \end{array}$$

5.
$$\begin{array}{r} 6x + 20y = -2 \\ 6x + 21y = -1\frac{1}{2} \\ \hline y = \frac{1}{2} \\ 10y = 5 \\ 3x + 5 = -1 \\ x = -2. \end{array}$$

7.
$$\begin{array}{r} 10x - 3y = 25 \\ 10x - 18y = -50 \\ \hline 15y = 75 \\ y = 5 \\ \hline 3y = 15 \\ 10x - 15 = 25 \\ x = 4. \end{array}$$

9. $4x+12y=36$
$13x-2y=35$
$4x+12y=36$
$78x-12y=210$
$82x=246$
$x=3$
$4x=12$
$12+12y=36$
$y=2.$

10. $4x-5y=-23$
$2x+5y=35$
$6x=12$
$x=2$
$2x=4$
$4+5y=35$
$y=6\frac{1}{5}.$

Page 120.

1. $x=13-2y$
$3x=39-6y$
$39-6y+y=14$
$y=5$
$x=13-10=3.$

4. $y=14x$
$\frac{1}{2}y=7x$
$7x-3x=2$
$x=\frac{1}{2}$
$y=14x\frac{1}{2}=7.$

6. $7x=19-8y$
$x=\frac{19-8y}{7}$
$5x=\frac{95-40y}{7}$
$\frac{95-40y}{7}+6y=13\frac{1}{2}$
$y=-\frac{1}{4}$
$x=\frac{19+2}{7}=3.$

8. $7y=5x+21$
$y=\frac{5x+21}{7}$
$9y=\frac{45x+189}{7}$
$21x-\frac{45x+189}{7}=75$
$147x-45x-189=525$
$x=7$
$y=\frac{5x+21}{7}=8.$

10. $5x-3y=30$
$2x+21y=567$
$x=21$
$y=25.$

Page 121.

1. $3y=70-7x$
$y=\frac{70-7x}{3}$
$-4y=7-5x$
$y=\frac{5x-7}{4}$
$\frac{70-7x}{3}=\frac{5x-7}{4}$
$x=7$
$y=7.$

3. $2y=11x-5$
$2y=16-10x$
$11x-5=16-10x$
$x=1$
$y=3.$

5. $y=\frac{43-5x}{5}$

$y=\frac{69-11x}{9}$

$\frac{43-5x}{5}=\frac{69-11x}{9}$

$x=3$
$y=4.$

7. $y=\frac{7x+41}{3}$

$y=2x-12$

$\frac{7x+41}{3}=2x-12$

$x=5$
$y=-2.$

9. $y=\frac{14x-39}{3}$

$y=\frac{35-6x}{17}$

$\frac{14x-39}{3}=\frac{35-6x}{17}$

$x=3$
$y=1.$

3. $3x+2y=84$
$2x+3y=66$

$x=24$
$y=6.$

5. $4x-3y=0$
$3x-7y=37$

$x=3$
$x=-4.$

8. $bx+ay=abm$
$ax+bx=abn$
$b^2x+aby=ab^2m$
$a^2x+aby=a^2bn$
$a^2x-b^2x=a^2bn-ab^2m$
$x(a^2-b^2)=ab(an-bm)$

$x=\frac{ab(an-bm)}{a^2-b^2}$
$y=\frac{ab(am-bn)}{a^2-b^2}.$

10. $y=\frac{c-ax}{b}$

$y=\frac{mx-d}{n}$

$\frac{c-ax}{b}=\frac{mx-d}{n}$

$nc-anx=bmx-bd$
$bmx+anx=nc+bd$
$x(bm+an)=nc+bd$

$x=\frac{nc+bd}{bm+an}.$

1. $x-y=5$
$5x-4y=40$
$4x-4y=20$
$x=20$
$2x-y=5$
$y=15.$

Page 122.

10. $x+2y+1=4x-2y+2$
$-3x+4y=1$
$3x-y+1=5x-5y+15$
$-2x+4y=14$

$x=13$
$y=10.$

11. $3x+3y=5x-5y$
$x+y+1=7x-7y-7$

$x=2$
$y=\frac{1}{2}.$

12. $x+ay=a^2+2ab$
$by-x=b^2$
$ay+by=a^2+2ab+b^2$
$y(a+b)=a^2+2ab+b^2$

$y=a+b$
$x=ab.$

13. $x+ay=-a^2$
$x+by=-b^2$
$ay-by=-a^2+b^2$
$y=-\frac{a^2-b^2}{a-b}=-(a+b)$
$x=ab.$

14. $x=4y$
$\frac{2x+7y}{5}-1=\frac{4x-12y+2}{3}$
$6x+21y-15=20x-60y+10$
$-14x+81y=25$
$-56y+81y=25$
$25y=25$
$y=1$
$x=4.$

15. $4x-5y=0$
$6x+y=34$
$x=5$
$y=4.$

16. $ax+bx+by=a^3-b^3$
$ax+by+ay=a^3-b^3$
$bx=ay$
$x=\frac{ay}{b}$
$\frac{a^2y}{b}+ay+by=a^3-b^3$
$a^2y+bay+b^2y=ba^3-b^4$
$y=\frac{b(a^3-b^3)}{a^2+ab+b^2}=b(a-b).$

17. $ax-bx+ay+by=2a^3-2ab^2$
$x-y=4ab$
$x=4ab+y$
$4a^2b+ay-4ab^2-by+ay+by=2a^3-2ab^2$
$2ay=2a^3+2ab^2-4a^2b$
$ay=a^3+ab^2-2a^2b=a(a^2+b^2-2ab)$
$y=(a-b)^2$
$x=(a+b)^2.$

Page 124.

1. $\frac{4}{x}+\frac{3}{y}=2$
$\frac{4}{x}-\frac{10}{y}=-14$
$\frac{13}{y}=16$
$y=\frac{13}{16}$
$\frac{4}{x}+\frac{48}{13}=2$
$\frac{4}{x}=-\frac{22}{13}$
$\frac{1}{x}=-\frac{22}{52}=-\frac{11}{26}$
$x=-\frac{26}{11}.$

3. $\frac{ac}{x}+\frac{bc}{y}=cm$
$\frac{ac}{x}+\frac{ad}{y}=an$
$\frac{bc}{y}-\frac{ad}{y}=cm-an$
$y=\frac{bc-ad}{cm-an}$
$x=\frac{ad-bc}{dm-bn}.$

4. $\frac{5}{3x}+\frac{2}{5y}=7$

$\frac{7}{3x}-\frac{1}{5y}=6=\frac{14}{3x}-\frac{2}{5y}=12$

$\frac{19}{3x}=19$

$\frac{1}{3x}=1$

$x=\frac{1}{3}$
$y=\frac{1}{5}.$

7. $\frac{2}{y}=\frac{1}{n}-n$

$\frac{2}{y}=\frac{1-n^2}{n}$

$\frac{1}{y}=\frac{1-n^2}{2n}$

$y=\frac{2n}{1-n^2}$

$\frac{2}{x}=\frac{1}{n}+n=\frac{1+n^2}{n}$

$\frac{1}{x}=\frac{1+n^2}{2n}$

$x=\frac{2n}{1+n^2}.$

9. $\frac{mn}{nx}+\frac{n^2}{my}=mn+n^2$

$\frac{mn}{nx}+\frac{m^2}{ny}=\frac{m(m^2+n^2)}{n}$

$\frac{m^2}{ny}-\frac{n^2}{my}=\frac{m^3+mn^2-mn-n^3}{n}$

$\frac{m^3-n^3}{mny}=\frac{m^3-n^3}{n}$

$\frac{1}{mny}=\frac{1}{n}$

$\frac{1}{my}=1$

$\frac{1}{y}=m$

$y=\frac{1}{m}$
$x=\frac{1}{n}.$

10. $\frac{m^2}{x}+\frac{n^2}{y}=m^2n+mn^2$

$\frac{n}{x}+\frac{m}{y}=m^2+n^2$

$\frac{m^2n}{x}+\frac{n^3}{y}=m^2n^2+mn^3$

$\frac{m^2n}{x}+\frac{m^3}{y}=m^4+m^2n^2$

$\frac{m^3-n^3}{y}=m^4+m^2n^2-m^2n^2-mn^3=m(m^3-n^3)$

$\frac{1}{y}=m$

$y=\frac{1}{m}$
$x=\frac{1}{n}.$

Page 125.

1. $3x+4y+5z=26$
$3x+4y+6z=31$
$y+z=5$
$6x+9y+12z=60$
$6x+8y+10z=52$
$y+2z=8$
$5-z=8-2z$
$z=3$
$y+3=5$
$y=2$
$2x+6+12=20$
$2x=2$
$x=1.$

3. $x+y-z=1$
$-4x-y+3z=1$
$-3x+2z=2$
$3x+3y-3z=3$
$8x+3y-6z=1$
$5x-3z=-2$
$-15x+10z=10$
$15x-9z=-6$
$z=4.$

4. $x+y+z=26$
$x-y=4$
$2y+z=22$
$x-y=4$
$x-z=6$
$-y+z=-2$
$2y+z=22$
$-y+z=-2$
$3y=24$
$y=8$
$x=12$
$z=6.$

Page 126.

6. $x+y+z=1$
$2x+y+16z=4$
$20x+9y-6z=12$
$2x+y+16z=4$
$x+y+z=1$
$x+15z=3$
$20x+9y-6z=12$
$9x+9y+9z=9$
$11x-15z=3$
$x+15z=3$
$12x=6$
$x=\frac{1}{2}.$

8. $\frac{2}{x}+\frac{2}{3y}+\frac{1}{z}=40$
$\frac{1}{x}+\frac{3}{y}-\frac{1}{z}=28$
$\frac{3}{x}+\frac{11}{y}=68$
$\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=36$
$\frac{1}{x}+\frac{3}{y}-\frac{1}{z}=28$
$\frac{2}{x}+\frac{4}{y}=64$
$\frac{3}{x}+\frac{11}{3y}=68$
$\frac{6}{x}+\frac{12}{y}=192$
$\frac{6}{x}+\frac{22}{3y}=136$
$\frac{14}{3y}=56$
$\frac{1}{3y}=4$
$3y=\frac{1}{4}$
$y=\frac{1}{12}.$

10. $\frac{x}{a}+\frac{y}{b}=1$

$\frac{x}{a}+\frac{z}{c}=1$

$\frac{y}{b}-\frac{z}{c}=0$

$\frac{y}{b}+\frac{z}{c}=1$

$\frac{2y}{b}=1$

$2y=b$

$y=\frac{b}{2}$

Page 127.

1. $2y+2u+2z=4$

$y+u+z=2$

$y+z=2$

$u=0$

$y+u+z=2$

$u+z=-1$

$y=3$

$y+u+z=2$

$y+u=3$

$z=-1.$

3. $2x+2y+2z=50$

$x+y+z=25$

$x+y=8$

$z=17.$

4. $2x+2y+2u+2w+2z=132$

$x+y+u+w+z=66$

$x+y=26$

$u+w+z=40$

$u+w=15$

$z=25.$

5. $2x+2y+2u+2z=a+b+c+d$

$x+y+u+z=\frac{a+b+c+d}{2}$

$-x+y+u+z=a$

$2x=\frac{a+b+c+d}{2}-a=\frac{a+b+c+d-2a}{2}$

$x=\frac{-a+b+c+d}{4}.$

Page 128.

1. $x=$ one number, $y=$ the other

$\frac{x+y}{19}=14$

$\frac{x-y}{2}=4.$

2. $\frac{x-2}{y-2}=\frac{3}{5}$

$\frac{x+1}{y+1}=\frac{2}{3}.$

3. $x=$ sum required

£5 = 100 s.

$100-x=4\,(10+x).$

Page 129.

4. $x=$ one part, $y=$ the other

$x+y=50$

$2x=3y.$

5. $x=$ B.'s money

$x-5=$ A.'s money

$2x-5=$ C.'s money

$x+x-5+2x-5=50$

$4x-10=50.$

6. x = time required
$70-x$ = age of older man at that time
$45-x$ = age of younger man at that time
$70-x=2(45-x)$
$x=20$ years ago.

7. $\left.\begin{aligned}\frac{x+1}{y}&=\tfrac{1}{3}\\ \frac{x}{y+1}&=\tfrac{1}{4}.\end{aligned}\right\}$

8. $\left.\begin{aligned}x+y&=a\\ x-y&=b\end{aligned}\right\}$
$\left.\begin{aligned}x&=\frac{a+b}{2}\\ y&=\frac{a-b}{2}.\end{aligned}\right\}$

t. u.

9. $x+y=9$
$10x+y-27=10y-x$
$\left.\begin{aligned}x&=6\\ y&=3\end{aligned}\right\}=63.$

10. x = A.'s present age
y = B.'s present age
$x-8=4(y-8)$
$x+12=2(y+12)$.

11. x = A.'s money
y = B.'s money
$x+100=y-100$
$2(x-100)=y+100$.

12. x = A.'s time
y = B.'s time
z = C.'s time
$A.+B.=\frac{1}{x}+\frac{1}{y}=\frac{1}{12}$
$B.+C.=\frac{1}{y}+\frac{1}{z}=\frac{1}{16}$
$A.+C.=\frac{1}{x}+\frac{1}{z}=\frac{1}{20}$.

13. x = selling price of a sheep
y = selling price of a calf
$5x+10y=90$
$10x+5y=75$.

Page 130.

14. x = first girl's roses
y = second girl's roses
$x+5=3(y-5)$
$5(x-2)=y+z$.

15. x = his rate in still water
y = rate of stream's flow
$\left.\begin{aligned}x+y&=7\\ 2\tfrac{1}{2}(x-y)&=7\end{aligned}\right\}$
$\left.\begin{aligned}x+y&=7\\ \tfrac{5}{2}x-\tfrac{5}{2}y&=7.\end{aligned}\right\}$

16. x = first capital

1st year

$x-500$
$\frac{4}{3}(x-500)$
$\frac{4x-2000}{3}$

2d year

$\frac{4x-2000}{3}-500$
$\frac{4}{3}\left(\frac{4x-3500}{3}\right)$
$\frac{16x-14000}{9}$

3d year

$\frac{16x-14000}{9}-500$
$\frac{4}{3}\left(\frac{16x-18500}{9}\right)$
$\frac{64x-74000}{27}=2x$
$x=7400$.

17. x = cavalry's required number of paces
In same time infantry will take $\frac{5}{4}x$ paces
$\frac{4}{3}x$ = cavalry's whole distance
$\frac{5}{4}x$ = infantry's whole distance
$\frac{4}{3}x - \frac{5}{4}x = 1200$.

18. x = length of 1st rectangle
y = width of 1st rectangle
xy = its area
$x+4$ = length of 2d rectangle
$y-2$ = width of 2d rectangle
$(x+4)\ (y-2)$ = its area
$x+3$ = length of 3d rectangle
$y-\frac{8}{5}$ = width of 3d rectangle
$(x+3)\ (y-\frac{8}{5})$ = its area
$xy = xy+4y-2x-8 = xy-\frac{8}{5}x+3y-\frac{24}{5}$.

19. x = A.'s time
y = B.'s time
z = C.'s time
u = D.'s time

A. + B. + C. $= \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{1}{10}$, working one day

B. + C. + D. $= \frac{1}{y} + \frac{1}{z} + \frac{1}{u} = \frac{1}{12}$, working one day

A. + C. + D. $= \frac{1}{x} + \frac{1}{z} + \frac{1}{u} = \frac{1}{15}$, working one day

A. + B. + D. $= \frac{1}{x} + \frac{1}{x} + \frac{1}{u} = \frac{1}{18}$, working one day

$\frac{3}{x} + \frac{3}{y} + \frac{3}{z} + \frac{3}{u} = \frac{1}{10} + \frac{1}{12} + \frac{1}{15} + \frac{1}{18} = \frac{11}{36}$

$\frac{3}{y} + \frac{3}{z} + \frac{3}{u} = \frac{9}{36}$

$\frac{3}{x} = \frac{11}{36} - \frac{9}{36} = \frac{2}{36} = \frac{1}{18}$

$\frac{1}{x} = \frac{1}{54}$

$x = 54$ days.

20. x = greater part
y = less part

1. $x+y = 100{,}000$
$\frac{5x}{100}$ = income from x
$\frac{4x}{100}$ = income from y

2. $\frac{5x+4x}{100} = 4640$.

Page 131.

E. F. G. H.

2. 1. $x+y+z+u=1000$

2. $y=\frac{x}{2}$

3. $z=u+\frac{x}{3}$

4. $y+100=z+u.$

3. 1. $x+y+z=10$

2. $y=x+z$ ·

3. $100z+10y+x=100x+10y+z+99.$

4. $x=$ ounces taken from the first ingot
$y=$ ounces taken from the second ingot
$z=$ ounces taken from the third ingot

$$\left.\begin{aligned}\frac{7x+12y+4z}{16}&=8\\ \frac{3x+3y+7z}{16}&=3\tfrac{3}{4}\\ \frac{6x+y+5z}{16}&=4\tfrac{1}{4}.\end{aligned}\right\}$$

5. $x=$ A.'s sum, $y=$ B.'s sum, $z=$ C.'s sum

First distribution

$x-y-z=$ what A. will have left
$2y=$ what B. will have
$2z=$ what C. will have

Second distribution

$3y-x-z=$ what B. will have left
$2x-2y-2z=$ what A. will have
$4z=$ what C. will have

Third distribution

$7z-x-y=$ what C. will have left
$4x-4y-4z=$ what A. will have
$6y-2x-2z=$ what B. will have

$-x-y+7z=6$
$4x-4y-4z=6$
$-2x+6y-2z=6.$

Page 134.

1. $32°-32°=0°$
$0x\frac{5}{9}=0°$ C.
$0x\frac{4}{9}=0°$ R.

2. $-10°-32°=-42°$
$-42°x\frac{5}{9}=-23\frac{1}{3}°$ C.
$-42°x\frac{4}{9}=-18\frac{2}{3}°$ R.

3.
$$\begin{array}{l|l} 4x^2-12xy+9y^2 & 2x-3y \\ \underline{2x^2} & \\ 4x-3y \;\big|\; -12xy+9y^2 & \\ \quad\; \underline{-12xy+9y^2} & \\ \qquad\quad 0. & \end{array}$$

4.
$$\begin{array}{l|l} x^4-4x^3+2x^2+4x+1 & x^2-2x-1 \\ \underline{x^4} & \\ 2x^2-2x \;\big|\; -4x^3+2x^2+4x+1 & \\ \quad\; \underline{-4x^3+4x^2} & \\ 2x^2-4x-1 \;\big|\; -2x^2+4x+1 & \\ \qquad\quad \underline{-2x^2+4x+1} & \\ \qquad\qquad\quad 0. & \end{array}$$

Page 143.

1.
$$\begin{array}{l|l} a^4-4a^3+8a+4 & a^2-2a-2 \\ \underline{a^4} & \\ 2a^2-2a \;\big|\; -4a^3+8a+4 & \\ \quad\; \underline{-4a^3+4a^2} & \\ 2a^2-4a-2 \;\big|\; -4a^2+8a+4 & \\ \qquad\quad \underline{-4a^2+8a+4} & \\ \qquad\qquad\quad 0. & \end{array}$$

2.
$$\begin{array}{l|l} 4x^4+12x^3+5x^2-6x+1 & 2x^2+3x-1 \\ \underline{4x^4} & \\ 4x^2+3x \;\big|\; 12x^3+5x^2-6x+1 & \\ \quad\; \underline{12x^3+9x^2} & \\ 4x^2+6x-1 \;\big|\; -4x^2-6x+1 & \\ \qquad\quad \underline{-4x^2-6x+1} & \\ \qquad\qquad\quad 0. & \end{array}$$

5.
$$\begin{array}{l|l} 16x^4+24x^3+89x^2+60x+100 & 4x^2+3x+10 \\ \underline{16x^4} & \\ 8x^2+3x \;\big|\; 24x^3+89x^2+60x+100 & \\ \quad\; \underline{24x^3+\;\;9x^2} & \\ 8x^2+6x+10 \;\big|\; 80x^2+60x+100 & \\ \qquad\quad \underline{80x^2+60x+100} & \\ \qquad\qquad\quad 0. & \end{array}$$

6.
$$\begin{array}{l|l} a^2-ab+\frac{1}{4}b^2 & a-\frac{b}{2} \\ \underline{a^2} & \\ 2a-\frac{b}{2} \;\big|\; -ab+\frac{1}{4}b^2 & \\ \qquad\quad \underline{-ab+\frac{1}{4}b^2} & \\ \qquad\qquad\quad 0. & \end{array}$$

7. $4a^4+4a^3-7a^2-4a+4 \mid 2a^2+a-2$

$4a^4$

$4a^2+a \mid 4a^3-7a^2-4a+4$

$4a^3+a^2$

$4a^2+2a-2 \mid -8a^2-4a+4$

$-8a^2-4a+4$

$0.$

8. $1+4x+10x^2+12x^3+9x^4 \mid 1+2x+3x^2$

1

$2+2x \mid 4x+10x^2+12x^3+9x^4$

$4x+4x^2$

$2+4x+3x^2 \mid 6x^2+12x^3+9x^4$

$6x^2+12x^3+9x^4$

$0.$

9. $4x^4+4x^3-\frac{1}{2}x+\frac{1}{16} \mid 2x^2+x-\frac{1}{4}$

$4x^4$

$4x^2+x \mid 4x^3-\frac{1}{2}x+\frac{1}{16}$

$4x^3+x^2$

$4x^2+2x-\frac{1}{4} \mid -x^2-\frac{1}{2}x+\frac{1}{16}$

$-x^2-\frac{1}{2}x+\frac{1}{16}$

$0.$

10. $4m^2+4m-11-\frac{6}{m}+\frac{9}{m^2} \mid 2m+1-\frac{3}{m}$

$4m^2$

$4m+1 \mid 4m-11-\frac{6}{m}+\frac{9}{m^2}$

$4m+1$

$4m+2-\frac{3}{m} \mid -12-\frac{6}{m}+\frac{9}{m^2}$

$-12-\frac{6}{m}+\frac{9}{m^2}$

$0.$

Page 146.

1. 2. $\sqrt{\frac{4}{16}}=\frac{2}{4}=\frac{1}{2}$.

4. $\sqrt{\frac{100}{1000}}=\frac{10}{100}=\frac{1}{10}$.

8. $\sqrt{\frac{2'25}{7'29}}=\frac{15}{27}=\frac{5}{9}$.

9. $\sqrt{\frac{23'04}{51'84}}=\frac{48}{72}=\frac{2}{3}$.

14. $\sqrt{\frac{1'00'00'00}{1'00'00'00'00}}=\frac{1000}{100000}=\frac{1}{10}$.

16. $\sqrt{\frac{4'93'72'84}{11'10'88'89}}=\frac{2222}{3333}=\frac{1}{3}$.

2. 1. $\sqrt{.09}=.3$

2. $\sqrt{.9}=\sqrt{.90'00'00'00}=.9486+$.

3. $\sqrt{.01'74} = .12.$

4. $\sqrt{.144} = \sqrt{.14'40'00'00} = .3821$

5. $\sqrt{.01'00} = .1.$

6. $\sqrt{.12'34'50} = .3513+.$

7. $\sqrt{.76'38'76} = .874.$

8. $\sqrt{.30'85'80'25} = .5555.$

9. $\sqrt{.00'00'98'01} = .0099.$

10. $\sqrt{.00'01'02'01} = .0101.$

2. $\sqrt{\frac{1}{8}} = \sqrt{.87'50} = .9354.$

3. $\sqrt{\frac{8}{3}} = \frac{\sqrt{8}}{3} = \frac{2.8284}{3} = .9428.$

4. $\sqrt{\frac{144}{1728}} = \sqrt{\frac{1}{12}} = \sqrt{\frac{12}{144}} = \frac{\sqrt{12}}{12} = \frac{3.4641}{12} = .2886+.$

5. $\sqrt{\frac{289}{2704}} = \frac{\sqrt{289}}{52} = \frac{17}{52}.$

Page 147.

6. $\sqrt{\frac{11}{16}} = \frac{\sqrt{11}}{4} = \frac{3.3166}{4} = .8291+.$

7. $\sqrt{\frac{12}{25}} = \frac{\sqrt{12}}{5} = \frac{3.4641}{5} = .6928+.$

7. $\sqrt{\frac{1}{2}} = \sqrt{.5} = .7071+.$

9. $\sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2} = \frac{1.73205}{2} = .86602+.$

10. $\sqrt{\frac{3}{4}+\frac{5}{5}+\frac{6}{9}} = \sqrt{\frac{29}{12}} = \sqrt{2.416666} = 1.5545+.$

Page 148.

3.

$$x^3+6x^2+12x+8 \,|\, x+2$$
$$x^3$$
$$3x^2+6x+4 \,|\, 6x^2+12x+8$$
$$6x^2+12x+8$$
$$0.$$

5.

$$64a^3-144a^2b+108ab^2-27b^3 \,|\, 4a-3b$$
$$64a^3$$
$$48a^2-36ab+9b^2 \,|\, -144a^2b+108ab^2-27b^3$$
$$-144a^2b+108ab^2-27b^3$$
$$0.$$

6.

$$x^6+6x^5-40x^3+96x-64 \,|\, x^2+2x-4$$
$$x^6$$
$$3x^4+6x^3+4x^2 \,|\, 6x^5-40x^3+96x-64$$
$$6x^5+12x^4+8x^3$$
$$3x^4+12x^3+12x^2 \,|\, -12x^4-48x^3+96x-64$$
$$-12x^2-24x+16$$
$$3x^4+12x^3-24x+16 \,|\, -12x^4-48x^3+96x-64$$

9.

$$8x^6-36x^5+42x^4+9x^3-21x^2-9x-1 \;\big|\; 2x^2-3x-1$$

$8x^6$

$$12x^4-18x^3+9x^2 \;\big|\; -36x^5+42x^4+9x^3-21x^2-9x-1$$
$$-36x^5+54x^4-27x^3$$

$$12x^4-36x^3+27x^2 \;\big|\; -12x^4+36x^3-21x^2-9x-1$$
$$-6x^2+9x+1 \;\big|\; -12x^4+36x^3-21x^2-9x-1$$
$$12x^4-36x^3-21x^2+9x+1 \;\big|\; 0.$$

10.

$$8a^6-12a^5-54a^4+59a^3+135a^2-75a-125 \;\big|\; 2a^2-a-5$$

$8a^6$

$$12a^4-6a^3+a^2 \;\big|\; -12a^5-54a^4+59a^3+135a^2-75a-125$$
$$-12a^5+6a^4-a^3$$

$$12a^4-12a^3+3a^2 \;\big|\; -60a^4+60a^3+135a^2-75a-125$$
$$-30a^2+15a+25 \;\big|\; -60a^4+60a^3+135a^2-75a-125$$
$$12a^4-12a^3-27a^2+15a+25 \;\big|\; 0.$$

Page 150.

2.

74′088(42
64
4800|10088
240|10088
4|0.
5044|

Page 151.

7.

1′860′867(123
1
300|860
60|728
4|132867
364|132867
43200|0.
1080|
9|
44289|

9.

145′531′576(526
125
7500|20531
300|15608
4|4923576
7804|4923576
811200|0.
9360|
36|
820596|

11.

1′879′080′904(1234
1
300|879
60|728
4|151080
364|132867
43200|18213904
1080|18213904
9|0.
44289|
4538700|
14760|
16|
4553476|

4. 1. $\sqrt[3]{.008} = .2$.
2. $\sqrt[3]{.08} = \sqrt[3]{.080'000} = .43+$.
3. $\sqrt[3]{.8} = \sqrt[3]{.800'000'000} = .928+$.
4. $\sqrt[3]{.125} = .5$.
5. $\sqrt[3]{.25} = \sqrt[3]{.250'000'000} = .627+$.
6. $\sqrt[3]{2.197} = 1.3$.

Page 152.

7. $\sqrt[3]{9.261} = 2.1$.
8. $\sqrt[3]{185.193} = 5.7$.
9. $\sqrt[3]{.1} = \sqrt[3]{.100'000} \ldots = .464+$.
10. $\sqrt[3]{6.} = \sqrt[3]{6.000000} \ldots = 1.816+$.
11. $\sqrt[3]{7.} = \sqrt[3]{7.000000} \ldots = 1.91+$.
12. $\sqrt[3]{34.965783} = 3.27$.
13. $\sqrt[3]{41.063625} = 3.45$.
14. $\sqrt{.000'001} = .01$.
15. $\sqrt{.000'000'100} \ldots = .00464+$.

1. $\sqrt[3]{\frac{3}{4}} = \sqrt[3]{.750} = .908+$.
2. $\sqrt[3]{\frac{9}{13}} = \sqrt[3]{.692'307} \ldots = .88+$.
3. $\sqrt[3]{\frac{3}{9}} = \sqrt[3]{\frac{1}{3}} = \sqrt[3]{.333'333} \ldots = .61+$.
4. $\sqrt[3]{\frac{5}{4}} = \sqrt[3]{1.250} = 1.07+$.
5. $\sqrt[3]{\frac{2}{3}} = \sqrt[3]{.666'666} \ldots = .87+$.
6. $\sqrt[3]{\frac{5}{6}} = \sqrt[3]{.833333} \ldots = .94+$.
7. $\sqrt[3]{\frac{1728}{1898}} = \sqrt[3]{\frac{864}{949}} = \sqrt[3]{.910432} \ldots = .96+$.
8. $\sqrt[3]{\frac{1492}{1776}} = \sqrt[3]{\frac{373}{444}} = .94+$.
9. $\sqrt[3]{\frac{5760}{7000}} = \sqrt[3]{\frac{144}{175}} = .93+$.
10. $\sqrt[3]{\frac{8000}{63366}} = \sqrt[3]{\frac{4000}{31683}} = .501+$.

1. $\sqrt{\sqrt{81x^4+108x^3+54x^2+12x+1}} = \sqrt{9x^2+6x+1} = 3x+1$.

2. $\sqrt{\sqrt{16x^4-96x^3y+216x^2y^2-216xy^3+81y^4}} = \sqrt{4x^2-12xy+9y^2} = 2x-3y$.

3. In like manner.

4. $\sqrt[3]{\sqrt{15x^2+20x^3+15x^4+6x^5+x^6+6x+1}} = \sqrt[3]{1+3x+3x^2+x^3} = 1+x$.

5. $\sqrt{\sqrt{\sqrt{}}}$ indicate the successive operations to be performed. Ans. $a-b$.

Page 153.

1. $\sqrt[3]{x^4} = x^{\frac{4}{3}}$.
3. $\sqrt{m} = m^{\frac{1}{2}}$.
5. $\sqrt[3]{p} = p^{\frac{1}{3}}$.
7. $\sqrt[3]{b^4}\ \sqrt[5]{x} = b^{\frac{4}{3}}x^{\frac{1}{5}}$.
9. $3a\sqrt[m]{b}\ \sqrt[n]{c^m} = 3ab^{\frac{1}{m}}c^{\frac{m}{n}}$.

1. $x^{\frac{1}{3}} = \sqrt[3]{x}$.
4. $xy^{\frac{3}{4}} = x\sqrt[4]{y^3}$.
6. $x^{\frac{3}{5}}y^{\frac{5}{3}} = \sqrt[5]{x^3}\ \sqrt[3]{y^5}$.
7. $(5m)^{\frac{4}{7}}n^{\frac{7}{2}} = \sqrt[7]{(5m)^4}\ \sqrt[2]{n^7}$.
8. $a\sqrt[3]{b}\ \sqrt[5]{c^4}\ \sqrt[3]{d^7}$.

Page 153.

Values:

1. $4^{\frac{5}{2}} = \sqrt[2]{4^5} = \sqrt[2]{1024} = 32$.
3. $27^{\frac{2}{3}} = \sqrt[3]{27^2} = \sqrt[3]{729} = 9$.

Values:

5. $36^{\frac{3}{2}} = \sqrt{36^3} = \sqrt{46656} = 216.$

7. $-1000^{\frac{2}{3}} = \sqrt[3]{-1000^2} = \sqrt[3]{1'000'000} = 100.$

9. $1^{\frac{1}{9}} = \sqrt[9]{1} = 1.$

10. $-125^{\frac{5}{3}} = \sqrt[3]{-125^5} = \sqrt[3]{-30'517'578'125} = -3125.$

Page 154.

1. $a^2y^{-3} = \frac{a^2}{y^3}.$

2. $x^0y^{-5} = \frac{x^0}{y^5} = \frac{1}{y^5}$

3. $p^3q^{-\frac{1}{3}} = \frac{p^3}{q^{\frac{1}{3}}} = \frac{p^3}{\sqrt[3]{q}}.$

4. $5x^2y^{-\frac{1}{2}} = \frac{5x^2}{y^{\frac{1}{2}}} = \frac{5x^2}{\sqrt{y}}.$

6. $4a^0b^{\frac{1}{2}}c^{-\frac{1}{3}} = \frac{4b^{\frac{1}{2}}}{c^{\frac{1}{3}}}.$

8. $6x^{-5}b^{-7}c^0 = \frac{6}{x^5b^7}.$

9. $7m^{-7}n^7 = \frac{7n^7}{m^7}.$

Page 155.

1. $\frac{1}{m} = m^{-1}.$

4. $\frac{a^0}{3x^{\frac{4}{5}}} = \frac{x^{-\frac{4}{5}}}{3}.$

5. $\frac{4c}{a^3b^{-2}} = \frac{4b^2c}{a^3} = 4a^{-3}b^2c.$

8. $\frac{x^3}{3a^{\frac{3}{4}}b^{\frac{4}{7}}} = \frac{a^{-\frac{3}{4}}b^{-\frac{4}{7}}x^3}{3}.$

9. $\frac{5x}{5m^{-5}n^{-\frac{5}{5}}} = xm^5n.$

10. $\frac{m^0n^0}{m^{\frac{4}{5}}n^{-\frac{5}{7}}x^0} = m^{-\frac{4}{5}}n^{\frac{5}{7}}.$

1. $\frac{m^3}{3} = \frac{1}{3m^{-3}}.$

5. $\frac{4b^{\frac{6}{7}}}{1} = \frac{4}{b^{-\frac{6}{7}}}.$

6. $\frac{6x^{-6}y}{c^{\frac{1}{3}}} = \frac{6}{c^{\frac{1}{3}}x^6y^{-1}}.$

8. $a^{-1}b^2c^3d^0 = \frac{1}{ab^{-2}c^{-3}}.$

9. $\frac{5a^{-3}b^{-\frac{3}{2}}}{7c^5} = \frac{5}{7a\,b^{\frac{3}{2}}c^5}.$

10. $x^0y^{-1}z^2u^{-3} = \frac{1}{yz^{-2}u^3}.$

Page 156.

1. $a^3xa^{-1} = a^{3-1} = a^2.$

3. $a^{-3}a^3 = a^{-3+3} = a^0 = 1.$

5. $3c^{\frac{1}{2}}xc^{-\frac{3}{2}} = 3c^{\frac{1}{2}-\frac{3}{2}} = 3c^{-\frac{2}{2}} = 3c^{-1} = \frac{3}{c}.$

7. $7a^{-5}b^{\frac{2}{3}}ab^{-1} = 7a^{-4}b^{\frac{2}{3}-\frac{3}{3}} = 7a^{-4}b^{-\frac{1}{3}} = \frac{7}{a^4b^{\frac{1}{3}}}.$

Values:

9. $11\sqrt[5]{a} \times 4\sqrt{a^2} = 44a^{\frac{1}{5}}a = 44a^{\frac{1+5}{5}} = 44a^{\frac{6}{5}}$.

10. $\dfrac{13xy}{a^{\frac{1}{3}}b^{-3}} \times \dfrac{2}{a^{-2}b^{\frac{1}{3}}} = 26a^{-\frac{1}{3}+2}b^{3-\frac{1}{3}}xy = 26a^{\frac{5}{3}}b^{\frac{8}{3}}xy$.

Multiplications:

1. $\left(m^{\frac{1}{3}}+n^{\frac{1}{3}}\right)\left(m^{\frac{1}{3}}-n^{\frac{1}{3}}\right) = m^{\frac{2}{3}}-n^{\frac{2}{3}}$.

3. $\left(m^{\frac{1}{3}}-n^{\frac{1}{3}}\right)\left(m^{\frac{1}{3}}-n^{\frac{1}{3}}\right) = m^{\frac{2}{3}}-2m^{\frac{1}{3}}n^{\frac{1}{3}}+n^{\frac{2}{3}}$.

5.
$$\begin{array}{l} x^{-2}+x^{-1}y^{-1}+y^{-2} \\ \underline{x^{-2}-x^{-1}y^{-1}+y^{-2}} \\ x^{-4}+x^{-3}y^{-1}+x^{-2}y^{-2} \\ \quad -x^{-3}y^{-1}-x^{-2}y^{-2}-x^{-1}y^{-3} \\ \underline{\qquad\qquad x^{-2}y^{-2}+x^{-1}y^{-3}+y^{-4}} \\ x^{-4}+x^{-2}y^{-2}+y^{-4}. \end{array}$$

6.
$$\begin{array}{l} x^{\frac{2}{3}}-x^{\frac{1}{3}}y^{\frac{1}{3}}+y^{\frac{2}{3}} \\ \underline{x^{\frac{1}{3}}+y^{\frac{1}{3}}} \\ x\ -x^{\frac{2}{3}}y^{\frac{1}{3}}+x^{\frac{1}{3}}y^{\frac{2}{3}} \\ \quad \underline{x^{\frac{2}{3}}y^{\frac{1}{3}}-x^{\frac{1}{3}}y^{\frac{2}{3}}+y} \\ x+y. \end{array}$$

8.
$$\begin{array}{l} a^{\frac{1}{2}}+a^{\frac{1}{4}}b^{\frac{1}{4}}+b^{\frac{1}{2}} \\ \underline{a^{\frac{1}{2}}-a^{\frac{1}{4}}b^{\frac{1}{4}}+b^{\frac{1}{2}}} \\ a\ +a^{\frac{3}{4}}b^{\frac{1}{4}}+a^{\frac{1}{2}}b^{\frac{1}{2}} \\ \quad -a^{\frac{3}{4}}b^{\frac{1}{4}}-a^{\frac{1}{2}}b^{\frac{1}{2}}-a^{\frac{1}{4}}b^{\frac{3}{4}} \\ \underline{\qquad\qquad a^{\frac{1}{2}}b^{\frac{1}{2}}+a^{\frac{1}{4}}b^{\frac{3}{4}}+b} \\ a+a^{\frac{1}{2}}b^{\frac{1}{2}}+b. \end{array}$$

Page 157.

9.
$$\begin{array}{l} x^{\frac{2}{3}}\ y^{-\frac{1}{2}}+2x^{\frac{1}{3}}\ -3y^{\frac{1}{2}} \\ \underline{\quad 2y^{-\frac{1}{2}}-4x^{-\frac{1}{3}}-6x^{-\frac{2}{3}}y^{\frac{1}{2}}} \\ 2x^{\frac{2}{3}}y^{-1}+4x^{\frac{1}{3}}y^{-\frac{1}{2}}-\ 6y^{0} \\ \quad -4x^{\frac{1}{3}}y^{-\frac{1}{2}}-\ 8x^{0}\ +12x^{-\frac{1}{3}}y^{\frac{1}{2}} \\ \underline{\qquad\qquad -\ 6x^{0}y^{0}-12x^{-\frac{1}{3}}y^{\frac{1}{2}}+18x^{-\frac{2}{3}}y} \\ 2x^{\frac{2}{3}}y^{-1}-20\qquad +18x^{-\frac{2}{3}}y. \end{array}$$

Multiplications:

10. $4a^{\frac{3}{4}}b^{-1} + a^{\frac{1}{4}} - 2a^{-\frac{1}{4}}b$

$7a^{\frac{1}{4}}b^{-1} - 2a^{-\frac{1}{4}} - 3a^{-\frac{3}{4}}b$

$28ab^{-2} + 7a^{\frac{1}{2}}b^{-1} - 14$

$- 8a^{\frac{1}{2}}b^{-1} - 2 + 4a^{-\frac{1}{2}}b$

$- 12 - 3a^{-\frac{1}{2}}b + 6a^{-1}b^2$

$28ab^{-2} - a^{\frac{1}{2}}b^{-1} - 28 + a^{-\frac{1}{2}}b + 6a^{-1}b^2.$

Divisions:

1. $x^3 \div a^{-2} = x^3a^2.$

2. $x \div x^4 = x^{-4} = x^{-3} = \frac{1}{x^3}.$

3. $x^{\frac{4}{9}} \div x^{\frac{4}{5}} = x^{\frac{4}{9} - \frac{4}{5}} = x\,\frac{20-36}{45} = x^{-\frac{16}{45}} = \frac{1}{x^{\frac{16}{45}}}.$

4. $x^{-\frac{1}{3}} \div x^{-\frac{3}{5}} = x^{-\frac{1}{3} + \frac{3}{5}} = x^{-\frac{5}{15} + \frac{9}{15}} = x^{\frac{4}{15}}.$

5. $5d^{-2} \div \sqrt[3]{k^5} = 5d^{-2} \div k^{\frac{5}{3}} = \frac{5d^{-2}}{k^{\frac{5}{3}}} = \frac{5}{d^2k^{\frac{5}{3}}}.$

6. $m^3 \div \sqrt[3]{m^{-2}} = m^3 \div m^{-\frac{2}{3}} = m^{3+\frac{2}{3}} = m^{\frac{11}{3}}.$

7. $a^{\frac{1}{3}} \div \frac{1}{\sqrt[5]{a^4}} = a^{\frac{1}{3}} \div \frac{1}{a^{\frac{4}{5}}} = a^{\frac{1}{3}} \div a^{-\frac{4}{5}} = a^{\frac{1}{3}+\frac{4}{5}} = a^{\frac{17}{15}}.$

8. $16x \div 4x^{-1}\sqrt[4]{y} = 4x^{1+1}y^{-\frac{1}{4}} = 4x^2y^{-\frac{1}{4}} = \frac{4x^2}{y^{\frac{1}{4}}}.$

9. $7a^{-1}x^{\frac{2}{3}} \div 3\tfrac{1}{2}ax^{-\frac{1}{5}} = 2a^{-1-1}x^{\frac{2}{3}+\frac{1}{5}} = 2a^{-2}x^{\frac{13}{15}} = \frac{2x^{\frac{13}{15}}}{a^2}.$

10. $x^{-n} \div x^{\frac{1}{n}} = x^{-n-\frac{1}{n}} = x^{\frac{-n^2-1}{n}} = x^{-\frac{n^2+1}{n}} = \frac{1}{x^{\frac{n^2+1}{n}}}.$

Page 158.

12. $a^{\frac{1}{3}} + b^{\frac{1}{3}}\big)a + b\ \big(a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}} + b^{\frac{2}{3}}$

$a + a^{\frac{2}{3}}b^{\frac{1}{3}}$

$- a^{\frac{2}{3}}b^{\frac{1}{3}} + b$

$- a^{\frac{2}{3}}b^{\frac{1}{3}} - a^{\frac{1}{3}}b^{\frac{2}{3}}$

$a^{\frac{1}{3}}b^{\frac{2}{3}} + b$

$a^{\frac{1}{3}}b^{\frac{2}{3}} + b$

$0.$

Divisions:

13. $a^{\frac{1}{4}}-b^{\frac{1}{4}})a- \quad b(a^{\frac{3}{4}}+a^{\frac{1}{2}}b^{\frac{1}{4}}+a^{\frac{1}{4}}b^{\frac{1}{2}}+b^{\frac{3}{4}}$

$a-a^{\frac{3}{4}}b$

$a^{\frac{3}{4}}b-b$

$a^{\frac{3}{4}}b-a^{\frac{1}{2}}b^{\frac{1}{2}}$

$a^{\frac{1}{2}}b^{\frac{1}{2}}-b$

$a^{\frac{1}{2}}b^{\frac{1}{2}}-a^{\frac{1}{4}}b^{\frac{3}{4}}$

$a^{\frac{1}{4}}b^{\frac{3}{4}}-b$

$a^{\frac{1}{4}}b^{\frac{3}{4}}-b$

0.

15. $3x^{\frac{1}{3}}+1)21x+\ x^{\frac{2}{3}}+x^{\frac{1}{3}}+1(7x^{\frac{2}{3}}-2x^{\frac{1}{3}}+1$

$21x+7x^{\frac{2}{3}}$

$-6x^{\frac{2}{3}}+\ x^{\frac{1}{3}}+1$

$-6x^{\frac{2}{3}}-2x^{\frac{1}{3}}$

$3x^{\frac{1}{3}}+1$

$3x^{\frac{1}{3}}+1$

0.

17. $3x^{\frac{1}{2}}-2-x^{-\frac{1}{2}})9x-12x^{\frac{1}{2}}-2+4x^{-\frac{1}{2}}+x^{-1}(3x^{\frac{1}{2}}-2-x^{-\frac{1}{2}}$

$9x-\ 6x^{\frac{1}{2}}-3$

$-\ 6x^{\frac{1}{2}}+1+4x^{-\frac{1}{2}}+x^{-1}$

$-\ 6x^{\frac{1}{2}}+4+2x^{-\frac{1}{2}}$

$-3+2x^{-\frac{1}{2}}+x^{-1}$

$-3+2x^{-\frac{1}{2}}+x^{-1}$

0.

18. $a^{\frac{1}{3}}-a^{\frac{1}{6}}b^{\frac{1}{6}}+b^{\frac{1}{3}})a^{\frac{2}{3}}+a^{\frac{1}{3}}b^{\frac{1}{3}}+\quad b^{\frac{2}{3}}(a^{\frac{1}{3}}+a^{\frac{1}{6}}b^{\frac{1}{6}}+b^{\frac{1}{3}}$

$a^{\frac{2}{3}}-a^{\frac{1}{2}}b^{\frac{1}{6}}+a^{\frac{1}{3}}b^{\frac{1}{3}}$

$a^{\frac{1}{2}}b^{\frac{1}{6}}+b^{\frac{2}{3}}$

$a^{\frac{1}{2}}b^{\frac{1}{6}}-a^{\frac{1}{3}}b^{\frac{1}{3}}+a^{\frac{1}{6}}b^{\frac{1}{2}}$

$a^{\frac{1}{3}}b^{\frac{1}{3}}-a^{\frac{1}{6}}b^{\frac{1}{2}}+b^{\frac{2}{3}}$

$a^{\frac{1}{3}}b^{\frac{1}{3}}-a^{\frac{1}{6}}b^{\frac{1}{2}}+b^{\frac{2}{3}}$

0.

20. $b^{\frac{1}{3}}-2b^{-\frac{1}{6}})5b^{\frac{2}{3}}-6b^{\frac{1}{3}}-4b^{-\frac{2}{3}}-4b^{-\frac{1}{3}}-5(5b^{\frac{1}{2}}+4b^{\frac{1}{6}}+3b^{-\frac{1}{6}}+2b^{-\frac{1}{2}}$

$5b^{\frac{2}{3}}-10b^{\frac{4}{3}}$

$4b^{\frac{1}{3}}-4b^{-\frac{2}{3}}-4b^{-\frac{1}{3}}-5$

$4b^{\frac{1}{3}}-8$

$3-4b^{-\frac{2}{3}}-4b^{-\frac{1}{3}}$

$3-6b^{-\frac{1}{3}}$

$2b^{-\frac{1}{3}}-4b^{-\frac{2}{3}}$

$2b^{-\frac{1}{3}}-4b^{-\frac{2}{3}}$

$0.$

Squares:

3. $\left(2x^{\frac{1}{2}}y^{\frac{1}{2}}+x^{-\frac{1}{2}}y^{-\frac{1}{2}}\right)^2=4xy+4x^0y^0+x^{-1}y^{-1}=4xy+4+x^{-1}y^{-1}.$

4. $(x^0-y^m)^2=(1-y^m)^2=1-2y^m+y^{2m}.$

5. $\left(\dfrac{1}{m^{-1}}-\dfrac{m^{-1}}{1}\right)^2=\dfrac{1}{m^{-2}}-2+\dfrac{m^{-2}}{1}=m^2-2+\dfrac{1}{m^2}.$

Cubes:

4. $\left(\frac{1}{2}x^{-\frac{1}{2}}-2x^{\frac{1}{4}}\right)^3=\frac{1}{8}x^{-\frac{3}{2}}-\frac{3}{2}x^{-\frac{3}{4}}+6-8x^{\frac{3}{4}}.$

5. $(\sqrt[3]{x}-\sqrt[3]{y})^3=\left(x^{\frac{1}{3}}-y^{\frac{1}{3}}\right)^3=x-3x^{\frac{2}{3}}y^{\frac{1}{3}}+3x^{\frac{1}{3}}y^{\frac{2}{3}}-y.$

Page 159.

1. $\dfrac{a^{2n}-b^{2n}}{a^n+b^n}=a^n-b^n.$

3. $(a^m)^{m-\frac{1}{m}}=(a^m)^{\frac{m^2-1}{m}}=a^{m^2-1}.$

2. $\sqrt[3]{\dfrac{1}{x^3y^2}}\left(x^2y^{\frac{2}{3}}\right)=\dfrac{1}{xy^{\frac{2}{3}}}\left(x^2y^{\frac{2}{3}}\right)=x.$

4. $\dfrac{x^{a+b}x^{a-b}x^{c-2a}}{x^{c-a}}=x^{a+b+a-b+c-2a-c+a}=x^a.$

5. $\left(a^2b^{-\frac{1}{2}}\right)^{-\frac{1}{4}}=a^{-\frac{1}{2}}b^{\frac{1}{8}}.$

6. $\dfrac{(a+x)^{\frac{1}{3}}\sqrt{b}}{\sqrt{(a+x)^{\frac{1}{2}}}}=\dfrac{(a+x)^{\frac{1}{3}}b^{\frac{1}{2}}}{(a+x)^{\frac{1}{4}}}=\dfrac{(a+x)^{\frac{4}{12}}b^{\frac{6}{12}}}{(a+x)^{\frac{3}{12}}}=\dfrac{\sqrt[12]{(a+x)^4b^6}}{\sqrt[12]{(a+x)^3}}=\sqrt[12]{(a+x)b^6}.$

7. $\left(\dfrac{1}{x^3y^2}\right)^{\frac{1}{6}}.(x^2)^{\frac{1}{5}}.y^{\frac{1}{3}}=\dfrac{1}{x^{\frac{1}{2}}y^{\frac{1}{3}}}.x^{\frac{2}{5}}.y^{\frac{1}{3}}=\dfrac{x^{\frac{2}{5}}}{x^{\frac{1}{2}}}=\dfrac{x^{\frac{4}{10}}}{x^{\frac{5}{10}}}=x^{-\frac{1}{10}}.$

8. $\left\{-(a^3)^{\frac{1}{2}}\right\}^{-\frac{1}{3}}.\left\{-(-a)^{-3}\right\}^{\frac{1}{2}}=-a^{-\frac{1}{2}}.a^{-\frac{3}{2}}=-a^{-\frac{1}{2}-\frac{3}{2}}=-a^{-2}.$

Page 160.

9. $\left(-(x^3)^{\frac{1}{2}}\right)^{-\frac{1}{3}}=-x^{-\frac{1}{2}}.$

10. $[\{(a^{-m})^{-n}\}^{-p}]\div a^{2m}=a^{-mnp}\div a^{2m}=a^{mnp}.$

11. $\left(a^2b^{-\frac{1}{2}}c^{\frac{2}{3}}\right)^{-\frac{1}{4}} = a^{-\frac{1}{2}}b^{\frac{1}{8}}c^{-\frac{1}{6}}$.

16.
$$\begin{array}{l|l} x^{-1}-4x^{-\frac{1}{2}}y^{\frac{1}{2}}+10y-12x^{\frac{1}{2}}y^{\frac{3}{2}}+9xy^2 & x^{-\frac{1}{2}}-2y^{\frac{1}{2}}+3x^{\frac{1}{2}}y \\ \underline{x^{-1}} & \\ -4x^{-\frac{1}{2}}y^{\frac{1}{2}}+10y-12x^{\frac{1}{2}}y^{\frac{3}{2}}+9xy^2 & \\ \underline{-4x^{-\frac{1}{2}}y^{\frac{1}{2}}+4y} & \\ 6y-12x^{\frac{1}{2}}y^{\frac{3}{2}}+9xy^2 & \\ \underline{6y-12x^{\frac{1}{2}}y^{\frac{3}{2}}+9xy^2} & \\ 0. & \end{array}$$

17.
$$\begin{array}{l|l} 9a^{-3}-30a^{-\frac{5}{2}}b+13a^{-2}b^2+20a^{-\frac{3}{2}}b^3+4a^{-1}b^4 & 3a^{-\frac{3}{2}}- \\ \underline{9a^{-3}} & 5a^{-1}b- \\ -30a^{-\frac{5}{2}}b+13a^{-2}b^2+20a^{-\frac{3}{2}}b^3+4a^{-1}b^4 & 2a^{-\frac{1}{2}}b^2 \\ \underline{-30a^{-\frac{5}{2}}b+25a^{-2}b^2} & \\ -12a^{-2}b^2+20a^{-\frac{3}{2}}b^3+4a^{-1}b^4 & \\ \underline{-12a^{-2}b^2+20a^{-\frac{3}{2}}b^3+4a^{-1}b^4} & \\ 0. & \end{array}$$

18.
$$\begin{array}{l|l} a^{-\frac{3}{2}}-3a^{-1}b^{\frac{2}{3}}+3a^{-\frac{1}{2}}b^{\frac{4}{3}}-b^2 & a^{-\frac{1}{2}}-b^{\frac{2}{3}} \\ \underline{a^{-\frac{3}{2}}} & \\ -3a^{-1}b^{\frac{2}{3}}+3a^{-\frac{1}{2}}b^{\frac{4}{3}}-b^2 & \\ \underline{-3a^{-1}b^{\frac{2}{3}}+3a^{-\frac{1}{2}}b^{\frac{4}{3}}-b^2} & \\ 0. & \end{array}$$

19.
$$\begin{array}{l|l} x-3x^{\frac{2}{3}}+9x^{\frac{1}{3}}-13+8x^{-\frac{1}{3}}-12x^{-\frac{2}{3}}+8x^{-1} & x^{\frac{1}{3}}-1+2x^{-\frac{1}{3}} \\ \underline{x} & \\ -3x^{\frac{2}{3}}+9x^{\frac{1}{3}}-13+8x^{-\frac{1}{3}}-12x^{-\frac{2}{3}}+8x^{-1} & \\ \underline{-3x^{\frac{2}{3}}+3x^{\frac{1}{3}}-1} & \\ 6x^{\frac{1}{3}}-12+8x^{-\frac{1}{3}}-12x^{-\frac{2}{3}}+8x^{-1} & \\ \underline{6x^{\frac{1}{3}}-12+8x^{-\frac{1}{3}}-12x^{-\frac{2}{3}}+8x^{-1}} & \\ 0. & \end{array}$$

20. $a-b=(\sqrt{a}+\sqrt{b})\ (\sqrt{a}-\sqrt{b})$ or $\left(a^{\frac{1}{2}}+b^{\frac{1}{2}}\right)\left(a^{\frac{1}{2}}-b^{\frac{1}{2}}\right)$.

21. $c^{-1}-d^{-1}=\left(c^{-\frac{1}{2}}+d^{-\frac{1}{2}}\right)\ \left(c^{-\frac{1}{2}}-d^{-\frac{1}{2}}\right)$.

22. $m^{\frac{1}{3}}-n^{\frac{1}{3}}=\left(m^{\frac{1}{6}}+n^{\frac{1}{6}}\right)\left(m^{\frac{1}{6}}-n^{\frac{1}{6}}\right)$.

23. $p^m-q^n=\left(p^{\frac{m}{2}}q+^{\frac{n}{2}}\right)\ \left(p^{\frac{m}{2}}-q^{\frac{n}{2}}\right)$.

24. $m^{-a}-n^{-b}=\left(m^{-\frac{a}{2}}+n^{-\frac{b}{2}}\right)\left(m^{-\frac{a}{2}}-n^{-\frac{b}{2}}\right)$.

25. $m+m^{\frac{1}{2}}n^{\frac{1}{2}}+n=\left(m^{\frac{1}{2}}+m^{\frac{1}{4}}n^{\frac{1}{4}}+n^{\frac{1}{2}}\right)\left(m^{\frac{1}{2}}-m^{\frac{1}{4}}n^{\frac{1}{4}}+n^{\frac{1}{2}}\right)$.

Page 161.

26. $a^{-4}+a^{-2}b^{-2}+b^{-4}=(a^{-4}+2a^{-2}b^{-2}+b^{-4})-a^{-2}b^{-2}=$
$(a^{-2}+b^{-2})^2-a^{-2}b^{-2}=(a^{-2}+a^{-1}b^{-1}+b^{-2})\ (a^{-2}-a^{-1}b^{-1}+b^{-2})$.

27. $a^2+ab+b^2=(a^2+2ab+b^2)-ab=(a+b)^2-ab=$
$(a+\sqrt{ab}+b)\ (a-\sqrt{ab}+b)$.

28. $x^3-8x^{-1}=\left(x-2x^{-\frac{1}{3}}\right)\left(x^2+2x^{\frac{2}{3}}+4x^{-\frac{2}{3}}\right)$.

29. $a^{\frac{6}{5}}+a^{\frac{3}{5}}b^{\frac{3}{5}}+b^{\frac{1}{5}}=\left(a^{\frac{3}{5}}+a^{\frac{3}{10}}b^{\frac{3}{10}}+b^{\frac{3}{5}}\right)\left(a^{\frac{3}{5}}+a^{\frac{3}{10}}b^{\frac{3}{10}}+b^{\frac{3}{5}}\right)$.

Page 162.

1. $\sqrt{288}=\sqrt{144x2}=12\sqrt{2}$.

3. $\sqrt[3]{256}=\sqrt[3]{64\times4}=4\sqrt[3]{4}$.

5. $3\sqrt{150}=3\sqrt{25\times6}=3\times5\sqrt{6}=15\sqrt{6}$.

7. $\sqrt[3]{-108x^4y^3}=\sqrt[3]{-27x^3y^3.4x}=-3xy\sqrt[3]{4x}$.

9. $\sqrt[mn]{x^{2mn}y^{3n}}=x^{\frac{2}{mn}}\sqrt[m]{y^{3n}}=x^2\sqrt[m]{y^3}$.

11. $\sqrt[3]{250x^2y^3z^7}=\sqrt[3]{125y^3z^6.2x^2z}=5yz^2\sqrt[3]{2x^2z}$.

13. $\sqrt{20a^2+60a+45}=\sqrt{5(4a^2+12a+9)}=(2a+3)\sqrt{5}$.

14. $\sqrt[p]{x^{a+p}x^{2p}}=y^2\sqrt[p]{x^{a+p}}$.

15. $\sqrt[3]{\frac{3}{4}}=\sqrt[3]{\frac{6}{8}}=\sqrt[3]{\frac{1}{8}\times6}=\frac{1}{2}\sqrt[3]{6}$.

16. $\sqrt{\frac{1}{2}}=\sqrt{\frac{2}{4}}=\sqrt{\frac{1}{4}\times2}=\frac{1}{2}\sqrt{2}$.

17. $\sqrt{\frac{1}{3}}=\sqrt{\frac{3}{9}}=\sqrt{\frac{1}{9}\times3}=\frac{1}{3}\sqrt{3}$.

18. $\sqrt[3]{\frac{5}{9}}=\sqrt[3]{\frac{15}{27}}=\sqrt[3]{\frac{1}{27}\times15}=\frac{1}{3}\sqrt{15}$.

19. $\sqrt[3]{\frac{77}{8}}=\sqrt[3]{\frac{1}{8}\times77}=\frac{1}{2}\sqrt[3]{77}$.

20. $\sqrt{\frac{20c^2}{9a^2b}}=\sqrt{\frac{4c^2.5}{9a^2.b}}=\frac{2c}{3a}\sqrt{\frac{5}{b}}$.

21. $\sqrt[3]{\frac{27x^4}{a^2}}=\sqrt[3]{\frac{27x^3.x}{a^2}}=3x\sqrt[3]{\frac{x}{a^2}}=3x\sqrt[3]{\frac{ax}{a^3}}=\frac{3x}{a}\sqrt[3]{ax}$.

22. $\sqrt[4]{\frac{b^4}{16a^3}}=\sqrt[4]{\frac{b^4}{16.a^3}}=\frac{b}{2}\sqrt[4]{\frac{1}{a^3}}=\frac{b}{2}\sqrt[4]{\frac{a}{a^4}}=\frac{b}{2a}\sqrt[4]{a}$.

23. $\sqrt[2n+1]{\frac{x^{2n+1}}{y^3}}=\sqrt[2n+1]{\frac{(x^{2n+1})\ (y^{2n-2})}{y^{2n+1}}}=\frac{x}{y}\sqrt[2n+1]{y^{2n-2}}$

24. $\frac{2}{5}\sqrt{12\frac{1}{2}}=\frac{2}{5}\sqrt{\frac{25}{2}}=\frac{2}{5}\sqrt{\frac{25}{4}\times2}=\frac{2}{5}\times\frac{5}{2}\sqrt{2}=\sqrt{2}$.

Page 163.

1. $7\sqrt{2}=\sqrt{49\times2}=\sqrt{98}$.

5. $\frac{2}{7}\sqrt{\frac{7}{9}}=\sqrt{\frac{4}{49}\times\frac{7}{9}}=\sqrt{\frac{28}{441}}=\sqrt{\frac{4}{63}}$.

6. $\frac{2xy}{c}\sqrt{\frac{10c^2}{9x^2y}}=\sqrt{\frac{4x^2y^2}{c^2}}\times\sqrt{\frac{10c^2}{9x^2y}}=\sqrt{\frac{40c^2x^2y^2}{9c^2x^2y}}=\sqrt{\frac{40y}{9}}$.

8. $\frac{x}{a^2}\sqrt{\frac{3a^3}{x}} = \sqrt{\frac{x^2}{a^4} \times \frac{3a^3}{x}} = \sqrt{\frac{3x}{a}}$.

10. $\frac{2x}{y}\sqrt[4]{\frac{y^4}{8x^3}} = \sqrt{\frac{16x^4}{7^4} \times \frac{y^4}{8x^3}} = \sqrt[4]{\frac{2x}{1}}$.

12. $(x+y)\sqrt{\frac{x-y}{x+y}} = \sqrt{\frac{(x+y)^2\,(x-y)}{x+y}} = \sqrt{\frac{x^2-y^2}{1}}$.

Page 164.

1. $3\sqrt{45}+7\sqrt{5}=9\sqrt{5}+7\sqrt{5}=16\sqrt{5}$.

3. $4\sqrt{63}+5\sqrt{7}=12\sqrt{7}+5\sqrt{7}=17\sqrt{7}$.

6. $\sqrt{44}-5\sqrt{176}+2\sqrt{99}=2\sqrt{11}-20\sqrt{11}+6\sqrt{11}=-12\sqrt{11}$.

7. $2\sqrt{363}-5\sqrt{243}+\sqrt{192}=22\sqrt{3}-45\sqrt{3}+8\sqrt{3}=-15\sqrt{3}$.

9. $3\sqrt[3]{875}-7\sqrt[3]{56}=15\sqrt[3]{7}-14\sqrt[3]{7}=\sqrt[3]{7}$.

11. $\sqrt{\frac{3}{5}}-\sqrt{\frac{5}{27}}=\sqrt{\frac{15}{25}}-\sqrt{\frac{15}{81}}=\sqrt{\frac{1}{25}.15}-\sqrt{\frac{1}{81}.15}=\frac{1}{5}\sqrt{15}-\frac{1}{9}\sqrt{15}=\frac{4}{45}\sqrt{15}$.

12. $4\sqrt{\frac{7}{3}}+12\sqrt{\frac{3}{4}}=4\sqrt{\frac{1}{9}\times 21}+12\sqrt{\frac{1}{4}\times 3}=\frac{4}{3}\sqrt{21}+6\sqrt{3}$.

Page 165.

1. $2\sqrt{15}\times 3\sqrt{5}=6\sqrt{75}=6\sqrt{25x3}=30\sqrt{3}$.

3. $\sqrt{x+2}\times\sqrt{x-2}=\sqrt{x^2-4}$.

6. $\frac{3}{x}\sqrt{\frac{a^2}{x}} \times \frac{4}{3}\sqrt{\frac{x^3}{2a^4}} = \frac{4}{x}\sqrt{\frac{a^2x^2}{2a^4x}} = \frac{4}{x}\sqrt{\frac{x^2}{2a^2}} = \frac{4}{a}\sqrt{\frac{1}{2}} = \frac{4}{a}\sqrt{\frac{1x2}{4}} = \frac{4}{2a}\sqrt{2} = \frac{2}{a}\sqrt{2}$.

7. $5\sqrt[5]{3}\times 5\sqrt{2}=25.3^{\frac{1}{5}}.2^{\frac{1}{2}}=25.3^{\frac{2}{10}}.2^{\frac{5}{10}}=25.9^{\frac{1}{10}}.32^{\frac{1}{10}}=25\sqrt[10]{288}$.

8. $\sqrt{3}\times\sqrt[3]{2}\times\sqrt[3]{\frac{1}{6}}=3^{\frac{1}{2}}\times 2^{\frac{1}{3}}\times(\frac{1}{6})^{\frac{1}{3}}=3^{\frac{3}{6}}\times 2^{\frac{2}{6}}\times(\frac{1}{6})^{\frac{2}{6}}=27^{\frac{1}{6}}\times 4^{\frac{1}{6}}\times(\frac{1}{36})^{\frac{1}{6}}=(\frac{108}{36})^{\frac{1}{6}}=\sqrt[6]{3}$.

9. $\sqrt[3]{\frac{1}{2}}\times\sqrt{\frac{3}{4}}\times\sqrt[3]{\frac{3}{4}}=(\frac{1}{2})^{\frac{1}{3}}\times(\frac{1}{2})^{\frac{1}{2}}=(\frac{1}{2})^{\frac{2}{6}}\times(\frac{1}{2})^{\frac{3}{6}}=(\frac{1}{4})^{\frac{1}{6}}\times(\frac{1}{8})^{\frac{1}{6}}=(\frac{1}{32})^{\frac{1}{6}}=\sqrt[6]{\frac{1}{32}}=\sqrt[6]{\frac{2}{64}}=\frac{1}{2}\sqrt[6]{2}$.

Page 166.

1. $(\sqrt{x+y}-1)\times\sqrt{x+y}=x+y-\sqrt{x+y}$.

2. $2\sqrt{7}-4\sqrt{3}$

$\sqrt{7}+5\sqrt{3}$

$2\times 7-4\sqrt{21}$

$10\sqrt{21}-20\times 3$

$14+6\sqrt{21}-60=6\sqrt{21}-46$.

3. $2\sqrt{x+1}+\sqrt{x}$

$\underline{\sqrt{x+1}-2\sqrt{x}}$

$2(x+1)+\sqrt{x^2+x}$

$\underline{-4\sqrt{x^2+x}-2x}$

$2x+2\quad -3\sqrt{x^2+x}-2x=2-3\sqrt{x^2+x}.$

5. $(a-\sqrt{a+x}+\sqrt{a})\ (\sqrt{a+x}-\sqrt{a+x})=(a-\sqrt{a+x}+\sqrt{a})\times 0=0.$

Divisions:

1. $\sqrt{54}\div\sqrt{3}=\sqrt{18}=\sqrt{9\times 2}=3\sqrt{2}.$

3. $\sqrt{18a^6}\div\sqrt{6a^2}=\sqrt{3a^4}=a^2\sqrt{3}.$

5. $\sqrt[3]{36}\div\sqrt{12}=36^{\frac{1}{3}}\div 12^{\frac{1}{2}}=36^{\frac{2}{6}}\div 12^{\frac{3}{6}}=1296^{\frac{1}{6}}\div 1728^{\frac{1}{6}}=\sqrt[6]{\frac{3}{4}}=$

$\sqrt[6]{\frac{4}{6}\frac{8}{4}}=\sqrt[6]{\frac{1}{8}\frac{1}{4}\times 48}=\frac{1}{2}\sqrt[6]{48}.$

6. $\sqrt[5]{4}\div\sqrt[4]{6}=4^{\frac{1}{5}}\div 6^{\frac{1}{4}}=4^{\frac{4}{20}}\div 6^{\frac{5}{20}}=256^{\frac{1}{20}}\div 7776^{\frac{1}{20}}=\sqrt[20]{\frac{8}{243}}.$

8. $\sqrt[3]{8a}\div\sqrt[4]{4a}=(8a)^{\frac{1}{3}}\div(4a)^{\frac{1}{4}}=(8a)^{\frac{4}{12}}\div(4a)^{\frac{3}{12}}=$

$\sqrt[12]{\frac{4096a^4}{64a^3}}=\sqrt[12]{64a}.$

9. $\sqrt[3]{6a^2b}\div\sqrt[5]{12a^3b^2}=(6a^3b)^{\frac{1}{3}}\div(12a^3b^2)^{\frac{1}{5}}=$

$(6a^3b)^{\frac{5}{15}}\div(12a^3b^2)^{\frac{3}{15}}=\sqrt[15]{\frac{7776a^{10}b^5}{1728a^9b^6}}=\sqrt[15]{\frac{9a}{2b}}=$

$\frac{1}{2b}\sqrt[15]{9a\times(2b)^{14}}.$

10. $\frac{\sqrt[6]{24a^3b^2c^5}}{\sqrt[4]{4a^2bc^3}}=\frac{\sqrt[12]{576a^6b^4c^{10}}}{\sqrt[12]{64a^6b^3c^9}}=\sqrt[12]{9bc}.$

Page 167.

1. $(\sqrt[3]{6})^5=\left(6^{\frac{1}{3}}\right)^5=6^{\frac{5}{3}}=\sqrt[3]{7776}=\sqrt[3]{216\times 36}=6\sqrt[3]{36}.$

3. $(\sqrt[3]{x^2a})^6=\left((x^2a)^{\frac{1}{3}}\right)^6\left(x^{\frac{2}{3}}a^{\frac{1}{3}}\right)^6=x^4a^2.$

5. $(\sqrt[4]{x}+y)^2=(x^{\frac{1}{4}}+y)^2=x^{\frac{1}{2}}+2x^{\frac{1}{4}}y+y^2=\sqrt{x}+2\sqrt[4]{x}y+y^2.$

7. $(\sqrt{m}+\sqrt{n})^2=m+2\sqrt{mn}+n.$

9. $(\sqrt[3]{1}-\sqrt[3]{2})^3=1-3\sqrt[3]{2}+3\sqrt[3]{4}-2=-1-3\sqrt[3]{2}+3\sqrt[3]{4}.$

11. $(\frac{1}{16}\sqrt[3]{256})^{\frac{1}{4}}=(\frac{1}{16})^{\frac{1}{4}}.(256)^{\frac{1}{12}}=\frac{1}{2}\sqrt[12]{256}=\frac{1}{2}\sqrt[4\times 3]{256}=\frac{1}{2}\sqrt[3]{4}.$

12. $\sqrt[3]{125^{\frac{1}{2}}}=5^{\frac{1}{2}}=\sqrt{5}.$

14. $\sqrt[3]{\sqrt[5]{27a^3}}=\sqrt[5]{3a}.$

15. $\sqrt[m]{\sqrt[n]{a^m}}=\sqrt[n]{a}.$

16. $\sqrt{\frac{2}{\sqrt[3]{2}}}=\frac{2^{\frac{1}{2}}}{2^{\frac{1}{6}}}=\frac{2^{\frac{3}{6}}}{2^{\frac{1}{6}}}=2^{\frac{1}{3}}=\sqrt[3]{2}.$

17. $\sqrt[3]{27a^3\sqrt{\frac{x^3}{y^6}}} = 3a\sqrt[6]{\frac{x^3}{y^6}} = 3a\frac{\sqrt{x}}{y} = \frac{3a}{y}\sqrt{x}.$

18. $\sqrt[4]{81\sqrt[3]{x^2}} = 3\sqrt[12]{x^2} = 3x^{\frac{2}{3\cdot 6}} = 3x^{\frac{1}{18}} = 3\sqrt[18]{x}.$

19. $\sqrt{8a^3x\sqrt[3]{(a-x)^2}} = \sqrt[6]{512a^9x^3(a-x)^2} = 2a\sqrt[6]{8a^3x^3(a-x)^2}.$

20. $\sqrt[3]{8x^4\sqrt{(x^2-y^2)^3}} = \sqrt[6]{64x^8(x^2-y^2)^3} = \sqrt[6]{2^6x^6.x^2(x^2-y^2)^3} =$
$2x\sqrt[6]{x^2(x^2-y^2)^3}.$

Page 168.

1. $\frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3} = \frac{1}{3}\sqrt{3}.$

3. $\frac{5}{4\sqrt[3]{4}} = \frac{5}{4\sqrt[3]{4}} \times \frac{\sqrt[3]{4}}{\sqrt[3]{4}} \times \frac{\sqrt[3]{4}}{\sqrt[3]{4}} = \frac{5\sqrt[3]{16}}{4\sqrt[3]{4^3}} = \frac{5\times 2\sqrt[3]{2}}{4\times 4} = \frac{10}{16}\sqrt[3]{2} = \frac{5}{8}\sqrt[3]{2}.$

5. $\frac{n}{b\sqrt{m}} = \frac{n}{b\sqrt{m}} \times \frac{\sqrt{m}}{\sqrt{m}} = \frac{n\sqrt{m}}{bm} = \frac{n}{bm}\sqrt{m}.$

6. $\frac{6}{\sqrt[4]{10d^3}} = \frac{6}{\sqrt[4]{10d^3}} \times \frac{\sqrt[4]{10d^3}}{\sqrt[4]{10d^3}} \times \frac{\sqrt[4]{10d^3}}{\sqrt[4]{30d^3}} \times \frac{\sqrt[4]{10d^3}}{\sqrt[4]{10d^3}} = \frac{6\sqrt[4]{1000d^9}}{\sqrt[4]{1000d^{12}}} =$
$\frac{6d^2\sqrt[4]{1000d}}{10d^3} = \frac{3\sqrt[4]{1000d}}{5d}.$

7. $\frac{3}{\sqrt[5]{9c^4}} = \frac{3}{\sqrt[5]{9c^4}} \times \frac{\sqrt[5]{9c^4}}{\sqrt[5]{9c^4}} \times \frac{\sqrt[5]{9c^4}}{\sqrt[5]{9c^4}} \times \frac{\sqrt[5]{9c^4}}{\sqrt[5]{9c^4}} \times \frac{\sqrt[5]{9c^4}}{\sqrt[5]{9c^4}} = \frac{3\sqrt[5]{6561c^{16}}}{\sqrt[5]{9^5c^{20}}} =$
$\frac{3\sqrt[5]{6561c^{16}}}{9c^4} = \frac{3\sqrt[5]{243\times 27c^{15}c}}{9c^4} = \frac{9c^3\sqrt[5]{27c}}{9c^4} = \frac{1}{c}\sqrt[5]{27c}.$

9. $\frac{\sqrt{1-a}}{\sqrt[3]{(1-a)^2}} = \frac{\sqrt{1-a}}{\sqrt[3]{(1-a)^2}} \times \frac{\sqrt[3]{(1-a)^2}}{\sqrt[3]{(1-a)^2}} \times \frac{\sqrt[3]{(1-a)^2}}{\sqrt[3]{(1-a)^2}} =$
$\frac{\sqrt{1-a}\,(\sqrt[3]{(1-a)^4})}{(1-a)^2} = \frac{\sqrt{1-a}\,(1-a)\sqrt[3]{1-a}}{(1-a)^2} = \frac{\sqrt{1-a}}{1-a}\sqrt[3]{1-a} =$
$\frac{\sqrt[6]{(1-a)^5}}{1-a}.$

Page 169.

1. $\frac{3}{3+\sqrt{2}} = \frac{3(3-\sqrt{2})}{(4+\sqrt{2})(3-\sqrt{2})} = \frac{3(3-\sqrt{2})}{9-2} = \frac{3(3-\sqrt{2})}{7}.$

3. $\frac{3+\sqrt{3}}{3-\sqrt{3}} = \frac{(3+\sqrt{3})(3+\sqrt{3})}{(3-\sqrt{3})(3+\sqrt{3})} = \frac{(3+\sqrt{3})(3+\sqrt{3})}{9-3} = \frac{9+6\sqrt{3}+3}{6} =$
$\frac{12+6\sqrt{3}}{6} = 2+\sqrt{3}.$

5. $\frac{2\sqrt{2}+2}{2\sqrt{2}-2} = \frac{(2\sqrt{2}+2)\ (2\sqrt{2}+2)}{(2\sqrt{2}-2)\ (2\sqrt{2}+2)} = \frac{8+8\sqrt{2}+4}{8-4} =$

$\frac{12+8\sqrt{2}}{4} = 3+2\sqrt{2}.$

6. $\frac{\sqrt{7}-1}{\sqrt{7}+1} = \frac{(\sqrt{7}-1)\ (\sqrt{7}-1)}{(\sqrt{7}+1)\ (\sqrt{7}-1)} = \frac{7-2\sqrt{7}+1}{7-1} = \frac{8-2\sqrt{7}}{6} = \frac{4-\sqrt{7}}{3}.$

8. $\frac{\sqrt{m-n}+\sqrt{n}}{\sqrt{m-n}-\sqrt{n}} = \frac{\sqrt{m-n}+\sqrt{n}}{\sqrt{m-n}-\sqrt{n}} \times \frac{\sqrt{m-n}+\sqrt{n}}{\sqrt{m-n}+\sqrt{n}} =$

$\frac{m-n+2\sqrt{n(m-n)}+n}{m-n-n} = \frac{m+2\sqrt{n(m-n)}}{m-2n}.$

9. $\frac{\sqrt{2x-1}+\sqrt{x+1}}{\sqrt{2x-1}-\sqrt{x+1}} = \frac{(\sqrt{2x-1}+\sqrt{x+1})\ (\sqrt{2x-1}+\sqrt{x+1})}{(\sqrt{2x-1}-\sqrt{x+1})\ (\sqrt{2x-1}+\sqrt{x+1})} =$

$\frac{2x-1+2\sqrt{(2x-1)\ (x+1}+x+1)}{2x-1-x-1} = \frac{3x+2\sqrt{(2x-1)\ (x+1)}}{x-2}$

10. $\frac{\sqrt{a^2-1}-\sqrt{a^2+1}}{\sqrt{a^2-1}+\sqrt{a^2+1}} = \frac{\sqrt{a^2-1}-\sqrt{a^2+1}}{\sqrt{a^2-1}+\sqrt{a^2+1}} \times \frac{\sqrt{a^2-1}-\sqrt{a^2+1}}{\sqrt{a^2-1}-\sqrt{a^2+1}} =$

$\frac{a^2-1-2\sqrt{(a^2-1)\ (a^2+1)}+a^2+1}{a^2-1-a^2-1} = \frac{2a^2-2\sqrt{a^4-1}}{-2} =$

$\sqrt{a^4-1}-a^2.$

1. $\frac{3}{\sqrt{3}+\sqrt{2}} = \frac{3\ (\sqrt{3}-\sqrt{2})}{(\sqrt{3}+\sqrt{2})\ (\sqrt{3}-\sqrt{2})} = \frac{3\ (\sqrt{3}-\sqrt{2})}{3-2}$

$3\ (\sqrt{3}-\sqrt{2}) = 3\ (1.73\ .\ .\ .\ -1.41\ .\ .\ .\ ) = 3(.31)\ .\ .\ .$

$.93\ .\ .\ .$

3. $\frac{4}{\sqrt{3}-1} = \frac{4\ (\sqrt{3}+1)}{(\sqrt{3}-1)\ (\sqrt{3}+1)} = \frac{4\ (\sqrt{3}+1)}{3-1} = \frac{4\ (\sqrt{3}+1)}{2} =$

$2\ (\sqrt{3}+1) = 2\ (1.732\ .\ .\ .\ +1) = 2\times 2.732\ .\ .\ .\ = 5.464\ .\ .\ .$

4. $\frac{7}{\sqrt[3]{7}} = \frac{7.\sqrt[3]{7}.\sqrt[3]{7}}{\sqrt[3]{7}.\sqrt[3]{7}.\sqrt[3]{7}} = \frac{7\sqrt[3]{49}}{7} = \sqrt[3]{49} = 3.66\ .\ .\ .$

Page 170.

7. $\frac{3+2\sqrt{7}}{4\sqrt{7}-5} = \frac{(3+2\sqrt{7})\ (4\sqrt{7}+5)}{(4\sqrt{7}-5)\ (4\sqrt{7}+5)} = \frac{22\sqrt{7}+71}{112-25} =$

$\frac{22\sqrt{7}+71}{87} = 1.4851.$

8. $\dfrac{\sqrt{6}}{\sqrt{2.5}+\sqrt{6}} = \dfrac{\sqrt{6}\,(\sqrt{2.5}-\sqrt{6})}{(\sqrt{2.5}+\sqrt{6})\,(\sqrt{2.5}-\sqrt{6})} = \dfrac{\sqrt{15}-6}{2.5-6} =$

$\dfrac{-2.127}{-3.5} = .608\ldots$

Page 171.

1. $\sqrt{-25} = \sqrt{25}\,(-1) = 5\sqrt{-1}$

$\sqrt{-81} = \sqrt{81}\,(-1) = 9\sqrt{-1}$

$14\sqrt{-1}.$

3. $\sqrt{-49} = \sqrt{49}\,(-1) = 7\sqrt{-1}$

$\sqrt{-144} = \sqrt{144}\,(-1) = 12\sqrt{-1}$

$19\sqrt{-1}.$

5. $\sqrt{-(a+b)^2} = \sqrt{(a+b)^2}\,(-1) = (a+b)\sqrt{-1}$

$\sqrt{-(a-b)^2} = \sqrt{(a-b)^2}\,(-1) = (a-b)\sqrt{-1}$

$2a\sqrt{-1}.$

9. $\sqrt{-243m^2} = \sqrt{243m^2}\,(-1) = 9m\sqrt{-3}$

$\sqrt{-75m^2} = \sqrt{75m^2}\,(-1) = 5m\sqrt{-3}$

$4m\sqrt{-3}.$

10. $\sqrt{-a^2-2ab-b^2} = \sqrt{(a^2+2ab+b^2)}\,(-1) = (a+b)\sqrt{-1}$

$\sqrt{-a^2+2ab-b^2} = \sqrt{a^2-2ab+b^2}\,(-1) = (a-b)\sqrt{-1}$

$2b\sqrt{-1}.$

Page 172.

1. $5\sqrt{-2} = 5\sqrt{2}\,(-1)$

$3\sqrt{-2} = 3\sqrt{2}\,(-1)$

$5\times3\times2\,(-1) = -30.$

2. $\sqrt{-b^2} = \sqrt{b^2}\,(-1)$

$\sqrt{-y^2} = \sqrt{y^2}\,(-1)$

$by\,(-1) = -by.$

3. $-4\sqrt{-x} = -4\sqrt{x}\,(-1)$

$5\sqrt{-y} = 5\sqrt{y}\,(-1)$

$-20\sqrt{xy}\,(-1) = 20\sqrt{xy}.$

5. $\left.\begin{array}{l}\sqrt{-a^2} = \sqrt{a^2}\,(-1)\\ \sqrt{-b^2} = \sqrt{b^2}\,(-1)\end{array}\right\} = ab\,(-1) = -ab$

$-ab\times\sqrt{c^2} = -abc.$

6. $\left.\begin{matrix}\sqrt{-4}=\sqrt{4(-1)}\\ \sqrt{-5}=\sqrt{5(-1)}\end{matrix}\right\}=\sqrt{20}\times-1=-\sqrt{20}$

$\sqrt{-6}=\sqrt{6(-1)}$, and

$$-\sqrt{20}\times\sqrt{6(-1)}=-\sqrt{120(-1)}=-2\sqrt{-30}.$$

7.
$$\begin{array}{r} 2-3\sqrt{-2}\\ 4+\sqrt{-2}\\ \hline 8-12\sqrt{-2}\\ \quad 2\sqrt{-2}+6\\ \hline 14-10\sqrt{-2}. \end{array}$$

8.
$$\begin{array}{r} 5+\sqrt{-8}\\ 9-3\sqrt{-8}\\ \hline 45+7\sqrt{-8}\\ \quad -15\sqrt{-8}-3(-8)\\ \hline 45-6\sqrt{-8}+24=69-6\sqrt{8(-1)}=69-12\sqrt{-2}. \end{array}$$

9.
$$\begin{array}{lll} 2\sqrt{-4}-4\sqrt{-3}=4\sqrt{-1}-4\sqrt{3(-1)} & & \\ \underline{5\sqrt{-4}+7\sqrt{-3}=10\sqrt{-1}+7\sqrt{3(-1)}} & & \\ -40 & 40(-1)-40(-1)\sqrt{3} & \\ & \underline{28(-1)\sqrt{3}-28x(-3)} & \\ -40 & -12(-1)\sqrt{3}+84=44+12\sqrt{3}. & \end{array}$$

10. $\sqrt{-1}\times2\sqrt{1}=-2$;

$-2\times\sqrt{25(-1)}=-10\sqrt{-1}$;

$-10\sqrt{-1}\times7\sqrt{-1}=-70\times(-1)=70.$

Expansion:

5.
$$\begin{array}{r} 2\sqrt{3}-\sqrt{5(-1)}\\ \underline{4\sqrt{3}-2\sqrt{5(-1)}}\\ 24-4\sqrt{15(-1)}-4\sqrt{15(-1)}+10(-1)=\\ 24-8\sqrt{-15}-10=14-8\sqrt{-15}. \end{array}$$

Page 173.

1. $\dfrac{\sqrt{-36}}{\sqrt{-9}}=\dfrac{\sqrt{36(-1)}}{\sqrt{9(-1)}}=\dfrac{6\sqrt{-1}}{3\sqrt{-1}}=2.$

2. $\dfrac{\sqrt{-9}}{\sqrt{-2\frac{1}{4}}}=\dfrac{\sqrt{9(-1)}}{\sqrt{2\frac{1}{4}(-1)}}=\sqrt{4}=2.$

4. $\dfrac{\sqrt{3}}{\frac{1}{2}\sqrt{5(-1)}}=2\sqrt{\frac{3}{5}}(-1)=\frac{2}{5}\sqrt{-15}.$

5. $1+\sqrt{-2}\,)1-\sqrt{-2}+4(1-2\sqrt{-2}$

$1+\sqrt{-2}$

$-2\sqrt{-2}+4$

$-2\sqrt{-2}+4$

$0.$

Page 175.

1. $\sqrt{9+2\sqrt{8}}=\sqrt{x}+\sqrt{y}$

$\sqrt{9-2\sqrt{8}}=\sqrt{x}-\sqrt{y}$

$\sqrt{81-32}\ =x-y$

$x-y=\sqrt{49}=7$

$9+2\sqrt{8}=x+2\sqrt{xy}+y$

$x+y=9$

$x-y=7$

$x+y=9$

$2x=16$

$x=8$

$2y=2$

$y=1$

$\sqrt{x}+\sqrt{y}=\sqrt{8}+\sqrt{1}=2\sqrt{2}+1.$

2. $\sqrt{3+2\sqrt{2}}=\sqrt{x}+\sqrt{y}$

$\sqrt{3-2\sqrt{2}}=\sqrt{x}-\sqrt{y}$

$\sqrt{9-8}\ =x-y$

$x-y=1$

$3-2\sqrt{2}=x+4\sqrt{xy}+y$

$x+y=3$

$x=2$

$y=1$

$\sqrt{x}+\sqrt{y}=\sqrt{2}+\sqrt{1}=\sqrt{2}+1.$

6. $\sqrt{1+\sqrt{1}}=\sqrt{x}+\sqrt{y}$

$\sqrt{1-\sqrt{1}}=\sqrt{x}-\sqrt{y}$

$1-1=x-y$

$x-y=0$

$x+y=1$

$x=\frac{1}{2}$

$y=\frac{1}{2}$

$\sqrt{x}+\sqrt{y}=\sqrt{\tfrac{1}{2}}+1=\tfrac{1}{2}\sqrt{2}+\sqrt{\tfrac{1}{2}}$

$\sqrt{x}+\sqrt{y}=\sqrt{\tfrac{1}{2}}+\sqrt{\tfrac{1}{2}}=\tfrac{1}{2}\sqrt{2}+\tfrac{1}{2}\sqrt{2}=\sqrt{2}.$

8. $\sqrt{75+12\sqrt{21}}=\sqrt{x}+\sqrt{y}$

$\sqrt{75-12\sqrt{21}}=\sqrt{x}-\sqrt{y}$

$\sqrt{5625-3024}\ =x-y$

$x-y=51$

$x+y=75$

$x=63$

$y=12$

$\sqrt{x}+\sqrt{y}=\sqrt{63}+\sqrt{12}=3\sqrt{7}+2\sqrt{3}.$

10. $\sqrt{4+\sqrt{15}}=\sqrt{x}+\sqrt{y}$

$\sqrt{4-\sqrt{15}}=\sqrt{x}-\sqrt{y}$

$\sqrt{16-15}=x-y$

$x-y=1$

$x+y=4$

$x=2\frac{1}{2}$

$y=1\frac{1}{2}$

$\sqrt{x}+\sqrt{y}=\sqrt{2\frac{1}{2}}+\sqrt{1\frac{1}{2}}=\sqrt{\frac{5}{2}}+\sqrt{\frac{3}{2}}=\frac{1}{2}\sqrt{10}+\frac{1}{2}\sqrt{6}.$

12. $\sqrt{-5+12\sqrt{-1}}=\sqrt{x}+\sqrt{y}$

$\sqrt{-5-12\sqrt{-1}}=\sqrt{x}-\sqrt{y}$

$\sqrt{25+144}=x-y$

$x-y=13$

$x+y=-5$

$x=4$

$y=-9$

$\sqrt{x}+\sqrt{y}=2+3\sqrt{-1}.$

14. $\sqrt{2m+2\sqrt{m^2-n^2}}=\sqrt{x}+\sqrt{y}$

$\sqrt{2m-2\sqrt{m^2-n^2}}=\sqrt{x}-\sqrt{y}$

$\sqrt{4m^2-4m^2+4n^2}=x-y$

$x-y=2n$

$x+y=2m$

$x=m+n$

$y=m-n$

$\sqrt{x}+\sqrt{y}=\sqrt{m+n}+\sqrt{m-n}.$

15. $\sqrt{2m-2\sqrt{m^2-x^2}}=\sqrt{x}-\sqrt{y}$

$\sqrt{2m+2\sqrt{m^2-x^2}}=\sqrt{x}+\sqrt{y}$

$\sqrt{4m^2-4m^2+4x^2}=x-y$

$x-y=2x$

$x+y=2m$

$x=x+m$

$y=x-m$

$\sqrt{x}-\sqrt{y}=\sqrt{x+m}-\sqrt{m-x}.$

Page 176.

1. $4\sqrt{x}-8=8$

$4\sqrt{x}=8+8=16$

$\sqrt{x}=4$

$x=16.$

3. $\sqrt{x+5}-\sqrt{x-1}=2$

$\sqrt{x+5}=2+\sqrt{x-1}$

$x+5=4+4\sqrt{x-1}+x-1$

$4\sqrt{x-1}=2$

$\sqrt{x-1}=\frac{1}{2}$

$x-1=\frac{1}{4}$

$x=\frac{1}{4}+\frac{4}{4}=\frac{5}{4}.$

6. $\sqrt{2x+\sqrt{x^4-x^2}}=x+1$

$2x+\sqrt{x^4-x^2}=x^2+2x+1$

$\sqrt{x^4-x^2}=x^2+1,$

$x^4-x^2=x^4+2x^2+1$

$3x^2=-1$

$x^2=-\frac{1}{3}$

$x=\sqrt{-\frac{1}{3}}=\frac{1}{3}\sqrt{-3}.$

9. $2+\sqrt{x+3}=\sqrt{x-2}+3$

$\sqrt{x+3}=\sqrt{x-2}+1$

$x+3=x-2+2\sqrt{x-2}+1$

$2\sqrt{x-2}=4$

$\sqrt{x-2}=2$

$x-2=4$

$x=6.$

11. $x-\frac{1}{2}\sqrt{x}=\sqrt{x^2-x}$

$x^2-x\sqrt{x}+\frac{1}{4}x=x^2-x$

$-4x\sqrt{x}+x=-4x$

$4x\sqrt{x}=4x+x=5x$

$4\sqrt{x}=5$

$\sqrt{x}=\frac{5}{4}$

$x=\frac{25}{16}.$

14. $\sqrt{x^2-5x+10}=x-1$

$x^2-5x+10=x^2-2x+1$

$3x=9$

$x=3.$

Page 177.

16. $\sqrt{b+x}+\sqrt{b-x}=c$

$b+x+2\sqrt{b^2-x^2}+b-x=c^2$

$2\sqrt{b^2-x^2}=c^2-2b$

$4x^2=4bc^2-c^4$

$x^2=\frac{4bc^2-c^4}{4}$

$x^2=\frac{c^2(4b-c^2)}{4}$

$x=\frac{c}{2}\sqrt{4b-c^2}.$

17. $\sqrt{8x+17}-\sqrt{2x}=\sqrt{2x+9}$

$8x+17-2\sqrt{16x^2+34x}+2x=2x+9$

$2\sqrt{16x^2+34x}=8x+8$

$16x^2+34x=(4x+4)^2=16x^2+32x+16$

$2x=16$

$x=8.$

19. $\sqrt{\{9+x\sqrt{x^2-3}\}}=x-3$

$9+x\sqrt{x^2-3}=x^2-6x+9$

$x\sqrt{x^2-3}=x^2-6x$

$\sqrt{x^2-3}=x-6$

$x^2-3=x^2-12x+36$

$12x=39$

$x=3\frac{1}{4}$.

20. $a+x=\sqrt{a^2+x\sqrt{b^2+x^2}}$

$a^2+2ax+x^2=a^2+x\sqrt{b^2+x^2}$

$2a+x=\sqrt{b^2+x^2}$

$4a^2+4ax+x^2=b^2+x^2$

$4ax=b^2-4a^2$

$x=\frac{b^2-4a^2}{4a}=$

$\frac{b^2}{4a}-a.$

Page 178.

1. $x^2-18=18$

$x^2=36$

$x=\pm 6.$

3. $7x^2+5=5x^2+55$

$2x^2=50$

$x^2=25$

$x=\pm 5.$

5. $\frac{x^2+1}{5}=10$

$x^2+1=50$

$x^2=49$

$x=\pm 7.$

7. $\frac{\sqrt{x}}{x(1-c)}=\frac{x}{\sqrt{x}}$

$\sqrt{x^2}=x^2(1-c)$

$x^2=x^4(1-c)^2$

$x^2(1-c)^2=1$

$x^2=\frac{1}{(1-c)^2}$

$x=\pm\frac{1}{1-c}.$

9. $\frac{6x^2-10}{4}=x^2-1$

$6x^2-10=4x^2-4$

$2x^2=6$

$x^2=3$

$x=\pm\sqrt{3}.$

11. $\frac{x-a}{x+a}+\frac{x+a}{x-a}=5$

$x^2-2ax+a^2+x^2+2ax+a^2=$

$5x^2-5a^2$

$3x^2=7a^2$

$x^2=\frac{7a^2}{3}$

$x=\pm\sqrt{\frac{7a^2}{3}}=\pm\frac{a}{3}\sqrt{21}.$

13. $\frac{2x^2-6}{2}-\frac{x^2-4}{4}-\frac{5x^2-10}{7}=0$

$28x^2-84-7x^2+28-20x^2+40=0$

$x^2=16$

$x=\pm 4.$

15. $x\sqrt{a+x^2}=b+x^2$

$x^2(a+x^2)=b^2+2bx^2+x^4$

$ax^2+x^4=b^2+2bx^2+x^4$

$ax^2+2bx^2=b^2$

$x^2(a-2b)=b^2$

$x^2=\frac{b^2}{a-2b}$

$x=\frac{\pm b}{\sqrt{a-2b}}.$

16. $x-a+\sqrt{x^2-2ax}=b$

$\sqrt{x^2-2ax}=b-x+a$

$x^2-2ax=$

$b^2+x^2+a^2-2bx+2ab-2ax$

$2bx=b^2+2ab+a^2=(a+b)^2$

$x=\frac{(a+b)^2}{2b}.$

Page 180.

4. $x^2-8x=20$

$x^2-8x+16=20+16=36$

$x-4=\pm 6$

$x=+6+4$ or $-6+4=10$ or -2.

6. $x^2+22x=48$

$x^2+22x+121=48+121=169$

$x+11=\pm 13$

$x=13-11$ or $-13-11=2$ or -24.

8. $(x+10)^2=28$

$x^2+20x+100=28$

$x^2+20x=-72$

$x^2+20x+100=-72+100=28$

$x+10=\pm\sqrt{28}=\pm 2\sqrt{7}$

$x=2\sqrt{7}-10$ or $-2\sqrt{7}-10=2(\sqrt{7}-5)$ or $-2(\sqrt{7}+5)$.

10. $x^2-\frac{5}{6}x=\frac{1}{6}$

$x^2-\frac{5}{6}x+\frac{25}{144}=\frac{1}{6}+\frac{25}{144}=\frac{49}{144}$

$x-\frac{5}{12}=\pm\frac{7}{12}$

$x=\frac{7}{12}+\frac{5}{12}$ or $-\frac{7}{12}+\frac{5}{12}=1$ or $-\frac{1}{6}$.

14. $x+22-6x^2=0$.

$6x^2-x=22$

$x^2-\frac{x}{6}=\frac{22}{6}$

$x=2$ or $-\frac{11}{6}$.

12. $\frac{1}{x-2}-\frac{2}{x+2}=\frac{3}{5}$

$\frac{x\times 2-2x+4}{x^2-4}=\frac{3}{5}$

$3x^2-12=30-5x$

$3x^2+5x=42$

$x=3$ or $-\frac{14}{3}$.

17. $15x^2-2ax=a^2$

$x^2-\frac{2a}{15}x=\frac{a^2}{15}$

$x^2-\frac{2a}{15}x=\frac{a^2}{225}=\frac{a^2}{15}+\frac{a^2}{225}=\frac{15a^2+a^2}{225}=\frac{16a^2}{225}$

$x-\frac{a}{15}=\pm\frac{4a}{15}$

$x=\frac{4a}{15}+\frac{a}{15}$ or $-\frac{4a}{15}+\frac{a}{15}=\frac{a}{3}$ or $-\frac{a}{5}$.

18. $12x^2 - cx - 20c^2 = 0$

$x^2 - \frac{c}{12}x = \frac{20}{12}c^2$

$x^2 - \frac{c}{12}x + \frac{c^2}{576} = \frac{20}{12}c^2 + \frac{c^2}{576} = \frac{960c^2 + c^2}{576} = \frac{961c^2}{576}$

$x - \frac{c}{24} = \frac{31c}{24}$

$x = \frac{4}{3}c$ or $-\frac{5}{4}c$.

19. $\frac{5x+7}{x-1} = 3x+2$

$5x+7 = 3x^2 - x - 2$

$3x^2 - 6x = 9$

$x = 3$ or -1.

20. $\frac{2x-1}{3} + \frac{3}{2x-1} = 2$

$\frac{4x^2 - 4x + 1 + 9}{6x-3} = 2$

$4x^2 - 4x + 10 = 12x - 6$

$x = 2$.

21. $\frac{2x-1}{x} - \frac{3x}{3x-1} + \frac{1}{2} = 0$

$\frac{6x^2 - 5x + 1 - 3x^2}{3x^2 - x} = -\frac{1}{2}$

$12x^2 - 10x + 2 - 6x^2 = -3x^2 + x$

$9x^2 - 11x = -2$

$x^2 - \frac{11}{9}x = -\frac{2}{9}$

$x = 1$ or $\frac{2}{9}$.

Page 182.

1. $x^2 - 6x + 8 = 0$

$(x-4)\ (x-2) = 0$

$x - 4 = 0$

$x = 4$

$x - 2 = 0$

$x = 2$.

2. $3x^2 - 5x - 7x - 420 = 0$

$3x^2 - 12x - 420 = 0$

$x^2 - 4x - 140 = 0$

$(x+10)\ (x-14) = 0$

$x = -10$

$x = 14$.

3. $2x^2 - x - 3 = 0$

$(2x-3)\ (x+1) = 0$.

4. $6x^2 - 18 = x - 3$

$6x^2 - x - 15 = 0$

$(3x-5)\ (2x+3) = 0$

$x = \frac{5}{3}$ or $-\frac{3}{2}$.

7. $x^2 - \frac{1}{30}ax - \frac{1}{30}a^2 = 0$

$(x - \frac{1}{6}a)\ (x + \frac{1}{5}a)$.

8. $9x^2 - 99x - 190 = 0$

$x^2 - 11x - \frac{190}{9} = 0$

$(x - \frac{38}{3})\ (x + \frac{5}{3}) = 0$.

9. $\frac{x-1}{x+1} - \frac{x+1}{x-1} = 1$

$(x-1)^2 - (x+1)^2 = x^2 - 1$

$x^2 - 2x + 1 - x^2 - 2x - 1 = x^2 - 1$

$x^2 + 4x + 1 = 0$

$(x + 2 + \sqrt{5})\ (x + 2 - \sqrt{5}) = 0$

$\therefore\ x = -2 - \sqrt{5}$

$x = -2 + \sqrt{5}$.

10. $x^2-2mx-1=0$

$[x-(m+\sqrt{1+m^2})]\ [x-(m-\sqrt{1+m^2}]$.

$x=m+\sqrt{1+m^2}$

$x=m-\sqrt{1+m^2}$.

Page 183.

1. $4x^2-x=3$

$$x=\frac{1\pm\sqrt{1+48}}{8}=\frac{1\pm7}{8}=\tfrac{8}{8}\text{ or }-\tfrac{6}{8}=1\text{ or }-\tfrac{3}{4}.$$

2. $3x^2+4x=4$

$$x=\frac{-4\pm\sqrt{16+48}}{6}=\frac{-4\pm8}{6}=\tfrac{4}{6}\text{ or }-\tfrac{12}{6}=\tfrac{2}{3}\text{ or }-2.$$

4. $7x^2-16x=-4$

$$x=\frac{16\pm\sqrt{256-112}}{14}=\frac{16\pm12}{14}=\tfrac{28}{14}\text{ or }\tfrac{4}{14}=2\text{ or }\tfrac{2}{7}.$$

6. $3x^2-11x=-6$

$$x=\frac{11\pm\sqrt{121-72}}{6}=\frac{11\pm7}{6}=\tfrac{18}{6}\text{ or }\tfrac{4}{6}=3\text{ or }\tfrac{2}{3}.$$

9. $5x^2-7x=6$

$$x=\frac{7\pm\sqrt{49+120}}{10}=\frac{7\pm13}{10}=\tfrac{20}{10}\text{ or }-\tfrac{6}{10}=2\text{ or }-\tfrac{3}{5}.$$

10. $11x^2-9x=-\tfrac{10}{9}$

$99x^2-81x=-10$

$$x=\frac{81\pm\sqrt{81^2-3960}}{198}=\tfrac{132}{198}\text{ or }\tfrac{30}{198}=\tfrac{2}{3}\text{ or }\tfrac{5}{33}.$$

MISCELLANEOUS EXERCISES.

1. $6x^2-25x=-21$

$$x=\frac{25\pm\sqrt{25^2-504}}{12}=\frac{25\pm11}{12}=\tfrac{36}{12}\text{ or }\tfrac{14}{12}=3\text{ or }\tfrac{7}{6}.$$

3. $12x^2-29x=-14$

$$x=\frac{29\pm\sqrt{29^2-672}}{24}=\frac{29\pm13}{24}=\tfrac{42}{24}\text{ or }\tfrac{16}{24}=\tfrac{7}{4}\text{ or }\tfrac{2}{3}.$$

5. $50x^2-15x=27$

$$x=\frac{15\pm\sqrt{15^2+5400}}{100}=\frac{15\pm75}{100}=\tfrac{90}{100}\text{ or }-\tfrac{60}{100}=\tfrac{9}{10}\text{ or }-\tfrac{3}{5}.$$

7. $21x^2-2ax=3a^2$

$$x=\frac{2a\pm\sqrt{4a^2+252a^2}}{42}=\frac{2a\pm16a}{42}=\frac{18a}{42}\text{ or }-\frac{14a}{42}=\frac{3a}{7}\text{ or }-\frac{a}{3}.$$

9. $x^2-2x+1-9x^2-48x-64-4x^2-20x-25=0$

$+12x^2+70x=-88$

$x=\frac{-70\pm\sqrt{4900-4224}}{24}=\frac{-70\pm26}{24}=\frac{44}{24}$ or $-\frac{96}{24}=-\frac{11}{6}$ or -4.

10. $\sqrt{4x-3}-\sqrt{x+1}=1$

$4x-3=1+2\sqrt{x+1}+x+1$

$3x-5=2\sqrt{x+1}$

$9x^2-34x=-21$

$x=\frac{34\pm\sqrt{39^2-756}}{18}=\frac{34\pm20}{18}=3$ or $\frac{7}{9}$.

11. $9\frac{1}{3}x^2-90\frac{1}{3}x+195=0$

$28x^2-271x=-585$

$x=\frac{271\pm\sqrt{271^2-585\times112}}{56}=\frac{271\pm89}{56}=\frac{360}{56}$ or $\frac{182}{56}=6\frac{3}{7}$ or $3\frac{1}{4}$.

14. $4a^2x=(a^2-b^2+x)^2$

$2a\sqrt{x}=a^2-b^2+x$

$x-2a\sqrt{x}=b^2-a^2$

$\sqrt{x}=\frac{2a\pm\sqrt{4a^2+4b^2-4a^2}}{2}=\frac{2a\pm2b}{2}=a\pm b$

$x=(a\pm b)^2$.

15. $\frac{x+c}{x-c}-\frac{x-c}{x+c}=1$

$(x+c)^2-(x-c)^2=x^2-c^2$

$x^2-4cx=c^2$

$x=\frac{4c\pm\sqrt{16c^2+4c^2}}{2}=\frac{4c\pm\sqrt{20c^2}}{2}=2c\pm2c\sqrt{5}$.

Page 184.

17. $\frac{x^2-5x}{x+3}=x-3+\frac{1}{x}$

$5x^2-8x=-3$

$x=\frac{8\pm\sqrt{64-60}}{10}=\frac{8\pm2}{10}=\frac{10}{10}$ or $\frac{6}{10}=1$ or $\frac{3}{5}$.

20. $\frac{x-6}{x-12}-\frac{x-12}{x-6}=-\frac{5}{6}$

$5x^2-262x=-1008$

$x=\frac{162\pm78}{10}=\frac{240}{10}$ or $\frac{84}{10}=24$ or $8\frac{2}{5}$.

2. x = one part
y = the other part
1. $x+y=m$
2. $xy=n$
Or
x = one part
$m-x$ = the other part
$x(m-x)=n$
$x^2-mx=-n$
$x=\frac{m\pm\sqrt{m^2-4n}}{2}$.

Page 185.

4. 1. $x+y=20$
2. $xy=24(x-y)$.

5. $x^2-119=(x-8)10$
$x^2-10x=39$.

6. $5x$ = man's age
x = son's age
$25x^2+x^2=2106$.

7. x = the number
$x+1$ = the next highest number
$x+x^2=9(x+1)$.

8. x and $x+1$ = the numbers
$\frac{1}{x}+\frac{1}{x+1}=\frac{21}{110}$.

9. x = length
y = breadth
$\left.\begin{array}{l}2x+2y=360\\ xy=7200.\end{array}\right\}$

10. x = cost of horse
$\frac{x}{100}$ = rate of gain
$\frac{x^2}{100}$ = gain
$x+\frac{x^2}{100}=171$.

11. x = the number
$(x-12)^2-(x-12)=30$.

Page 186.

12. x = one number
$\frac{3}{4}x$ = the other
$x-\frac{3}{4}x+\frac{3}{4}x^2=196$
$x=16$, $\frac{3}{4}x=12$.

13. x = cost of horse
$\frac{x}{100}$ = rate of loss
$x-\frac{x^2}{100}=24$
$x=60$ or 40.

14. x = the number
$(10-x)x=21$.

15. x = the number of yards
$\frac{240}{x}$ = cost per yard
$\frac{240}{x-3}=\frac{240}{x}+4$.

16. x = the number of yards
$\frac{240}{x}$ = cost per yard
$\frac{240}{x+3}=\frac{240}{x}-4$
$\frac{24}{2}$ or $-\frac{30}{2}=12$ or -15.

17. x, $x+1$, $x+2$ = the consecutive numbers

$2x^2+4x=x^2+2x+63$.

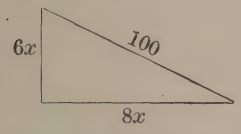
18. x^2 = the number of dollars

$\frac{x^2}{2}-x=180$.

19. x = the number of hours

$6x$ = the distance north

$8x$ = the distance east



$64x^2+36x^2=100^2$.

20. x = the number of yards

$\frac{100}{x}$ = cost per yard

$(x-5)\left(\frac{100}{x}+2\right)=120$.

Page 187.

21. $2x$ = the time down

$3x$ = the time back

y = the rate of the stream

$2x+3x=10$

$x=2$

$2x=4$ hours

$3x=6$ hours

$\frac{20}{4}-y=\frac{20}{6}+y$

$24y=20$

$y=\frac{5}{6}$ mi. per hour.

22. x = breadth of rectangle

y = length of rectangle

$xy=1200$

$(x+3)\,(y-1\frac{1}{2})=1260$.

23. A. $x-y=3$

B. $x^3-y^3=513$

A. $x^2-2xy+y^2=9$

A., B. $x^2+xy+y^2=171$

$3xy=162$

$xy=54$

$y=\frac{54}{x}$.

Page 189.

1. $x^4-21x^2=100$

$x^2=\frac{21\pm\sqrt{441+400}}{2}=\frac{21\pm29}{2}=\frac{50}{2}$ or $-\frac{8}{2}=25$ or -4

$x=\sqrt{25}$ or $\sqrt{-4}=\pm5$ or $\pm2\sqrt{-1}$.

3. $x^3-x^{\frac{3}{2}}=56$

$x^{\frac{3}{2}}=\frac{1\pm\sqrt{1+224}}{2}=\frac{1\pm15}{2}=\frac{16}{2}$ or $-\frac{14}{2}=8$ or -7

$x^{\frac{1}{2}}=\sqrt[3]{8}$ or $\sqrt[3]{-7}=2$ or $\sqrt[3]{-7}$

$x=2^2$ or $(\sqrt[3]{-7})^2=4$ or $\sqrt[3]{49}$.

4. $x^6 - 65x^3 = -64$

$x^3 = \frac{65 \pm \sqrt{65^2 - 256}}{2} = \frac{65 \pm 63}{2} = 1\frac{2}{2}\frac{8}{2}$ or $\frac{2}{2} = 64$ or 1

$x = \sqrt[3]{64}$ or $\sqrt[3]{1} = 4$ or 1.

Page 190.

5. $x + x^{\frac{1}{2}} = 6$

$x^{\frac{1}{2}} = \frac{-1 \pm \sqrt{1+24}}{2} = \frac{-1 \pm 5}{2} = \frac{4}{2}$ or $-\frac{6}{2} = 2$ or -3

$x = 2^2$ or $-3^2 = 4$ or 9.

6. $x^{-1} - x^{-\frac{1}{2}} = 6$

$x^{-\frac{1}{2}} = \frac{1 \pm \sqrt{1+24}}{2} = \frac{1 \pm 5}{2} = \frac{6}{2}$ or $-\frac{4}{2} = 3$ or -2.

$x^{-1} = 3^2$ or $-2^2 = 9$ or 4

$x = \frac{1}{9}$ or $\frac{1}{4}$.

7. $5\sqrt{x+4} + (x+4) = 24$

$(x+4) + 5(x+4)^{\frac{1}{2}} = 24$

$(x+4)^{\frac{1}{2}} = \frac{-5 \pm \sqrt{25+96}}{2} = \frac{-5 \pm 11}{2} = \frac{6}{2}$ or $-\frac{1}{2}\frac{6}{2} = 3$ or -8

$x + 4 = 9$ or 64

$x = 5$ or 60.

8. $\frac{15}{4}x^2 + x - 3\sqrt{\frac{15}{4}x^2 + x} = 40$

$\sqrt{\frac{15}{4}x^2 + x} = \frac{3 \pm \sqrt{9+160}}{2} = \frac{3 \pm 13}{2} = \frac{16}{2}$ or $-\frac{10}{2} = 8$ or -5

$\frac{15x^2}{4} + x = 64$ or 25

$15x^2 + 4x = 256$ or 100

$x = \frac{-4 \pm \sqrt{16+1024}}{30} = \frac{-4 \pm 124}{30} = \frac{120}{30}$ or $-\frac{128}{30} = 4$ or $-4\frac{4}{15}$.

9. $3x^{-\frac{2}{3}} - 7x^{-\frac{1}{3}} = 6$

$x^{-\frac{1}{3}} = \frac{7 \pm \sqrt{49+72}}{6} = \frac{7 \pm 11}{6} = \frac{18}{6}$ or $-\frac{4}{6} = 3$ or $-\frac{2}{3}$

$x^{-1} = 27$ or $-\frac{8}{27}$

$x = \frac{1}{27}$ or $-\frac{27}{8}$.

10. $\left(\frac{6}{y}+y\right)^2+\left(\frac{6}{y}+y\right)=30$

$\frac{6}{y}+y=\frac{-1\pm\sqrt{1+120}}{2}=\frac{-1\pm 11}{2}=\frac{10}{2}$ or $-\frac{12}{2}=5$ or -6

$y^2+6=5y$ or $-6y$

$y^2-5y=-6$

$y=\frac{5\pm\sqrt{25-24}}{2}=\frac{5\pm 1}{2}=\frac{6}{2}$ or $-\frac{4}{2}=3$ or -2

Also

$y^2+6y=-6$

$y=\frac{-6\pm\sqrt{36-24}}{2}=\frac{-6\pm 2\sqrt{3}}{2}=-3\pm\sqrt{3}.$

12. $ax^4+2x^2=b$

$x^2=\frac{-2\pm\sqrt{4+4ab}}{2a}=-1\pm\frac{\sqrt{4+4ab}}{2a}=\frac{-1\pm\sqrt{1+ab}}{a}$

$x=\pm\sqrt{\frac{-1\pm\sqrt{1+ab}}{a}}$

This answer may be simplified by the method of page 174.

13. $2x^{-\frac{1}{2}}-9x^{-\frac{1}{4}}=-4$

$x^{-\frac{1}{4}}=\frac{9\pm\sqrt{81-32}}{4}=\frac{9\pm 7}{4}=\frac{16}{4}$ or $\frac{2}{4}=4$ or $\frac{1}{2}$

$x^{-1}=256$ or $\frac{1}{16}$

$x=\frac{1}{256}$ or 16.

14. $x^2-5x+2\sqrt{x^2-5x+3}=12$

$(x^2-5x+3)+2\sqrt{x^2-5x+3}=15$

$(x^2-5x+3)^{\frac{1}{2}}=\frac{-2\pm\sqrt{4+60}}{2}=\frac{-2\pm 8}{2}=\frac{6}{2}$ or $-\frac{10}{2}=3$ or -5

$(x^2-5x+3)^1=3^2$ or $-5^2=9$ or 25

$x^2-5x=6$ or 22

$x=\frac{5\pm\sqrt{25+24}}{2}$ or $\frac{5\pm\sqrt{88}}{2}=\frac{5\pm 7}{2}$ or $\frac{5\pm\sqrt{113}}{2}=6$ or -1, or $\frac{5\pm\sqrt{113}}{2}$.

15. $x^4-8x^3+10x^2+24x+5=0$

$x^4-8x^3+16x^2-6x^2+24x=-5$

$(x^2-4x)^2-6(x^2-4x)=-5$

$x^2-4x=\frac{6\pm\sqrt{36-20}}{2}=\frac{6\pm 4}{2}=5$ or 1

$x=5,\ -1,$ or $2\pm\sqrt{5}.$

16. $\frac{x^2-6}{x}+\frac{5x}{x^2-6}=6$

Substitute y for $\frac{x^2-6}{x}$,

then, $y^2-6y+5=0$

and $y=5$, or 1

Wherefore $\frac{x^2-6}{x}=5$, or 1

Hence $x^2-5x-6=0$

and $x^2-x-6=0$;

from which, $x=6$, -1; or $x=3$, -2.

17. $x^{\frac{3}{2}}-x^{\frac{3}{4}}=2$

$x^{\frac{3}{4}}=\frac{1\pm\sqrt{1+8}}{2}=\frac{1\pm 3}{2}=2$ or -1

$x^{\frac{1}{4}}=2^{\frac{1}{3}}$, or $-1^{\frac{1}{3}}$

$x=2^{\frac{4}{3}}$, or $-1^{\frac{4}{3}}$

That is, $x=\sqrt[3]{16}=2\sqrt[3]{2}$, or $\sqrt[3]{1}\ (=1)$.

18. $x^4-14x^3+61x^2-84x=-20$

$(x^4-14x^3+49x^2)+(12x^2-84x)=-20$

$(x^2-7x)^2+12(x^2-7x)=-20$

$x^2-7x=\frac{-12\pm\sqrt{144-80}}{2}=-2$ or -10

$x=\frac{7\pm\sqrt{49-40}}{2}=5$ or 2.

Page 193.

1. $\left.\begin{array}{l} x+y=5 \\ x^2-2y^2=1 \end{array}\right\}$

$y=5-x$

$2y^2=2(5-x)^2$

$x^2-2(5-x)^2=1$.

2. $\left.\begin{array}{l} x+y=9 \\ xy=20 \end{array}\right\}$

$y=\frac{20}{x}$

$x+\frac{20}{x}=9$

$x^2+20=9x$.

4. $\left.\begin{array}{l} 2x^2-xy=6y \\ x+2y=7 \end{array}\right\}$

$\left.\begin{array}{l} x=7-2y \\ xy=7y-2y^2 \end{array}\right\}$

$2x^2=2(7-2y)^2$

$2(7-2y)^2-7y+2y^2=6y$

$98-5by+8y^2-7y+2y^2=6y$

$10y^2-69y=-98$

$y=\frac{69\pm\sqrt{69^2-3920}}{20}=\frac{69\pm 29}{20}=\frac{98}{20}$ or $\frac{40}{20}=4\frac{2}{5}$ or $+2$

$x=3$ or $2\frac{3}{5}$.

6. $x^2+y^2=50$
$x-y=8$
$x^2-2xy+y^2=64$
$x^2+y^2=50$
$-2xy=14.$

Page 194.

7. $x^3+y^3=133$
$x^2-xy+y^2=19$
$x+y=7$
$x^2+2xy+y^2=49$
$y^2-xy+y^2=19$
$3xy=30.$

8. $x+y=10$
$x^2+y^2=58$
$x^2+2xy+y^2=100$
$x^2+y^2=58$
$2xy=42$
$xy=21$
$y=\frac{21}{x}.$

9. $x+y=5$
$x^3+y^3=35$
$x^2-xy+y^2=7$
$x^2+2xy+y^2=25$
$3xy=18$
$xy=6$
$y=\frac{6}{x}.$

10. $x-y=8$
$x^3-y^3=728$
$x^2+xy+y^2=91$
$x^2-2xy+y^2=64$
$3xy=27$
$xy=9.$

11. $2x^2-xy=6$
$2y^2+3xy=8$
$x=my$
$xy=my^2$
$x^2=m^2y^2$
$2m^2y^2-my^2=6$
$y^2=\frac{6}{2m^2-m}$
$2y^2+3my^2=8$
$y^2=\frac{8}{2+3m}$
Hence $\frac{6}{2m^2-m}=\frac{8}{2+3m}$
and $m=2$
$\therefore\ y^2=\frac{6}{6}=1$
$y=\pm 1$
$x=\pm 2.$

12. $2x-y=1$
$x^2+2y^2=22$
$x=\frac{1+y}{2}$
$x^2=\frac{1+2y+y^2}{4}$
$\frac{1+2y+y^2}{4}+2y^2=22$
$y=3$ or $-3\frac{2}{3}$
$x=2$ or $-1\frac{1}{3}.$

13. $x^2-xy=6$
$x^2+xy=66$
$xy=x^2-6$
$xy=66-x^2$
$x^2-6=66-x^2.$

14. $\frac{xy}{\sqrt{\frac{x}{y}}} = 48$

$\frac{xy}{\sqrt{x}} = 24$

$\sqrt{x}$

$\frac{xy}{\sqrt{\frac{x}{y}}} \times \frac{\sqrt{x}}{xy} = \frac{48}{24}$

$\sqrt{y} = 2$; $y = 4$

$\therefore x = 36$.

15. $\left.\begin{array}{l} \frac{x}{y} + \frac{y}{x} = 2\frac{1}{2} \\ x + y = 6 \end{array}\right\}$

$x^2 - 6x = -8$.

16. $\left.\begin{array}{l} 2x - 4y = 2 \\ 2x^2 - xy - 3y^2 = 112 \end{array}\right\}$

$x - 2y = 1$

$x = 1 + 2y$

$x^2 = 1 + 4y + 4y^2$

$2 + 8y + 8y^2 - y - 2y^2 - 3y^2 = 112$

$3y^2 + 7y = 110$.

17. $\left.\begin{array}{l} x + y = 1 \\ x^2 + y^2 = 25 \end{array}\right\}$

$y = 1 - x$

$y^2 = 1 - 2x + x^2$

$x^2 + 1 - 2x + x^2 = 25$

$x^2 - x = 12$.

18. $\left.\begin{array}{l} x - y = 1 \\ x^3 - y^3 = 19 \end{array}\right\}$

$x^2 + xy + y^2 = 19$

$x^2 - 2xy + y^2 = 1$

$3xy = 18$

$xy = 6$

$x^2 - x = 6$.

19. $\left.\begin{array}{l} xy = a^2 \\ x - y = 0 \end{array}\right\}$

$y = \frac{a^2}{x}$

$x - \frac{a^2}{x} = 0$.

20. $\left.\begin{array}{l} xy + x^2 = 14 \\ y^2 + xy = 35 \end{array}\right\}$

See No. 11 above.

21. $x^2 + y^2 = 25$

$x - y = -1$

$x^2 + y^2 = 25$

$x^2 - 2xy + y^2 = 1$

$2xy = 24$

$xy = 12$.

22. $\left.\begin{array}{l} x - y = c \\ xy = d \end{array}\right\}$

$y = \frac{d}{x}$

$x^2 - cx = d$

$x = \frac{c \pm \sqrt{c^2 + 4d}}{2}$

$y = \frac{-c \pm \sqrt{c^2 + 4d}}{2}$.

24. $x + y = 13$

$\sqrt{x} + \sqrt{y} = 5$

$x + 2\sqrt{xy} + y = 25$

$x + y = 13$

$2\sqrt{xy} = 12$

$\sqrt{xy} = 6$

$xy = 36$.

25. $x^4 - y^4 = m$

$x^2 - y^2 = n$

$x^2 + y^2 = \frac{m}{n}$

$x^2 - y^2 = n$

$2x^2 = \frac{m}{n} + n = \frac{m + n^2}{n}$

$x^2 = \frac{m + n^2}{2n}$

$\left.\begin{array}{l} x = \pm\sqrt{\frac{m + n^2}{2n}} \\ y = \pm\sqrt{\frac{m - n^2}{2n}}. \end{array}\right\}$

26. $3x^2+4xy=20$
$2y^2+5xy=12$
$x=my$
$x^2=m^2y^2$
$xy=my^2$
$3m^2y^2+4my^2=20$
$2y^2+5my^2=12$
$y^2=\frac{20}{3m^2+4m}$
$y^2=\frac{12}{2+5m}$.

27. $x+4y=14$
$4x-2y+y^2=11$
$x=14-4y$
$4x=56-16y$
$56-16y-2y+y^2=11$
$y^2-18y=-45$.

28. $\begin{cases} 6x-5y=9 \\ x^2+2y+1=-2+5\sqrt{x^2+2y+3} \end{cases}$
$x^2+2y+3=5\sqrt{x^2+2y+3}$
$\sqrt{x^2+2y+3}=5$
$x^2+2y+3=25$
$x^2+2y=22$
$6x-5y=9$
$x^2+2y=22$
Whence $x=4$ and $y=3$.

29. $x+2y=9$
$3y^2-5x^2=43$
$x=9-2y$
$x^2=81-36y+4y^2$
$5x^2=405-18y+20y^2$
$3y^2-405+180y-20y^2=43$
$17y^2-180y=-448$.

Page 195.

30. $2x-3=y$
$10-2x^2=xy$
$xy=2x^2-3x$
$2x^2-3x=10-2x^2$.

31. $3x^2+165=16xy$
$7xy+3y^2=132$
$3x^2-16xy=-165$
$3y^2+7xy=132$
$x=my$.

32. $3x^2+xy+y^2=15$
$31xy-3x^2-5y^2=45$
$x=my$.

33. $x^2+y^2-3xy=9$
$2x^2+y^2=18$
$x^2+3xy=9$
$2x^2+6xy=18$
$\therefore 2x^2+6xy=2x^2+y^2$
$6xy-y^2=0$
$y(6x-y)=0$
$\therefore y=0$
and $2x^2+0=18$
$x^2=9$
$x=\pm 3$.

34. $x^4+y^4=706$
$x+y=8$

$x=m+n$
$y=m-n$

$x+y=2m=8$
$m=4$

$x^4+y^4=(m+n)^4+(m-n)^4=2m^4+12m^2n^2+2n^4=706$
$n^4+96n^2=97$
$n=\pm 1$

$x=m+n=4+1=5$
$y=m-n=4-1=3$

The other values are easily obtainable.

1. $x=$ one number
$y=$ the other
$xy=54$
$5x+10y=105.$

2. $x+y=20$
$xy=p$
$y=\frac{p}{x}$
$x+\frac{p}{x}=20$
$x^2+p=20x.$

3. $x+y=9$
$x^3+y^3=189.$

4. $x+y=a$
$x^3+y^3=c$

$x=m+n$
$y=m-n$

$x+y=2m=a$
$m=\frac{a}{2}$

$x^3+y^3=(m+n)^3+(m-n)^3=$
$2m^3+6mn^2=c$
$a^3+12an^2=4c$
$n=\pm\sqrt{\frac{4c-a^3}{12a}}$

$x=m+n=\frac{a}{2}\pm\sqrt{\frac{4c-a^3}{12a}}=\frac{a}{2}\pm\frac{1}{2}\sqrt{\frac{4c-a^3}{3a}}$

$y=m-n=\frac{a}{2}\mp\sqrt{\frac{4c-a^3}{12a}}=\frac{a}{2}\mp\frac{1}{2}\sqrt{\frac{4c-a}{3a}}.$

5. $(x+y)\,y=77$
$(x-y)\,x=12.$

6. $x+y=100$
$\sqrt{x}+\sqrt{y}=14.$

7. $x+y=24$
$xy=(x-y)\,35.$

8. $xy=255$
$x^2+y^2=514.$

Page 196.

9. x = B.'s age
$x+4$ = A.'s age
$x^2+(x+4)^2=976$.

10. x = a side of the smaller square
$x+4$ = a side of the larger square
$x^2+(x+4)^2=1066$.

11. x = length, y = breadth

$$\left.\begin{aligned} 2x+2y&=88\\ xy&=363. \end{aligned}\right\}$$

12. $\frac{x}{y}$ = the fraction

$$\left.\begin{aligned} x+y&=8\\ \left(\frac{x+1}{y+1}\right)\frac{x}{y}&=\frac{2}{5} \end{aligned}\right\}$$

$\frac{x}{y}=\frac{3}{5}$ or $-\frac{16}{24}$.

13. x = the number of passengers
$\frac{2}{19}x$ = the number of crew
$x-\frac{2}{19}x=425$.

14. x = rate in still water
$x+2$ = rate down the river
$x-2$ = rate up the river
$\frac{7}{2}\div(x+2)$ = time down
$\frac{7}{2}\div(x-2)$ = time up

$$\frac{7}{2x+4}=\frac{7}{2x-4}=\tfrac{5}{3} \text{ hours.}$$

15. x = revolutions of fore wheel
y = revolutions of hind wheel

$$\left.\begin{aligned} \frac{5280}{x}-\frac{5280}{y}&=132\\ \frac{5280}{x+2}-\frac{528}{y+2}&=88. \end{aligned}\right\}$$

16. x = the number
$5x^2+100=10x+500$.

17. x = days for 1st man
$x+5$ = days for 2d man
$\frac{1}{x}$ = work of 1st man in 1 day
$\frac{1}{x+5}$ = work of 2d man in 1 day
$\frac{1}{x}+\frac{1}{x+5}=\frac{1}{6}$, work of both in 1 day.

18. m = one fraction, n = the other
$m+n=mn$
$m^2-n^2=\frac{5}{6}mn$
By dividing,
$m-n=\frac{5}{6}$
$m=\frac{5}{6}+n$
$\frac{5}{6}+n+n=\frac{5n}{6}+n^2$
$n=\frac{5}{3}$ $m=\frac{5}{6}+\frac{5}{3}=\frac{15}{6}=\frac{5}{2}$.

Page 197.

19. $\left.\begin{array}{l} x+y+z=14 \\ x^2+y^2+z^2=84 \\ xz=y^2 \end{array}\right\}$

$x^2+y^2+z^2+2xy+2xz+2yz=196$

$x^2+y^2+z^2 \quad\quad =84$

$2xy+2xz+2yz=112$

$xy+xz+yz=56$

From the first, $\left.\begin{array}{l} xy+y^2+yz=56 \\ xy+y^2+yz=14y \end{array}\right\}$

$14y=56$

$y=4$

The values of x and z are easily obtained.

20. $x=$ cost

$\frac{x}{100}=$ rate of gain

$\frac{x^2}{100}=$ gain

$x+\frac{x^2}{100}=75.$

21. $x=$ father's age

$y=$ son's age

$\left.\begin{array}{l} x+y=100 \\ \frac{xy}{10}-x=180. \end{array}\right\}$

22. $x=$ left hand digit

$y=$ right hand digit

$x^2+y^2=58$

$10x+y-12y=10y+x$

$9x-9y-12y=0$

$3x-7y=0$

$3x=7y$

$x^2=\frac{49}{9}y^2.$

23. $x-y=6$

$x^3-y^3=342.$

24. $x=$ the num. of yds. A. sold.

$x+3=$ the num. of yds. B. sold.

$\frac{24}{x+3}=$ A.'s price per yd.

$\frac{12\frac{1}{2}}{x}=$ B.'s price per yd.

$\frac{24x}{x+3}+\frac{12\frac{1}{2}(x+3)}{x}=35.$

Page 201.

6. Roots, $-\frac{7}{2}, \frac{4}{9}$

Sum, $-\frac{6}{1}\frac{3}{8}+\frac{8}{1}\frac{}{8}=-\frac{5}{1}\frac{5}{8}$; coef., $+\frac{5}{1}\frac{5}{8}$

Product, $-\frac{7}{2}\times\frac{4}{9}=-\frac{2}{1}\frac{8}{8}=-\frac{1}{9}\frac{4}{}$; 2d mem., $\frac{1}{9}\frac{4}{}$

Equation, $x^2+\frac{5}{1}\frac{5}{8}x=\frac{1}{9}\frac{4}{}$.

9. Roots, $a+2b,\ a-b$

Sum, $2a+b$; coef., $-(2a+b)$

Product, $a^2+ab-2b^2$; 2d mem., $-(a^2+ab-2b^2)$

Equation, $x^2-(2a+b)x=-(a^2+ab-2b^2)$.

10. Roots, $\frac{p+\sqrt{q}}{2}$, $\frac{p-\sqrt{q}}{2}$

Sum, p; coef., $-p$

Product, $\frac{p^2-q}{4}$; 2d mem., $-\frac{p^2-q}{4}$

Equation, $x^2-px=-\frac{p^2-q}{4}$.

Page 202.

1. $x^2+7x+12=0$

$(x+3)\ (x+4)=0$

$x=-3$ }

$x=-4.$ }

3. $x^2-x-20=0$

$(x-5)\ (x+4)=0$

$x=+5$ }

$x=-4.$ }

5. $x^3-27x^2+50x=0$

$x\,(x^2-27x+50)=0$

$x\,(x-25)\ (x-2)=0$

$x=25$ }

$x=\ 2$ }

$x=\ 0.$ }

7. $x^3-8\ =0$

$x^3-2^3=0$

$(x-2)\ (x^2+2x+4)$

$x^2+2x+4=0$

$x=\frac{-2\pm\sqrt{4-16}}{2}=\frac{-2\pm2\sqrt{-3}}{2}=$

$-1\pm\sqrt{-3}$ }

$x=+2.$ }

8. $x^6-1=0$

$(x^3+1)\ (x^3-1)=0$

$(x+1)\ (x^2-x+1)\ (x-1)\ (x^2+x+1)=0.$

10. $12x^3+8x^2-27x-18=0$

$4x^2\,(3x+2)-9\,(3+2)=0$

$(3x+2)\ (4x^2-9)=0$

$(3x+2)\ (2x+3)\ (2x-3)=0.$

Page 208.

1. $x:3=15:9$

$9x=45$

$x=\ 5.$

2. $\frac{2}{3}:\frac{5}{7}=\frac{2}{15}:x$

$\frac{2}{3}x=\frac{10}{105}=\frac{2}{21}$

$42x=6$

$x=\frac{6}{42}=\frac{1}{7}.$

Page 209.

2. 1. $x^3 : xy = 5x^2y : 4\text{th}$

$4\text{th}\ x^3 = 5x^3y^2$

$4\text{th} \quad = 5y^2.$

3. $1\frac{5}{7} : 1\frac{4}{5} = 1\frac{1}{2} : 4\text{th}$

$1\frac{2}{7} : \frac{9}{5} = \frac{3}{2} : 4\text{th}$

$4\text{th} \times 1\frac{2}{7} = \frac{27}{10}$

$4\text{th} = \frac{27}{10} \times \frac{7}{12} = \frac{189}{120} = \frac{63}{40} = 1\frac{23}{40}.$

6. $a^2 - b^2 : a^2 + ab + b^2 = a - b : x$

$$x = \frac{(a^2+ab+b^2)\ (a-b)}{a^2-b^2} = \frac{a^2+ab+b^2}{a+b}.$$

3. 1. $x^2y : xy = xy : \text{Third}$

$\text{Third}\ x^2y = x^2y^2$

$$\text{Third} = \frac{x^2y^2}{x^2y} = y.$$

2. $(x+4)\,5 = (x+4)\,13$

$5x + 20 = 13x - 52$

$8x = 72$

$x = 9.$

4. 1. $x^2 : m = m : y^2$

$m^2 = x^2y^2$

$m = \sqrt{x^2y^2} = xy.$

3. $\sqrt{36b^4y^2} = 6b^2y$

5. 1. $a : c = d : b.$

4. $a : a^3 = d : b^3.$

5. $(\frac{1}{2} + \sqrt{a})\ (\sqrt{a} - 2a) = (\frac{1}{4} - a)\,x$

$\frac{1}{2}\sqrt{a} - 2a\sqrt{a} = \frac{1}{4}x - ax$

$x\,(\frac{1}{4} - a) = (\frac{1}{2} - 2a)\sqrt{a}$

$$x = \frac{(\frac{1}{2} - 2a)\sqrt{a}}{\frac{1}{4} - a} = 2\sqrt{a}.$$

Page 210.

6. $\left.\begin{array}{l} x : y = x + y : 42 \\ x : y = x - y : 6 \end{array}\right\}$

$x + y : 42 = x - y : 6$

$6x + 6y = 42x - 42y$

$36x = 48y$

$3x = 4y$

$x = \frac{4}{3}y$

$6x = xy - y^2$

$2\frac{4}{3}y = \frac{4}{3}y^2 - y^2$

$24y = 4y^2 - 3y^2$

$\left.\begin{array}{l} y = 24 \\ x = \frac{4}{3}y = 32. \end{array}\right\}$

8. $(x+y)\ (a-b)=(x-y)\ (a+b)$

$x^2+y^2=a^2b^2\ (a^2+b^2)$

$ax+ay-bx-by=ax-ay+bx-by$

$2ay-2bx=0$

$y=\frac{bx}{a}$

$y^2=\frac{b^2x^2}{a^2}$

$x^2+\frac{b^2x^2}{a^2}=a^2b^2\ (a^2+b^2)$

$a^2x^2+b^2x^2=a^4b^2\ (a^2+b^2)$

$x^2\ (a^2+b^2)=a^4b^2\ (a^2+b^2)$

$x^2=\frac{a^4b^2\ (a^2+b^2)}{a^2+b^2}=a^4b^2$

$x=\pm a^2b$

$\therefore y=\pm ab^2.$

9. $\begin{cases} 3x^3y=81 \\ x^3+y^3 : x^3-y^3=14:13 \end{cases}$

$3x^3y=81$

$x^3=\frac{27}{y}$

$351+13y^4=378-14y^4.$

10. By composition and division:

$2\sqrt{x+1} : 2\sqrt{x-1}=4x+1:4x-3$

By squaring:

$4x+4:4x-4=16x^2+8x+1:16x^2-24x+9$

$x+1:\ x-1=16x^2+8x+1:16x^2-24x+9$

$8x=10$

$x=\frac{10}{8}=\frac{5}{4}.$

1. If $a:b=c:d$

then $\frac{a}{b}=\frac{c}{d}$

Dividing a unit by each ratio,

$\frac{b}{a}=\frac{d}{c}.$

2. If $a:b=c:d,$

then $\frac{a}{b}=\frac{c}{d}$

Since $\frac{a\times a}{b\times a}=\frac{c\times c}{d\times c}$

then $\frac{a^2}{ab}=\frac{c^2}{dc}$

and $a^2:ab=c^2:dc$

By alternation, $a^2:c^2=ab:dc.$

3. 1. If $a:b=c:d$
2. then $a:c=b:d$,
3. and $a+c:c=b+c:d$,
4. also $a+c:b+d=c:d$

Again $\dfrac{c}{d}=\dfrac{a}{b}$

and $bc=ad$;

also, $\left.\begin{aligned}\dfrac{abc}{abd}&=\dfrac{a^2d}{b^2c}\end{aligned}\right\}$

and $\dfrac{c}{d}=\dfrac{a^2d}{b^2c}$

Wherefore by 4, $a+c:b+d=a^2d:b^2c$.

4. If $a:b=c:d$,
then, by Inv., $b:a=d:c$;
and, by Div., $b-a:b=d-c:d$.

5. If $a:b=c:d$
then $a^2:b^2=c^2:d^2$
and $a^2:c^2=b^2:d^2$;
also, $a^2+c^2:a^2=b^2+d^2:b^2$.
Whence, $a^2+c^2:b^2+d^2:a^2:b^2$
or $a^2+b^2=a^2+c^2:b^2+d^2$.

6. $x:y=x+y:42=x-y:6$

$\left.\begin{aligned}x:y&=x+y:42\\x:y&=x-y:\ 6\end{aligned}\right\}$

$x+y:x-y=42:6$
$x+y:x-y=\ 7:1$
$x+y=7x-7y$
$x=\frac{4}{3}y$
$\frac{4}{3}y:\ y=\frac{4}{3}y-y:6$
$8y=\dfrac{4y^2}{3}-y^2$
$24y=y^2$
$y=24$
$x=\frac{4}{3}y=32.$

7. $x^2-x-2\ :x-2=4x^2+5x-6:5x-6$
$x^2-2x:x-2=4x^2:5x-6$
$x^2-2x:4x^2=x-2:5x-6$
$x-2\ :4x=x-2:5x-6$
$5x^2-16x+12=4x^2-8x$
$x^2-8x=12$
$x=\ 6$ or $+2$.

8. $\left\{\begin{aligned}x+y&=14\\xy:x^2+y^2&=10:29\end{aligned}\right\}$

$x=14-y$
$x^2=196=28y+y^2$
$xy=14y-y^2$

$14y-y^2:196-28y+2y^2=10:29$

By composition:

$196-14y+y^2:14y-y^2=39:10$
$196:14y-y^2=49:10$
$y^2-14y=-40$
$y=10$ or 4
$\therefore x=\ 4$ or 10.

9. $\sqrt{3\frac{6}{11}\times\frac{6}{18}}=\sqrt{\frac{39}{11}\times\frac{6}{18}}=\sqrt{\frac{3}{11}\times\frac{6}{1}}=\sqrt{\frac{18}{11}}=\sqrt{\frac{9\times2\times11}{121}}=\frac{3}{11}\sqrt{22}.$

10. $xy=96$
$x^3-y^3:(x-y)^3=19:1$
$x^3-y^3:x^3-3x^2y+3xy^2-y^3=19:1$
$3x^2y-3xy^2:x^3-y^3=18:19$
$3xy\,(x-y):x^3-y^3=18:19$
$288:x^2+y^2+96=18:19$
$x^2+\frac{9216}{x^2}=208$
$x=12$ or 8
$y=8$ or $12.$

11. $2x=$ age of older
$x=$ age of younger
$2x+20:x+20=4:3$
$6x+60=4x+80.$

Page 212.

1. $l=a+(n-1)\,d$
$l=5+(27-1)\,6$
$l=5+156=161.$

3. $l=a-(n-1)\,d$
$l=7-(15-1)\,4$
$l=7-56=-49.$

5. $l=a+(n-1)\,d$
$l=x+(25-1)\,x$
$l=x+24x=25x.$

Page 213.

1. $l=a+(n-1)\,d$
$l=7+(20-1)\,9$
$l=7+171=178$
$S=\frac{a+b}{2}\times n$
$S=\frac{7+178}{2}\times20=185\times10=1850.$

3. $l=a+(n-1)\,d$
$l=5+(19-1)\,4$
$l=5+72=77$
$S=\frac{a+l}{2}\times n=\frac{5+77}{2}\times19=779.$

5. $l=a+(n-1)\,d$
$l=4+(37-1)\,1\frac{1}{4}=4+45=49.$
$S=\frac{a+l}{2}\times n=\frac{4+49}{2}\times37=980\frac{1}{2}.$

7. $l=a+(n-1)\,d=1+(100-1)\,2=199$
$S=\frac{a+l}{2}\times n=\frac{1+199}{2}\times100=10000.$

9. $l=a+(n-1)\,d=1+(n-1)\,1=n$
$S=\frac{a+l}{2}\times n=\frac{1+n}{2}\times n=\frac{n+n^2}{2}.$

10. $l=a+(n-1)\,d=a+(a-1)\,(-a)=a+(-a^2+a)=$
$a-a^2+a=2a-a^2$

$$S=\frac{a+l}{2}\times n=\frac{a+2a-a^2}{2}\times a=\frac{(3a-a^2)\,a}{2}=\frac{3a^2-a^3}{2}=\frac{a^2\,(3-a)}{2}.$$

Page 216.

2. $a=l-(n-1)d$
$a=67-(17-1)4=67-64=3$
$S=\frac{1}{2}n[2l-(n-1)d]$
$S=\frac{1}{2}\times 17[2\times 67-(17-1)4]$
$S=8\frac{1}{2}[134-64]=595.$

4. $n=\dfrac{2l+d\pm\sqrt{(2l+d)^2-8ds}}{2d}$

$n=\dfrac{42+2\pm\sqrt{(42+2)^2-8\cdot 2\cdot 120}}{4}$

$n=\dfrac{44\pm 4}{4}=\frac{40}{4}=10$, or $\frac{48}{4}=12$

$a=l-(n-1)d$
$a=21-(10-1)2=3$
or
$a=21-(12-1)2=-1.$

5. $d=\dfrac{2(S-an)}{n(n-1)}$

$d=\dfrac{2(90-45)}{15(15-1)}=\frac{90}{210}=\frac{3}{7}$

$l=\dfrac{2S}{n}-a=\frac{180}{15}-3=9.$

6. $n=\dfrac{2S}{a+l}=\dfrac{-\frac{9}{2}}{\frac{7}{12}+(-\frac{5}{6})}=\dfrac{-\frac{9}{2}}{\frac{7}{12}-\frac{10}{12}}=\dfrac{-\frac{9}{2}}{-\frac{1}{2}}=-\frac{9}{2}\times(-4)=\frac{36}{2}=18.$

$d=\dfrac{(l+a)(l-a)}{2S-(l+a)}=\dfrac{(-\frac{5}{6}+\frac{7}{12})(-\frac{5}{6}-\frac{7}{12})}{-\frac{9}{2}-(-\frac{5}{6}+\frac{7}{12})}=-\frac{1}{12}.$

Problems.

1. $a=l-(n-1)d$
$S=\frac{1}{2}n[2l-(n-1)d].$

2. $d=\dfrac{l-a}{n-1}=\dfrac{5-2}{11-1}=\frac{3}{10}=.3$

$\therefore$ the series is 2, 2.3, 2.6, 2.9, 3.2, 3.5, 3.8, 4.1, 4.4, 4.7, 5.

3. $l=a+(n-1)d=1+(n-1)1=1+n-1=n$

$$S=\frac{a+l}{2}\times n=\frac{1+n}{2}\times n=\frac{n+n^2}{2}=\frac{n(1+n)}{2}.$$

4. $n=\dfrac{d-2a\pm\sqrt{(d-2a)^2+8ds}}{2d}=\dfrac{.50-4\pm\sqrt{(.50-4)^2+8\times.5\times 42.50}}{1}=$

$-3.5\pm\sqrt{12.25+170}=-3.5+13.5=10$ yds.

Page 217.

5. x, y, z, u are the numbers

$$\left.\begin{aligned} x+u &= 46 \\ y^2+z^2 &= 1060 \\ y-x &= u-z \end{aligned}\right\}$$

$$\begin{aligned} y+z &= 46 \\ y &= 46-z \\ y^2 &= (46-z)^2 \end{aligned}$$

$$\begin{aligned} (46-z^2)+z^2 &= 1060 \\ z^2-46z &= -528 \\ z &= 24 \\ \therefore\ y &= 22 \\ x &= 20 \\ u &= 26. \end{aligned}$$

6. The series is 4, 8, 16, etc.

$l = a+(n-1)d = 4+(100-1)4 = 4+396 = 400$

$S = \dfrac{a+l}{2} \times n = \dfrac{4+400}{2} \times 100 = 202 \times 100 = 20{,}200$ yds. $= 11$ m. 840 yds.

Page 218.

1. $l = ar^{n-1}$

$l = 3 \times 2^4 = 3 \times 16 = 48.$

2. $l = ar^{n-1} = 256 \times (\frac{1}{2})^9 = 256 \times \frac{1}{512} = \frac{256}{512} = \frac{64}{128} = \frac{1}{2}.$

3. $l = ar^{n-1} = 64 \times (\frac{1}{2})^6 = 64 \times \frac{1}{64} = 1.$

4. $l = ar^{n-1} = x \cdot x^{p-1} = x^p.$

5. $l = ar^{n-1} = \frac{1}{8} \times (\frac{3}{2})^7 = \frac{1}{8} \times \frac{2187}{128} = \frac{729}{128}.$

Page 219.

1. $l = ar^{n-1} = 1 \cdot 3^7 = 2187$

$S = \dfrac{ar^n-a}{r-1} = \dfrac{1 \cdot 3^8-1}{3-1} = \dfrac{6561-1}{2} = \frac{6560}{2} = 3280.$

2. $l = ar^{n-1} = 4 \cdot (-\frac{1}{2})^9 = 4 \cdot (-\frac{1}{512}) = -\frac{1}{128}$

$S = \dfrac{a-ar^n}{1-r} = \dfrac{4-4 \cdot (-\frac{1}{2})^{10}}{1-(-\frac{1}{2})} = \dfrac{4-\frac{1}{1024} \cdot 4}{\frac{3}{2}} = \frac{1023}{256} \times \frac{2}{3} = \frac{341}{128}.$

4. $l = ar^{n-1} = 3 \times (\frac{3}{2})^6 = \frac{729}{64}$

$S = \dfrac{ar^n-a}{r-1} = \dfrac{3 \times (\frac{3}{2})^7-3}{\frac{3}{2}-1} = 96\frac{33}{64}.$

7. $l = ar^{n-1} = 1 \times (\frac{3}{2})^4 = \frac{81}{16}$

$S = \dfrac{ar^n-a}{r-1} = \dfrac{1 \times (\frac{3}{2})^5-1}{\frac{3}{2}-1} = \frac{422}{32} = 2\frac{11}{16}.$

9. $l = ar^{n-1} = 5x\,(-2)^9 = -2510$

$S = \dfrac{ar^n-a}{1-r} = \dfrac{5x\,(-2)^{10}-5}{1-(-2)} = -1705.$

10. $l = ar^{n-1} = \frac{1}{\sqrt{2}} \times \left(\frac{-4}{\sqrt{2}}\right)^6 = \frac{1}{\sqrt{2}} \times \frac{4096}{2^3} = \frac{512}{\sqrt{2}} = \frac{512\sqrt{2}}{2} = 256\sqrt{2}$

$$S = \frac{a - ar^{n+}}{1-r} = \frac{\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\left(\frac{-4}{\sqrt{2}}\right)^7}{\frac{-4}{1-\sqrt{2}}} = \frac{585\sqrt{2} - 292}{2}.$$

Page 222.

5. $l = ar^{n-1} = 1 \times 3^{p-1} = 3^{p-1}$

$S = \frac{ar^n - a}{r-1} = \frac{1 \times 3^p - 1}{3-1} = \frac{3^p - 1}{2}.$

6. $r^n - \frac{S}{a}r = \frac{a-S}{a} = \left[r^3 - \frac{1300}{100} \times 3 = \frac{100 - 1300}{100}\right] = [r^3 - 39 = -12] =$

27; $r = 3$

$l = ar^{n-1} = 100 \times 3^2 = 100 \times 9 = 900.$

8. $a = 3$, $r = -\frac{2}{3}$, $n = 6$

$l = ar^{n-1} = l = 3 \times (-\frac{2}{3})^5 = 3 \times (-\frac{32}{243}) = -\frac{96}{243} = -\frac{32}{81}$

$S = \frac{a - ar^n}{1-r} = \frac{3 - 3 \times (-\frac{2}{3})^6}{\frac{5}{3}} = \frac{3 - \frac{192}{729}}{\frac{5}{3}} = \frac{\frac{1905}{729}}{\frac{5}{3}} = \frac{133}{81}.$

9. $a = -4$, $r = \frac{4}{3} \div (-4) = -\frac{1}{3}$, n?

$l = -\frac{4}{729}$, $S = -\frac{243}{729}$.

Page 223.

2. $S = \frac{a}{1-r} = \frac{60}{1-\frac{1}{4}} = \frac{60}{\frac{3}{4}} = 60 \times \frac{4}{3} = 80.$

3. $S = \frac{a}{1-r} = \frac{1}{1-\frac{1}{3}} = \frac{1}{\frac{2}{3}} = \frac{3}{2}.$

4. $S = \frac{a}{1-r} = \frac{.9}{1-\frac{1}{10}} = \frac{\frac{9}{10}}{\frac{29}{30}} = \frac{9}{10} \times \frac{30}{29} = \frac{27}{29}.$

5. $S = \frac{a}{1-r} = \frac{.8}{1+\frac{1}{2}} = \frac{.8}{\frac{3}{2}} = \frac{8}{10} \times \frac{2}{3} = \frac{16}{30} = \frac{8}{15}.$

Problems.

2. $m = \sqrt{ab} = \sqrt{5 \times 405} = 45.$

3. $m = \sqrt{ab} = \sqrt{\frac{14}{3} \times \frac{56}{27}} = \sqrt{\frac{784}{81}} = \frac{28}{9} = 3\frac{1}{9}.$

Page 224.

6. $r = \sqrt[n-1]{\frac{l}{a}} = \sqrt[4]{\frac{6}{486}} = \sqrt[4]{\frac{1}{81}} = \frac{1}{3}$

Series, 486, 162, 54, 18, 6.

7. $r = \sqrt[n-1]{\frac{l}{a}} = \sqrt[7]{\frac{-\frac{7}{16}}{56}} = \sqrt[7]{-\frac{1}{128}} = -\frac{1}{2}$

Series, $56, \underline{-28}, \underline{14}, \underline{-7}, \underline{\frac{7}{2}}, \underline{-\frac{7}{4}}, \underline{\frac{7}{8}}, \underline{-\frac{7}{16}}$.

8. $r = \sqrt[n-1]{\frac{l}{a}} = \sqrt[3]{\frac{y^6}{x^3}} = \frac{y^2}{x} = x^{-1}y^2$

Series, $x^3, \underline{x^2y^2}, \underline{xy^4}, x^0y^6$.

9. $r = \sqrt[4]{\frac{320}{20}} = \sqrt[4]{\frac{16}{1}} = 2$

$20 = ar^{n-1} = a \cdot 2^5 = 32a$

$32a = 20,\ a = \frac{20}{32} = \frac{5}{8}$

Series, $\frac{5}{8}, \frac{5}{4}, \frac{5}{2}, 5, 10, \ldots\ldots$

10. $r = \sqrt[3]{\frac{-384}{48}} = \sqrt[3]{\frac{-64}{8}} = \frac{-4}{2} = -2$

$48 = ar^{n-1} = a \times (-2)^4 = 16a$

$a = \frac{48}{16} = 3$.

11. $r = \sqrt[8]{\frac{1536}{6}} = \sqrt[8]{\frac{256}{1}} = \frac{2}{1} = 2$

$l = ar^{n-1} = a \cdot 2^3 = 8a$

$a = \frac{3}{4}$.

$S = \frac{ar^n - a.}{r-1} = \frac{\frac{3}{4} \times 2^{11} - \frac{3}{4}}{1} = \frac{3}{4} \times 2048 - \frac{3}{4} = 1536 - \frac{3}{4} = 1535\frac{1}{4}$.

12. $x + y + z = 14$

$\underline{x^2 + y^2 + z^2 = 84}$

$y = \sqrt{xz}$

$y^2 = xz$

Dividing B. by A.,

$\underline{x - \sqrt{xz} + z = 6}$

Adding A. and C.

$2x + 2z = 20$

A. $x + \sqrt{xz} + z = 14$

B. $x^2 + xz + z^2 = 84$

D. $x + z = 10$

Subtracting D. from A.,

$\sqrt{xz} = 4$

Hence $y = 4$

x and z are now easily obtained.

13. $S = \frac{a}{1-r} = \frac{\frac{3}{4}}{1-\frac{2}{3}} = \frac{3}{4} \times \frac{3}{1} = \frac{9}{4} = 2\frac{1}{4}$.

14. $\frac{14}{25} - a = 2\text{d term} = \frac{14 - 25a}{25}$, and $r = \frac{14 - 25a}{25a}$

$\frac{2}{3} = \frac{a}{1-r} = \frac{a}{1 - \frac{14 - 25a}{25a}} = \frac{25a^2}{50a - 14}$

Hence $75a^2 - 100a = -28$

Whence $a = \frac{14}{15}$.

15. $\sqrt{\sqrt[3]{a^2x} \cdot \sqrt[3]{ax^2}} = \sqrt{a^{\frac{2}{3}}x^{\frac{1}{3}} \cdot a^{\frac{1}{3}}x^{\frac{2}{3}}} = \sqrt{ax}$.

16. $r=\dfrac{-2}{a}$

$$\frac{9}{2}=\frac{a}{1-\dfrac{-2}{a}}=\frac{a}{\dfrac{a+2}{a}}=\frac{a^2}{a+2}$$

$$9a+18=2a^2$$
$$2a^2-9a=18$$
$$a=6.$$

17. $l=ar^{n-1}$, and $S=\dfrac{ar^n-a}{a-1}$

Solve this problem, and also the next, No. 18.

Page 226.

2. 1. Literal part, x^3m^5

2. Last factor of denominator, 5

3. Denominator, $1\cdot2\cdot3\cdot4\cdot5$

4. Numerator, $8\cdot7\cdot6\cdot5\cdot4$

5. 6th term $=\dfrac{8\cdot7\cdot6\cdot5\cdot4}{1\cdot2\cdot3\cdot4\cdot5}-x^3m^5=56x^3m^5.$

Page 227.

3. 8th term $=-\dfrac{11\cdot10\cdot9\cdot8\cdot7\cdot6\cdot5}{1\cdot\ 2\cdot3\cdot4\cdot5\cdot6\cdot7}\times1^4\times\left(\dfrac{1}{a}\right)^7=-\dfrac{330}{a^7}.$

5. 4th term $=\dfrac{10\cdot9\cdot8}{1\cdot2\cdot3}\times(4a)^7\times\left(\dfrac{x}{2}\right)^3=245760a^7x^3.$

GENERAL REVIEW.

1. $\left(\dfrac{a+2b}{2a+b}-\dfrac{a-2b}{2a-b}\right)\left(\dfrac{1}{b}+\dfrac{b}{2ab}\right)=$

$$\left[\frac{(a+2b)(2a-b)-(a-2b)(2a+b)}{4a^2-b^2}\right]\left[\frac{2a+b}{2ab}\right]=$$

$$\frac{6ab}{4a^2-b^2}\times\frac{2a+b}{2ab}=\frac{3}{2a-b}.$$

5. $\dfrac{a}{a^2-b^2}\times\dfrac{a-b-2b\left(\dfrac{a-b}{a+b}\right)}{\dfrac{a-b}{b}\times\dfrac{a}{a+b}}$

$$\frac{a}{a^2-b^2}\times\frac{a^2-2ab+b^2}{a+b}\times\frac{b}{a-b}\times\frac{a+b}{a}=\frac{b}{a+b}.$$

6. $a-[2b+3a-(3b-2a-a-b+2a)-b-3a]$

$a-[2b+3a\ \ 3b+2a+a+b-2a-b-3a]$

$a-2b-3a+3b-2a-a-b+2a+b+3a=b.$

7. $\left(\frac{x}{1+x}+\frac{1-x}{x}\right)\div\left(\frac{x}{1+x}-\frac{1-x}{x}\right)=\frac{1}{x+x^2}\times\frac{x+x^2}{2x^2-1}=\frac{1}{2x^2-1}.$

9. $\left(\frac{a^2-ax}{a^2-x^2}\times\frac{a^3-x^3}{a^2-2ax+x^2}\right)\left(\frac{a^2-x^2}{a^2}\right)=\frac{a^2+ax+x^2}{a}.$

12. $\frac{x^2+xy+y^2}{x+\sqrt{xy}+y}\times\frac{x^2+1}{x-\sqrt{xy}+y}=\frac{x^2+xy+y^2}{x+\sqrt{xy}+y}\times\frac{x^2+1}{x\,(x-\sqrt{xy}+y)}=$
$\frac{x^2+1}{x}=x+\frac{1}{x}.$

14. $\left(\frac{3a+b}{2a-b}-\frac{3a-b}{2a+b}\right)\left(\frac{4a^2-b^2}{2a}\right)=\frac{10ab}{4a^2-b^2}\times\frac{4a^2-b^2}{2a}=\frac{10ab}{2a}=5b.$

Page 228.

16. $\frac{a^4-2a^2b^2+b^4}{ab\,(a^2+b^2)}\times\frac{a^2+b^2}{a^2-b^2}\times\frac{4a^2b^2}{a^2-b^2}=4ab.$

18. $\left(\frac{2}{x}-\frac{1}{a+x}+\frac{1}{a-x}\right)\div\left(\frac{a+x}{a-x}-\frac{a-x}{a+x}\right)=$
$\frac{2a^2}{x\,(a^2-x^2)}\times\frac{a^2-x^2}{4ax}=\frac{a}{2x^2}.$

19. $\frac{b\,(a-b)}{a\,(a-b)}=\frac{b}{a};\ \frac{b}{a}+\frac{a^2-b^2}{ab}=\frac{b^2+a^2-b^2}{ab}=\frac{a^2}{ab}=\frac{a}{b};$
$\frac{a}{b}\times\frac{b\,(a-b)}{a\,(a+b)}=\frac{a-b}{a+b};\ \frac{a-b}{a+b}\times\frac{(a+b)^2}{a^2-b^2}=1.$

21. *a.* $8a^3-b^3=(2a-b)\ (4a^2+2a^2b+b^2)$
$a^4+a^2b^2+b^4=a^4+2a^2b^2+b^4-a^2b^2=(a^2+b^2)^2-a^2b^2=$
$(a^2+b^2+ab)\ (a^2+b^2-ab)$

b. $6m^2+5mn-6n^2=(3m-2n)\ (2m+3n).$

22. $9a^2-\frac{4}{9}=(3x+\frac{2}{3})\ (3x-\frac{2}{3})$
$ab+bc-b^2-ac=ab-b^2-ac+bc=b\,(a-b)-c\,(a-b)=$
$(a-b)\ (b-c).$

Page 229.

23. 1. $2ac-3bd-6bc+ad=2ac+ad-3bd-6bc=$
$(2ac+ad)-(6bc+3bd)=a\,(2c+d)-(2c+d)\,3b=$
$(2c+d)\ (a-3b).$

3. $x^6-64=(x^3+8)\ (x^3-8)=(x+2)\ (x-2)\ (x^2-2x+4)\ (x^2+2x+4).$

24. 3. $1-x-x^2+x^3=(1-x)-x^2\,(1-x)=(1-x)\ (1-x^2)=$
$(1-x)\ (1+x)\ (1-x).$

4. $x^8+x^4+1=x^8+2x^4+1-x^4=(x^4+1)^2-x^4)=$
$(x^4+1+x^2)\ (x^4+1-x^2)=(x^4+2x^2+1-x^2)\ (x^4-x^2+1)=$
$(x^2+1+x)\ (x^2+1-x)\ (x^4-x^2+1).$

25. 1. $16+4x^2+x^4=16+8x^2+x^4-4x^2=(4+x^2+2x)\ (4+x^2-2x)$.

4. $x^{3m}+y^{3n}=(x^m+y^n)\ (x^{2m}-x^my^n+y^{2n})$.

26. 1. $x^9-y^9=(x^3+y^3)\ (x^3-y^3)=$
$(x+y)\ (x-y)\ (x^2-xy+y^2)\ (x^2+xy+y^2)$.

29. 2. $256-x^8=2^8-x^8=(2^4+x^4)\ (2^4-x^4)=$
$(16+x^4)\ (4+x^2)\ (2+x)\ (2-x)$.

30. 2. $a^3-3a^2-10a=a\ (a+2)\ (a-5)$.

31. $12x+72-88x+792+66x+99=704+33x+44$
$43x=215$. Whence $x=5$.

32. $\left.\begin{array}{l}\frac{1}{x}+\frac{2a}{y}=\frac{3b+4a^2}{6ab}\\ \frac{b}{x}-\frac{1}{y}=\frac{3b^2-2a}{6ab}\end{array}\right\}\quad\left.\begin{array}{l}\frac{1}{x}+\frac{2a}{y}=\frac{3b+4a^2}{6ab}\\ \frac{2ab}{x}-\frac{2a}{y}=\frac{3b^2-2a}{3b}\end{array}\right\}$

$\frac{1}{x}+\frac{2ab}{x}=\left[\frac{3b+4a^2+6ab^2-4a^2}{6ab}=\frac{3b+6ab^2}{6ab}\right]=\frac{1+2ab}{2a}$.

$\frac{1+2ab}{x}=\frac{1+2ab}{2a}$ $\quad\therefore x=2a$, and $y=3b$.

33. $\left.\begin{array}{l}\frac{a}{x}+\frac{b}{y}=c\\ \frac{a}{y}-\frac{b}{x}=d\end{array}\right\}\quad\left.\begin{array}{l}ay+bx=cxy\\ ax-by=dxy\end{array}\right\}$

$\left.\begin{array}{l}cxy-ay=bx\\ y=\frac{bx}{cx-a}\end{array}\right\}\quad\left.\begin{array}{l}dxy+by=ax\\ y=\frac{ax}{dx+b}\end{array}\right.$

$\therefore\ \frac{bx}{cx-a}=\frac{ax}{dx+b}$

and $x=\frac{a^2+b^2}{ac-bd}$

By eliminating x, $y=\frac{a^2+b^2}{ad+bc}$.

34. $\left.\begin{array}{l}\frac{1}{x}-\frac{1}{y}=a\\ \frac{1}{y}+\frac{1}{z}=b\\ \frac{1}{z}-\frac{1}{x}=c\end{array}\right\}\quad\left.\begin{array}{l}\frac{1}{x}+\frac{1}{z}=a+b\\ \frac{1}{z}-\frac{1}{x}=c\end{array}\right\}\quad\left.\begin{array}{l}\frac{2}{x}=a+b-c\\ \frac{1}{x}=\frac{a+b-c}{2}\end{array}\right.$

The values of y and z are easily found.

$x=\frac{2}{a+b-c}$.

35. $\left.\begin{matrix} cx+by=3bc \\ 2bx+cy=2(b^2+c^2) \end{matrix}\right\}$ $y=\dfrac{3bc-cx}{b},\ y=\dfrac{2b^2+2c^2-2bx}{c}$

$\therefore 3bc^2-c^2x=2b^3+2bc^2-2b^2x$

$2b^2x-c^2x=2b^3-bc^2$

$\left.\begin{matrix} x=b \\ y=2c. \end{matrix}\right\}$

36. $\left.\begin{matrix} abx+cdy=m \\ ay-cx=n \end{matrix}\right\}$

$y=\dfrac{m-abx}{cd}$

$y=\dfrac{n+cx}{a}$

$\therefore \dfrac{m-abx}{cd}=\dfrac{n+cx}{a}$

$\left.\begin{matrix} x=\dfrac{am-cdn}{c^2d+a^2b} \\ y=\dfrac{abn+mc}{a^2b+c^2d}. \end{matrix}\right\}$

37. $\left.\begin{matrix} \dfrac{a}{x}-\dfrac{b}{y}=c \\ \dfrac{c}{x}+\dfrac{1}{y}=ab \end{matrix}\right\}$

$\dfrac{bc}{x}+\dfrac{b}{y}=ab^2$

$\dfrac{a}{x}-\dfrac{b}{y}=c$

$\dfrac{bc+a}{x}=ab^2+c$

$\dfrac{1}{x}=\dfrac{ab^2+c}{bc+a}$

$x=\dfrac{bc+a}{ab^2+c}$

$y=\dfrac{a+bc}{a^2b-c^2}.$

Page 230.

38. $\left.\begin{matrix} \dfrac{a}{x}+\dfrac{b}{y}=\dfrac{c}{d} \\ \dfrac{m}{x}+\dfrac{n}{y}=\dfrac{r}{s} \end{matrix}\right\}$ $\left.\begin{matrix} \dfrac{an}{x}+\dfrac{bn}{y}=\dfrac{cn}{d} \\ \dfrac{bm}{x}+\dfrac{bn}{y}=\dfrac{br}{s} \end{matrix}\right\}$ $\dfrac{an}{x}-\dfrac{bm}{x}=\dfrac{cn}{d}-\dfrac{bn}{s}$

$\dfrac{an-bm}{x}=\dfrac{cns-brd}{ds}$

$\left.\begin{matrix} \dfrac{1}{x}=\dfrac{cns-brd}{(an-bm)\,ds} \end{matrix}\right\}$ $x=\dfrac{(an-bm)\,ds}{cns-brd}.$

39. $\left.\begin{matrix} \dfrac{x+y}{4}+\dfrac{x-y}{3}=5 \\ 2x-3(x-2y)=9 \end{matrix}\right\}$ $\left.\begin{matrix} 7x-y=60 \\ -x+6y=9 \end{matrix}\right\}$ $41x=369$

$x=9,\ y=3.$

40. $\dfrac{2x+a}{b}-\dfrac{2a-x}{a}=\dfrac{3x-b}{b}$

$2ax+a^2-2ab+bx=3ax-ab$

$x=\dfrac{ab-a^2}{b-a}=\dfrac{a(b-a)}{b-a}=a.$

41. $\frac{a^2c}{b}+x^2=(a+x)\ (b+x)-a\,(b+c)$

$a^2c+bx^2=b\,(a+x)\ (b+x)-ab\,(b+c)$

$a^2c+bx^2=ab^2+b^2x+abx+bx^2-ab^2-abc$

$b^2x+abx=a^2c+abc$

$x\,(b^2+ab)=ac\,(a+b)$

$x=\frac{ac\,(a+b)}{b\,(a+b)}=\frac{ac}{b}.$

48. $4x^4+2x^3+4x^2+39x-9)\ 8x^4+20x^2+51x\ +\ 9(2$

$\underline{8x^4+\ 4x^3+\ 8x^2+78x-18}$

$-4x^3+12x^2-27x+27$

$4x^3-12x^2+27x-27)4x^4+\ 2x^3+\ 4x^2+\ 39x-\ 9(x+14$

$\underline{4x^4-12x^3+\ 27x^2-\ 27x}$

$14x^3-\ 23x^2+\ 66x-\ 9$

$56x^3-\ 92x^2+264x-\ 36$

$\underline{56x^3-168x^2+378x-378}$

$38)\ 76x^2-114x+342$

$2x^2-\ 3x+\ 9$

G. C. D. $=2x^2-3x+9)4x^3-12x^2+27x-27(2x-3$

$\underline{4x^3-\ 6x^2+18x}$

$-\ 6x^2+\ 9x-27$

$\underline{-\ 6x^2+\ 9x-27}$

$0.$

51. $\frac{1}{4}a+\frac{1}{4}a^{\frac{1}{2}}b^{\frac{1}{2}}-\frac{1}{2}b=\left(\frac{1}{2}a^{\frac{1}{2}}+b^{\frac{1}{2}}\right)\left(\frac{1}{2}a^{\frac{1}{2}}-\frac{1}{2}b^{\frac{1}{2}}\right)$

$\frac{1}{8}a-\frac{3}{8}a^{\frac{1}{2}}b^{\frac{1}{2}}+b^{\frac{1}{4}}=\left(\frac{1}{4}a^{\frac{1}{2}}-\frac{1}{2}b^{\frac{1}{2}}\right)\left(\frac{1}{2}a^{\frac{1}{2}}-\frac{1}{2}b^{\frac{1}{2}}\right)$

$\therefore$ the G. C. D. $=\frac{1}{2}a^{\frac{1}{2}}-\frac{1}{2}b^{\frac{1}{2}}.$

52. 1. $\sqrt{a^2b-2ab^2+b^3}=\sqrt{b\,(a^2-2ab+b^2}=(a-b)\sqrt{b}.$

2. $3\sqrt{\frac{2}{5}}-2\sqrt{\frac{1}{10}}=3\sqrt{\frac{10}{25}}-2\sqrt{\frac{100}{100}}=\frac{3}{5}\sqrt{10}-\frac{1}{5}\sqrt{10}=\frac{2}{5}\sqrt{10}.$

53. 1. $(\sqrt{a^3b^2})\ (\sqrt[3]{a^2b^3})=(a^3b^2)^{\frac{1}{2}}.(a^2b^3)^{\frac{1}{3}}=(a^3b^2)^{\frac{3}{6}}.(a^2b^3)^{\frac{2}{6}}=$

$(a^9b^6)^{\frac{1}{6}}.(a^4b^6)^{\frac{1}{6}}=(a^{13}b^{12})^{\frac{1}{6}}=\sqrt[6]{a^{13}b^{12}}=a^2b^2\sqrt[6]{a}.$

2. $\sqrt{\frac{a^2b}{3}}+a\sqrt{12b}-\sqrt{3b^3}=\frac{a}{3}\sqrt{3b}+2a\sqrt{3b}-b\sqrt{3b}=$

$\left(\frac{a}{3}+2a-b\right)\sqrt{3b}=\frac{7a-3b}{3}\sqrt{3b}.$

54. 2. $\sqrt[3]{54a^7b^4}=\sqrt[3]{27a^6b^3.2ab}=3a^2b\sqrt[3]{2ab}.$

3. $\frac{2a}{3x}\left(\sqrt{\frac{27x^4}{a^2}}\right)=\frac{2a}{3x}\sqrt{\frac{9x^4\cdot3}{a^2}}=\frac{2a}{3x}\cdot\frac{3x^2}{a}\sqrt{3}=2x\sqrt{3}.$

Page 231.

55. $\sqrt[3]{\frac{128x^6y^2}{81z^4}} \div \sqrt{\frac{16x^3y^2}{3z^7}} = \frac{4x^2}{3z}\sqrt[3]{\frac{2y^2}{3z}} \div \frac{4xy}{z^3}\sqrt{\frac{x}{3z}} = \frac{z^2}{3y}\sqrt{12x^3y^4z}.$

56. 1. $\frac{1}{2a+b}\sqrt{b(4a^2+4ab+b^2)} = \frac{2a+b}{2a+b}\sqrt{b} = \sqrt{b}.$

2. $2\sqrt{16a^3b^4c^5} = 2\times 4ab^2c^2\sqrt{ac} = 8ab^2c^2\sqrt{ac}.$

3. $\frac{a}{b}\sqrt[5]{\frac{32a}{b^3}} = \frac{a}{b}\cdot\frac{2}{1}\sqrt[5]{\frac{a}{b^3}} = \frac{2a}{b}\sqrt[5]{\frac{a}{b^3}} = \frac{2a}{b^2}\sqrt[5]{ab^2}.$

4. $\sqrt{a}\times\sqrt[3]{a} = a^{\frac{1}{2}}\times a^{\frac{1}{3}} = a^{\frac{5}{6}} = \sqrt[6]{a^5}.$

57. 2. $5\sqrt{\tfrac{3}{5}} = 5\sqrt{\tfrac{1\cdot 5}{2\cdot 5}} = \tfrac{5}{5}\sqrt{15} = \sqrt{15}.$

4. $(16a^4b^5-8a^6b^3)^{\frac{1}{3}} = \sqrt[3]{8a^3b^3(2ab^2-a^3)} = 2ab\sqrt[3]{2ab^2-a^3}.$

58. $\sqrt[n]{\frac{ab}{(a+b)^{-n}}} + \sqrt[n]{\frac{(a-b)^n}{(ab)^{-1}}} = \sqrt[n]{(a+b)^n ab} + \sqrt[n]{(a-b)^n ab} =$

$(a+b)\sqrt[n]{ab} + (a-b)\sqrt[n]{ab} = 2a\sqrt[n]{ab}$

$\left(\frac{\sqrt{x^3-y^3}}{\sqrt{(x-y)^3}}\right)\left(\frac{\sqrt[3]{x-y^4}}{\sqrt{x^2+xy+y^2}}\right) = \frac{\sqrt[6]{(x^3-y^3)(x^3-y^3)(x^3-y^3)}}{\sqrt[6]{(x-y)^3(x-y)^3(x-y)^3}} \times$

$\frac{\sqrt[6]{(x-y)^4(x-y)^4}}{\sqrt[6]{(x^2+xy+y^2)^3}} = \sqrt[6]{(x-y)^2} = \sqrt[3]{x-y}.$

59. 1. $\sqrt[2x]{a^{3x}b^{2x}} = \sqrt[2]{a^3b^2} = \sqrt{a^2b^2}.a = ab\sqrt{a}.$

2. $\sqrt[b]{m^bn^{2b}} = mn^2.$

3. $\frac{a}{b^n}\sqrt{\frac{b^{2n}}{a^5}} = \frac{a}{b^n}\cdot\frac{b^n}{a^2\sqrt{a}} = \frac{1}{a\sqrt{a}} = \frac{\sqrt{a}}{a^2}.$

4. $\frac{a}{x}\sqrt[n]{\frac{x^n}{a^{-2n}}} = \frac{a}{x}\cdot\frac{x}{a^{-2}} = a^3.$

60. 1. $\sqrt{\frac{1}{a}} \div \sqrt[3]{\frac{1}{b}} = \left(\frac{1}{a}\right)^{\frac{1}{2}} \div \left(\frac{1}{b}\right)^{\frac{1}{3}} = \left(\frac{1}{a}\right)^{\frac{3}{6}} \div \left(\frac{1}{b}\right)^{\frac{2}{6}} = \sqrt[6]{\frac{1}{a^3}\times\frac{b^2}{1}} = \frac{1}{a}\sqrt[6]{a^3b^2}.$

2. $(\sqrt{2}+1)\ (\sqrt{2}-1) = \sqrt{4}-1 = \sqrt{3}.$

61.
$$\left.\begin{aligned} x^2+3xy &= 28 \\ xy+4y^2 &= 8 \end{aligned}\right\}$$
$$x = my$$
$$xy = my^2$$
$$x^2 = m^2y^2$$
$$my^2+4y^2 = 8$$
$$m^2y^2+3my^2 = 28.$$

63.
$$x^2+xy = 12$$
$$xy-y^2 = 2$$
$$x = my$$
$$x^2 = m^2y^2$$
$$xy = my^2$$
$$m^2y^2+my^2 = 12$$
$$my^2-y^2 = 2.$$

64. $3x^2-11x = 70$

$$x = \frac{11\pm\sqrt{121+840}}{6} = \frac{11\pm 31}{6} = \frac{42}{6} \text{ or } \frac{-20}{6} = 7 \text{ or } -3\tfrac{1}{3}.$$

64. $3x^2-11x=70$

$$x=\frac{11\pm\sqrt{121+840}}{6}=\frac{11\pm 31}{6}=\frac{42}{6}\text{ or }\frac{-20}{6}=7\text{ or }-3\tfrac{1}{3}.$$

65. $x^3+y^3=91$

$x^2-xy+y^2=13$

$x+y=7$

$x^2+2xy+y^2=49$

$3xy=36$

$xy=12$

$y=\frac{12}{x}$

$x+\frac{12}{x}=7.$

67. $2x^2+3xy=32$

$3y^2-4xy=16$

$x=my$

$xy=my^2$

$x^2=m^2y^2.$

68. $x=\frac{-6a\pm\sqrt{36a^2+160a^2}}{16}=\frac{a}{2}\text{ or }-\frac{5}{4}a.$

69. $x^2+xy=15$ A.

$xy-y^2=2$ B.

Adding $xy+y^2$ to A,

$x^2+2xy+y^2=25$ presumably

$\therefore x+y=5$

A. $x(x+y)=15$

$5x=15$

$x=3$

$\therefore y=2.$

70. $5x^2-12x=108$

$x=\frac{12\pm\sqrt{144+2160}}{10}=6\text{ or }-3\frac{3}{5}.$

71. $\left.\begin{array}{l}x^2-y^2=8\frac{3}{4}\\ xy+y^2=\frac{7}{4}\end{array}\right\}$

$\left.\begin{array}{l}4x^2-4y^2=35\\ 4xy+4y^2=7.\end{array}\right\}$

Page 232.

72. $\left.\begin{array}{l}x^2+3xy=27\\ 3y^2+2xy=24\end{array}\right\}$

Assume $x=my$.

Or mul. by 8 and 9,

$8x^2+24xy=216$

$27y^2+18xy=216$

Hence $8x^2-27y^2+6xy=0$

Factoring, $(4x+9y)(2x-3y)=0$

$2x-3y=0$

$x=\frac{3}{2}y$

Substituting, $\frac{9}{4}y^2+\frac{9}{2}y^2=27$

$\frac{3}{4}y^2=3$ $\quad y^2=4$ $\quad y=\pm 2$ $\quad x=\pm 3.$

75. $abx^2-(a^2+b^2)x+ab=0$

$$x=\frac{a^2+b^2\pm\sqrt{(a^2+b^2)^2+4a^2b^2}}{2ab}.$$

77. $3x^2-5ax=112a^2$

$$x=\frac{5a\pm\sqrt{25a^2+1344a^2}}{6}=\frac{5a\pm 37a}{6}=\frac{42a}{6}\text{ or }\frac{-32a}{6}=7a\text{ or }-5\tfrac{1}{3}a.$$

83. $\frac{x^2}{a^2}-\frac{x}{a}+1\Big)\frac{x^4}{a^4}+\frac{x^2}{a^2}+1\Big(\frac{x^2}{a^2}+\frac{x}{a}+1$

$$\frac{\frac{x^4}{a^4}-\frac{x^3}{a^3}+\frac{x^2}{a^2}}{\frac{x^3}{a^3}+1}$$

$$\frac{\frac{x^3}{a^3}-\frac{x^2}{a^2}+\frac{x}{a}}{\frac{x^2}{a^2}-\frac{x}{a}+1}$$

$$\frac{\frac{x^2}{a^2}-\frac{x}{a}+1}{0.}$$

84. $3a^{\frac{1}{2}}-2-a^{-\frac{1}{2}})9a-12a^{\frac{1}{2}}-2+4a^{-\frac{1}{2}}+a^{-1}(3a^{\frac{1}{2}}-2-a^{-\frac{1}{2}}$

$$\frac{9a-6a^{\frac{1}{2}}-3}{-6a^{\frac{1}{2}}+7+4a^{-\frac{1}{2}}+a^{-1}}$$

$$\frac{-6a^{\frac{1}{2}}+4+2a^{-\frac{1}{2}}}{3+2a^{-\frac{1}{2}}+a^{-1}}$$

$$\frac{3+2a^{-\frac{1}{2}}+a^{-1}}{0.}$$

87. $\sqrt{2x+3}-\sqrt{2x-2}=\sqrt{8x-23}$

$\sqrt{(2x-2)(2x+3)}=12-2x$

$50x=150$

$x=3.$

88. $\sqrt{x+a}+\sqrt{x-a}=-\sqrt{x}$

$x+a+2\sqrt{x^2-a^2}+x-a=x$

$2\sqrt{x^2-a^2}=-x$

$\sqrt{x^2-a^2}=-\frac{x}{2}$

$x^2-a^2=\frac{x^2}{4}$

$4x^2-4a^2=x^2$

$3x^2=4a^2$

$x^2=\frac{4}{3}a^2$

$x=\frac{2a}{\sqrt{3}}=\frac{2a\sqrt{3}}{3}.$

89. $x\sqrt{x}-2\sqrt{x}=x$

$x-2=\sqrt{x}$

$x^2-4x+4=x$

$x^2-5x=-4$

$\therefore x=4$ or $-1.$

90. $\sqrt{\frac{x}{c}+\frac{a}{b}}+\sqrt{\frac{x}{c}-\frac{a}{b}}=\sqrt{\frac{4x}{c}-\frac{2a}{b}}$

$\sqrt{\frac{x^2}{c^2}-\frac{a^2}{b^2}}=\frac{x}{c}-\frac{a}{b}$

Whence $x=\frac{ac}{b}$.

91. $\sqrt{2x-1}+\sqrt{2x+1}=8$

$\sqrt{4x^2-1}=32-2x$

$128x=1025$

$x=8\frac{1}{128}$.

92. $\sqrt{4x-11}=2\sqrt{x}-1$

$4\sqrt{x}=12$

$\sqrt{x}=3$

$x=9$.

93. $\sqrt{b^2+x}-\sqrt{a^2-x}=a-b$

$\sqrt{(b^2+x)(a^2-x)}=-ab$

$x^2-a^2x+b^2x=0$

$x^2-x(a^2-b^2)=0$

$x=a^2-b^2$.

Page 233.

94. $\sqrt{x+1}+\sqrt{x-2}=\sqrt{2x+3}$

$\sqrt{(x+1)(x-2)}=2$

$(x+1)(x-2)=4$

$x^2-x=6$

$x=3$ or -2.

95. $x^{\frac{1}{2}}-a^{\frac{1}{2}}=(x-b)^{\frac{1}{2}}$.

$\sqrt{x}-\sqrt{a}=\sqrt{x-b}$

$x-2\sqrt{ax}+a=x-b$

$x=\frac{(a+b)^2}{4a}$.

97.

$$\begin{array}{l|l} a^2+b^2+\frac{c^2}{z^2}+2ab-\frac{2ac}{z}-\frac{2bc}{z} & a-\frac{c}{z}+b \\ \hline a^2 & \\ \hline 2a-\frac{c}{z} & b^2+\frac{c^2}{z^2}+2ab-\frac{2ac}{z}-\frac{2bc}{z} \\ & +\frac{c^2}{z}\quad -\frac{2ac}{z} \\ \hline 2a-\frac{2c}{z}+b & b^2\quad +2ab-\frac{2bc}{z} \\ & b^2\quad +2ab-\frac{2bc}{z} \\ \hline & 0. \end{array}$$

102. $\left(2a^2-\frac{b}{2}\right)^7=(2a^2)^7-7.(2a^2)^6.\frac{b}{2}+21.(2a^2)^5.\left(\frac{b}{2}\right)^2-35.(2a^2)^4.\left(\frac{b}{2}\right)^3+35.(2a^2)^3.\left(\frac{b}{2}\right)^4-21.(2a^2)^2.\left(\frac{b}{2}\right)^5+7.2a^2.\left(\frac{b}{2}\right)^6-\left(\frac{b}{2}\right)^7.=128a^{14}-224a^{12}b+168a^{10}b^2-70a^8b^3+\frac{35}{2}a^6b^4-\frac{21}{8}a^4b^5+\frac{7}{32}a^2b^6-\frac{b^7}{128}$.

109. Third term of $(x-3y)^{10} = \frac{10.9}{1.2} \cdot \times x^8 (3y)^2 = \frac{10.9}{1.2} x^8 y^2 = 45x^8y^2$.

Page 234.

111. Twenty-first term of $(2-x)^{22} =$

$$\frac{22 \cdot 21 \cdot 20 \cdot 19 \cdot 18 \cdot 17 \cdot 16 \cdot 15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8 \cdot 9 \cdot 10 \cdot 11 \cdot 12 \cdot 13 \cdot 14 \cdot 15 \cdot 16 \cdot 17 \cdot 18 \cdot 19 \cdot 20} \cdot 2^2 \cdot x^{20} =$$

$11 \cdot 21 \cdot 4x^{20} = 924x^{20}$.

112. The middle term is the 5th.

Problems.

1. x = cost of horse

$\frac{x}{100}$ = rate of gain

$\frac{x^2}{100}$ = the gain

$\therefore x + \frac{x^2}{100} = 144$

$x = 80$.

2. x = length, y = breadth

$$\left.\begin{aligned} xy &= 2400 \\ (x-10)\,(y+10) &= 2500 \end{aligned}\right\}$$

$$\left.\begin{aligned} x &= 60 \\ y &= 40. \end{aligned}\right\}$$

3. x = 1st digit

y = 2d digit

z = 3d digit

$$\begin{cases} 1.\ x = \frac{1}{3}z \\ 2.\ y = 2x \\ 3.\ 100x + 10y + z + 396 = 100z + 10y + x \end{cases}$$

$\therefore x = 2$, $y = 4$, $z = 6$, and the number = 246.

4. A. $x + y = 12$

$10x + y$ = original number

$10y + x$ = resulting number

B. $10x + y = 10y + x + 36$.

5. $$\left.\begin{aligned} x - y &= 2 \\ x^2 + y^2 &= 100. \end{aligned}\right\}$$

6. x = length, y = breadth

$$\left.\begin{aligned} 2x + 2y &= 220 \\ xy &= 2925. \end{aligned}\right\}$$

7. $$\left.\begin{aligned} x + y &= 17 \\ x^2 - y^2 &= 119. \end{aligned}\right\}$$

8. x = length

y = breadth

$$\left.\begin{aligned} 2x + 2y &= 140 \\ xy &= 1200. \end{aligned}\right\}$$

9. x = sum, y = rate

$$\left.\begin{aligned} x + \frac{xy}{100} \cdot \frac{8}{12} &= 496 \\ x + \frac{xy}{100} \cdot \frac{15}{12} &= 510. \end{aligned}\right\}$$

Page 235.

10. $$\left.\begin{aligned} x - y &= 3 \\ x^3 - y^3 &= 513. \end{aligned}\right\}$$

11. x = father's age

y = son's age

$$\left.\begin{aligned} x - 5 &= 6\,(y-5) + 2 \\ x + 5 &= 4\,(y+5) - 20. \end{aligned}\right\}$$

12. $x^2 = \frac{-5 \pm \sqrt{25 + 1656}}{4} = \frac{-5 \pm 41}{4} = \frac{36}{4}$ or $\frac{-46}{4} = 9$ or $-11\frac{1}{2}$

Hence $x = \sqrt{9} = \pm 3$

and $x = \sqrt{-11\frac{1}{2}} = \sqrt{-\frac{23}{2}} = \pm \frac{1}{2}\sqrt{-46}$.

13. Interest $= p.\frac{r}{100}.n = \frac{prn}{100}$.

14. $\left.\begin{array}{l} x + y = 10 \\ x^2 + y^2 = 58. \end{array}\right\}$

15. x = sister's age

$x + 5$ = boy's age

$6x + 30$ = father's age

$8x + 35 = 51$.

16. x = 1st number

$x + 1$ = 2d number

$x^2 + x = 306$

$x = \frac{-1 \pm \sqrt{1 + 1224}}{2} = \frac{-1 \pm 35}{2} = \frac{34}{2}$ or $\frac{-36}{2} = 17$ or -18.

17.

$$x^6 + 6x^5 - 40x^3 + 96x - 64 \;\big|\; x^2 + 2x - 4$$

$$x^6$$

$$3x^4 + 6x^3 + 4x^2 \;\big|\; 6x^5 - 40x^3 + 96x - 64$$

$$6x^5 + 12x^4 + 8x^3$$

$$3x^4 + 12x^3 + 12x^2 \;\big|\; -12x^4 - 48x^3 + 96x = 64$$

$$-12x^2 - 24x + 16 \;\big|\; -12x^4 - 48x^3 + 96x - 64$$

$$3x^4 + 12x^3 \quad -24x + 16 \;\big|\; 0.$$

18. x = numerator

$x + 5$ = denominator

$x + \frac{x+5}{3} = 15$

$x = 10$, $x + 5 = 15$

$\therefore$ the fraction is $\frac{10}{15}$.

20. x = sum, y = rate

$$\left.\begin{array}{l} x + \frac{xy}{100}3 = 460 \\ x + \frac{x}{4} + \left(x + \frac{x}{4}\right)\left(\frac{y+1}{100}\right)3 = 770. \end{array}\right\}$$

21. x = tens, y = units

$10x + y$ = the number

$10y + x$ = resulting number

$x + y = 4x$

$10y + x = 10x + y + 36$.

Page 236.

22. x = length

y = breadth

$\left.\begin{array}{l} 2x + 2y = 46 \\ x^2 + y^2 = 17^2. \end{array}\right\}$

$x = 15$

$y = 8$.

23. See page 174, text-book.

$\sqrt{x} - \sqrt{y} = \sqrt{63} - \sqrt{12} = 3\sqrt{7} - 2\sqrt{3}$.

24. See proof on page 205.

25. Page 218 explains the derivation of the formula $S=\frac{ar^n-a}{r-1}$.

26. $x^{\frac{2}{n}}-5x^{\frac{1}{n}}+6=0$

$$x^{\frac{1}{n}}=\frac{5\pm\sqrt{25-24}}{2}=\frac{5\pm1}{2}=3 \text{ or } 2$$

$\therefore x=3^n$ or 2^n.

27. Find interpretation on pages 96 and 97.

29. Find proof on pages 153 and 154.

30. $3\frac{1}{2}:6=6:x$.

31. $$\frac{1}{\sqrt[3]{a+b}}\times\frac{a^{\frac{2}{3}}-a^{\frac{1}{3}}b+b^2}{a^{\frac{2}{3}}-a^{\frac{1}{3}}+b^2}=\frac{a^{\frac{2}{3}}-a^{\frac{1}{3}}b+b^2}{a+b^3}.$$

32. See page 212, text-book.

34. $$\frac{1}{x+\frac{1}{1+\frac{x+1}{3-x}}}=\frac{1}{x+\frac{1}{\frac{x+\frac{1}{4}}{3-x}}}=\frac{1}{x+\frac{3-x}{4}}=\frac{1}{\frac{4x+3-x}{4}}=\frac{1}{\frac{3x+3}{4}}=\frac{4}{3x+3}.$$

Page 237.

35. $x=$number bought for 20 shillings

$x-40=$number bought for 20 shillings at advanced price

$\frac{20}{x}=$price per piece

$\frac{20}{x-40}=$advanced price per piece

$\frac{5}{4}$ of $\frac{20}{x}=\frac{20}{x-40}$

$5x-200=4x$

$x=200$

If 200 cost 20 shillings

1 cost $\frac{1}{10}$ shilling

20 cost 2 shillings.

36. $x=$number of days A. worked

A. can do $\frac{1}{20}$ of the work in 1 day

B. can do $\frac{1}{12}$ of the work in 1 day

If A. worked x days, B. worked $14-x$ days

Hence $\frac{x}{20}+\frac{14-x}{12}=1$

$x=5$ days.

37. A. + B. = $\frac{1}{4}$ in 1 day
A. + C. = $\frac{1}{6}$ in 1 day
B. + C. = $\frac{1}{12}$ in 1 day
2A. + 2B. + 2C. = $\frac{1}{4}+\frac{1}{6}+\frac{1}{12}$ in 1 day = $\frac{1}{2}$ in one day
A. + B. + C. = $\frac{1}{2}$ of $\frac{1}{2}$ in 1 day, or $\frac{1}{4}$, and $\frac{4}{4}$ in 4 days.

38. x = the common quotient
$2x$ = 1st part
$3x$ = 2d part
$4x$ = 3d part
$2x+3x+4x=343$
$9x=343$
$x=27$
$2x=54$
$3x=81$
$4x=108.$

39. x = one number
$\frac{3}{5}x$ = the other
$x^2-\frac{9}{25}x^2=16.$

40. Bicycler from C. has gone $27+18=45$ miles in 3 hours
Bicycler from S. has gone $27+9=36$ miles in 3 hours
The rate of the man from C. is 15 miles per hour
The rate of the man from S. is 12 miles per hour
x = their distance from C. when they first met
y = their distance from S. when they first met
$x+y=27$
$x:y=15:12$
$12x=15y$
$x=\frac{5}{4}y$
$\frac{5}{4}y+y=27$
15 miles being the first man's rate per hour, they met for the first time at 1 o'clock P. M.

41. Let x = width of frame
The frame may be treated as composed of four sections, two of them $40+x$ inches long each, the other two $25+x$ inches long each, making a total length of $130+4x$ inches
$\therefore (130+4x)\,x=40\times 25$
$x=6.4$ inches.

42. The consecutive numbers = x, $(x+1)$, $(x+2)$, $(x+3)=840$
$x(x+1)(x+2)(x+3)=840$
$(x^2+3x)^2+2(x^2+3x)+1=840$
$x^2+3x+1=29$
$x=4.$

43. $x^{-n}=\frac{1\pm\sqrt{1+80}}{2}=\frac{1\pm 9}{2}=\frac{10}{2}=5$

$x^{-n}=5$, $\therefore x^n=\frac{1}{5}$, and $x=\sqrt[n]{\frac{1}{5}}$.

44. $(x-2)\ (x+2)\ (x-3)\ x=0$

$\therefore\ x^4-3x^3-4x^2+12x=0.$

45. Eighth term of $\left(2a^{\frac{1}{3}}+\frac{1}{2a}\right)^{11}=\frac{11.10.9.8.7.6.5}{1.\ 2.3.4.5.6.7}\left(2a^{\frac{1}{3}}\right)^4\left(\frac{1}{2a}\right)^7=$

$\frac{11.10.3.16}{128}.a^{\frac{4}{3}}.a^{-\frac{21}{3}}=\frac{1680}{4}a^{-\frac{17}{3}}.$

46. $3x^2+15x-2\sqrt{x^2+5x+1}=2$

$x^2+5x+1-\frac{2}{3}\sqrt{x^2+5x+1}=\frac{2}{3}+1=+\frac{5}{3}$

$(x^2+5x+1)^{\frac{1}{2}}=\frac{\frac{2}{3}\pm\sqrt{\frac{4}{9}+\frac{60}{9}}}{2}=\frac{\frac{2}{3}\pm\frac{8}{3}}{2}=\frac{5}{3}$ or $=-\frac{3}{3}$ or -1

$x^2+5x+1=\frac{25}{9}$ or 1

$x^2+5x=\frac{16}{9}$ or 0

$x=\frac{-5\pm\sqrt{25+0}}{2}=\frac{-5\pm5}{2}=\frac{0}{2}$ or $\frac{-10}{2}=0$ or -5

Again, $x=\frac{-5\pm\sqrt{25+64}}{2}=\frac{-5\pm\frac{17}{3}}{2}=\frac{-\frac{15}{3}\pm\frac{17}{3}}{2}=\frac{1}{3}$ or $-\frac{16}{3}$.

Page 238.

47. $x^2+px+q=0$

$x=\frac{-p\pm\sqrt{p^2-4q}}{2}$

Sum:

$(-p+\sqrt{p^2-4q})\div2$

$\underline{(-p-\sqrt{p^2-4q})\div2}$

$-2p\div2=-p.$

Product:

$(-p+\sqrt{p^2-4q})\div2$

$\underline{(-p-\sqrt{p^2-4q})\div2}$

$p^2-(p^2-4q)\div4$

$p^2-p^2+4q\div4=q.$

For $(a+b)\ (a-b)=a^2-b^2.$

48. $r=\sqrt[4]{\frac{18}{2\frac{9}{9}}}=\sqrt[4]{\frac{18}{1}\times\frac{9}{29}}=\sqrt[4]{\frac{162}{29}}=\sqrt[4]{\frac{81}{1}\times\frac{2}{29}}=3\sqrt[4]{\frac{2}{29}}$

$\frac{29}{9}$, $\frac{29}{9}\cdot3\sqrt[4]{\frac{2}{29}}=\frac{29}{3}\sqrt[4]{\frac{2}{29}}$, $\frac{29}{3}\times3\sqrt[4]{\frac{2}{29}}=29\sqrt[4]{\left(\frac{2}{29}\right)^2}$,

$87\sqrt[4]{\left(\frac{2}{29}\right)^3}$, $87\times3\sqrt[4]{\left(\frac{2}{29}\right)^4}=87\times3\times\frac{2}{29}=\frac{522}{29}=18.$

49. $\left[\frac{(a^{m-n})^{m+n}\ (a^n)^{n+p}}{(a^m)^{m-n}}\right]^{\frac{1}{n}}=\left[\frac{a^{m^2-n^2}\ a^{n^2+np}}{a^{m^2-mn}}\right]^{\frac{1}{n}}=\left[\frac{a^{m^2+np}}{a^{m^2-mn}}\right]^{\frac{1}{n}}=$

$[a^{m^2+np-m^2+mn}]^{\frac{1}{n}}=[a^{np+mn}]^{\frac{1}{n}}=a^{p+m}.$

50. $x^3-6x^2+3x=18$

$x^3-6x^2+3x-18=0$

$x^2(x-6)+3(x-6)=0$

$(x^2+3)(x-6)=0$

$x=6$

$x^2=-3$

$x=\pm\sqrt{-3}$.

51. Put $x=my$ then $xy=my^2$ and $x^2=m^2y^2$.

52. 1. $\dfrac{y^4-7y^3+8y^2-12y}{2y^2-2y-60}$

H. C. F. $=y-6$.

2. $\dfrac{\left(1+\frac{1}{x}\right)\times\left(1-\frac{1}{x}\right)^2}{x-\frac{1}{x}}=\dfrac{x+1}{x}\cdot\left(\dfrac{x-1}{x}\right)^2\cdot\dfrac{1}{x^2-1}=\dfrac{x-1}{x^2}$.

53.

	x	=	A.'s time
A.'s distance		B.'s distance	
$\frac{17x}{4}$	=	$\frac{12x}{4}$	$+\frac{9}{2}$
	x	=	$3\frac{3}{5}$ hr. $=3$ hr. 36 min.

55. $n=400,\ a=27,\ d=1$

$l=a+(n-1)\,d=27+399=\$4.26$

$S=\dfrac{a+l}{2}.n=\dfrac{27+426}{2}\times 400=\$906.$

56. $\dfrac{2\sqrt{10}}{3\sqrt{27}}\times\dfrac{15\sqrt{21}}{4\sqrt{15}}\div\dfrac{5\sqrt{14}}{7\sqrt{48}}=\dfrac{30\sqrt{210}}{36\sqrt{45}}\times\dfrac{7\sqrt{48}}{5\sqrt{14}}=\dfrac{\sqrt{210}}{6\sqrt{45}}\times\dfrac{28\sqrt{3}}{\sqrt{14}}=$

$\dfrac{\sqrt{15}}{6\sqrt{45}}\times 28\sqrt{3}=\frac{28}{6}=4\frac{2}{3}$.

57. $(x^2-5x+2)^2-(x^2-5x+2)=+20$

$x^2-5x+2=\dfrac{1\pm\sqrt{1+80}}{2}=\dfrac{1\pm 9}{2}=5$ or -4

$x^2-5x=3$

$x=\dfrac{5\pm\sqrt{25+12}}{2}=\dfrac{5\pm\sqrt{37}}{2}$

$x^2-5x=-6$

$x=\dfrac{5\pm\sqrt{25-24}}{2}=\dfrac{5\pm 1}{2}=3$ or 2.

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5. $\left.\begin{aligned} x+y&=100\\ \sqrt{x}+\sqrt{y}&=14.\end{aligned}\right\}$

6. $\begin{aligned} x-2y&=-3000\\ x+y&=36000.\end{aligned}$

7. $\begin{aligned} \tfrac{1}{8}x&=1420\\ x&=11360.\end{aligned}$

8. $\left.\begin{aligned} \frac{2x}{y+7}&=\tfrac{2}{3}\\ \frac{x+2}{2y}&=\tfrac{3}{5}.\end{aligned}\right\}$

9. $\frac{240}{x}-\frac{240}{x+3}=4.$

10. $\left.\begin{aligned} x^2+y^2&=89\\ (x+y)x&=104.\end{aligned}\right\}$

13. $2m^2+(7m^2+2bc-[3m^2-bc-x]-2x)$
$2m^2+(7m^2+2bc-\ 3m^2+bc+x\ -2x)$
$2m^2+\ 7m^2+2bc-\ 3m^2+bc+x\ -2x$
$6m^2+3bc-x.$

16. $\left.\begin{aligned} \frac{3x}{7}+\frac{2y}{3}&=5\\ x&=y+4\end{aligned}\right\}$
$\frac{3y+12}{7}+\frac{2y}{3}=5.$

18. *b.* $\left.\begin{aligned} x+100&=y-100\\ y+100&=(x-100)2.\end{aligned}\right\}$

19. *a.* $\left.\begin{aligned} \frac{x}{2}+\frac{2y}{3}&=65\\ \tfrac{1}{2}(x+y)&=120-65.\end{aligned}\right\}$

b. $\left.\begin{aligned} x+y+z&=59\\ \frac{x-y}{2}&=5\\ \frac{x-z}{2}&=9.\end{aligned}\right\}$

20. $\frac{3a^{\frac{1}{2}}\times b^{\frac{2}{3}}\times cx\times 2a^{\frac{1}{4}}\times b^2\times c}{(-2a^2b^{\frac{2}{3}}c)^2}=\frac{6a^{\frac{5}{6}}b^{\frac{8}{3}}c^2x}{+4a^4b^{\frac{4}{3}}c^2}=\frac{3b^{\frac{4}{3}}x}{2a^{\frac{19}{6}}}.$

23. $\frac{x+c}{3}-\frac{3(x+c)}{4}=\tfrac{1}{3}(x+c)-c$
$\tfrac{1}{3}-\tfrac{3}{4}=\tfrac{1}{3}-\frac{c}{x+c}.$

24. $\left.\begin{aligned} x+y&=a\\ mx&=y.\end{aligned}\right\}$

25. $x=$ time all can do the work
$2x+3x+4x=1.$

26. (1) $x-y=1$
(2) $x^2-y^2=x+y$
(2) divided by $x+y$ gives (1) for a quotient.

28. $x+\sqrt{a^2+x^2}=\frac{2a^2}{\sqrt{a^2+x^2}}$
$x\sqrt{a^2+x^2}+a^2+x^2=2a^2$
$x\sqrt{a^2+x^2}=a^2-x^2$
$x^2(a^2+x^2)=a^4-2a^2x^2+x^4$
$a^2x^2+x^4=a^4-2a^2x^2+x^4.$

29. $3:4=x:y$
$3y=4x\quad y=\tfrac{4}{3}x$
$x+y:x^2+y^2=7:50$
$x+\tfrac{4}{3}x:x^2+\tfrac{16}{9}x^2=7:50$
$1+\tfrac{4}{3}:x+\tfrac{16}{9}x=7:50$
$50+\tfrac{200}{3}=7x+\tfrac{112}{9}x.$

32. $x+y=100$
$\frac{x}{y}=2+\frac{19}{y}.$

33. $x+y=900$
$x+z=800$
$y+z=700.$

34. $3x=$ A.'s money
$x=$ B.'s money
$3x+400=(x+150)\,2.$

35. $x+y=72$
$\sqrt[3]{x}+\sqrt[3]{y}=6$
$x^{\frac{1}{3}}+y^{\frac{1}{3}}=6$
$x+3x^{\frac{2}{3}}y^{\frac{1}{3}}+3x^{\frac{1}{3}}y^{\frac{2}{3}}+y=216$
$x+y=72$

$3x^{\frac{2}{3}}y^{\frac{1}{3}}+3x^{\frac{1}{3}}y^{\frac{2}{3}}=144$
$x^{\frac{2}{3}}y^{\frac{1}{3}}+x^{\frac{1}{3}}y^{\frac{2}{3}}=48$
$x^{\frac{1}{3}}y^{\frac{1}{3}}\left(x^{\frac{1}{3}}+y^{\frac{1}{3}}\right)=48$
$6\left(x^{\frac{1}{3}}y^{\frac{1}{3}}\right)=48$
$x^{\frac{1}{3}}y^{\frac{1}{3}}=8$
$xy=512$
$y=\frac{512}{x}$
$x+\frac{512}{x}=72$
$x^2+512=72x$
$x^2-72x=-512$
$x=64$
$y=8.$

38. $\frac{x}{y+6}=\frac{1}{2}$
$\frac{x-5}{y}=\frac{1}{4}.$

42. $\frac{x}{6}=$ hours with current
$\frac{x}{3}=$ hours against current
$\frac{x}{6}+\frac{x}{3}=8.$ Ans. 16.

43. $x+y+x^2+y^2=18$
$10xy=60$
$xy=6$
$2xy=12$
$(x+y)+(x^2+2xy+y^2)=30$
$m+m^2=30$
$m^2+m+\frac{1}{4}=30+\frac{1}{4}=1\frac{2}{4}1$
$m+\frac{1}{2}=1\frac{1}{2}$
$m=1\frac{0}{2}=5$
$x+y=5$
$xy=6.$

46. *a.* $x^6-y^6=(x^3+y^3)\,(x^3-y^3)=$
$(x+y)\,(x^2-xy+y^2)\,(x-y)\,(x^2+xy+y^2).$

47. $\frac{5x}{2}+\frac{8y}{3}=171$
$\frac{(x+y)\,36}{12}-171=27.$

48. $x+y+z=13$

$\dfrac{100x+10y+z-8}{y+z}=25$

$100x+10y+z+99=100z+10y+x$

$99x-99z=-99$

$x-z=-1$

$100x+10y+z-8=25y+25z$

$100x-15y-24z=8$

$24x-24z=-24$

$76x-15y=32$

$30x+15y=180$

$106x=212.$

49. $x=$ time required

$x=9\times 5\frac{5}{11}$ minutes

$x=49\frac{1}{11}$ minutes past 9.

50. $1+x\sqrt{x^2+12}=1+x$

$x\sqrt{x^2+12}=x$

$\sqrt{x^2+12}=1.$

51. $\$5x=$ cost per acre

$\$90x=$ total cost

$\dfrac{90x}{5x}=18$, the number of acres

$5x.4x=18\times 160.$

52. $x=$ pounds

$y=$ cost per pound

$\dfrac{216}{y}=x$

$\dfrac{216}{y+1}=x-3$

$216=xy$

$216=xy+x-3y-3.$

53. $x^2+y^2-x-y=18$

$xy+x+y=19$

$2xy+2x+2y=38$

$x^2+y^2-x-y=18$

$(x^2+2xy+y^2)+(x+y)=56$

$m^2+m=56.$

54. B. travelled $\dfrac{x}{19}$ miles every hour

He had travelled $\dfrac{x}{19}\cdot\dfrac{x}{19}=\dfrac{x^2}{361}$ miles when he met A.

A. had travelled when he met B.,

$28+\dfrac{3x}{19}$ miles,

Hence $28+\dfrac{3x}{19}+\dfrac{x^2}{361}=x.$

55. $a-d,\ a,\ a+d=$ the series

$a-d \qquad a^2-2ad+d^2$

$a \qquad a^2$

$a+d \qquad a^2+2ad+d^2$

$3a=18 \qquad 3a^2+2d^2=158$

$a=6 \qquad 108+2d^2=158.$

$3a^2=108$

57. $xy=2400$

$(x-10)\ (y+10)=2500$

$x=\dfrac{2400}{y}$

$\left(\dfrac{2400}{y}-10\right)\ (y+10)=2500$

$\left(\dfrac{2400-10y}{y}\right)\ (y+10)=2500.$

58. $\left.\begin{aligned} x &= \tfrac{1}{3}z \\ 2x &= y \end{aligned}\right\}$

$100x + 10y + z + 396 = 100z + 10y + x$

$99x - 99z = -396$

$x - z = -4$

$\tfrac{1}{3}z - z = -4$

$z - 3z = -12.$

60. $\left(3a^2 - \frac{b}{2}\right)^5 = (3a^2)^5 - 5\cdot(3a^2)^4.\frac{b}{2} + 10\cdot(3a^2)^3.\left(\frac{b}{2}\right)^2 -$

$10\cdot(3a^2)^2.\left(\frac{b}{2}\right)^3 + 5\cdot 3a^2\cdot\left(\frac{b}{2}\right)^4 - \left(\frac{b}{2}\right)^5 =$

$243a^{10} - \frac{405}{2}a^8b + \frac{2\,70}{4}a^6b^2 - \frac{90}{8}a^4b^3 + \frac{15}{16}a^2b^4 - \frac{b^5}{32} =$

$243a^{10} - 405a^8b + 135a^6b^2 - \frac{45}{4}a^4b^3 + \frac{15}{16}a^2b^4 - \frac{b^5}{32}.$

61. $x = \text{cost}$

$\frac{x}{100} = \text{gain per cent.}$

$x + \frac{x^2}{100} = 144.$

63. $\sqrt{7 - \sqrt{48}} = \sqrt{7 - 2\sqrt{12}}$ See also text-book, page 174.

$7 = 4 + 3$

and $12 = 4 \times 3$

$\therefore\ \sqrt{7 - 2\sqrt{12}} = \sqrt{4} - \sqrt{3} = 2 - \sqrt{3}.$

64. $\left.\begin{aligned} \frac{a^4}{a^5} &= a^{4-5} = a^{-1} \\ \frac{a^4}{a^5} &= \frac{1}{a^1}. \end{aligned}\right\}\ \therefore\ a^{-1} = \frac{1}{a^1}.$ See also No. 573.

65. *a.* $a : b = c : d$

$\frac{a}{b} = \frac{c}{d}$

Multiplied by $\frac{b}{c}$, $\frac{a}{c} = \frac{b}{d}$

$\therefore\ a : c = b : d.$

b. $3\tfrac{1}{2} : 6 = 6 . x.$

66. $\left.\begin{array}{l} x+y=z \\ 100x+10y+z+396=100z+10y+x \end{array}\right\}$

$$\left.\begin{array}{r} 99x-99z=-396 \\ x-z=-4 \\ x+y-z=0 \\ x\quad -z=-4. \end{array}\right\}$$

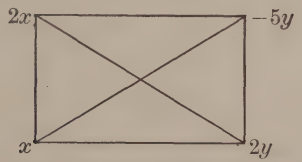
67. $x-\frac{x^2}{100}=16.$

73. $\frac{a^2-ax}{a^2-x^2}\times\frac{a^3-x^3}{a^2-2ax+x^2}\times\frac{a^2-x^2}{a^2}=\frac{a^2+ax+x^2}{a}=a+x+\frac{x^2}{a}.$

74. $\left.\begin{array}{l} abx+cdy=m \\ -cx+ay=n \end{array}\right\}$

$$\left.\begin{array}{l} a^2bx+acdy=am \\ -c^2dx+acdy=cdn. \end{array}\right\}$$

76. *a.* $2x^2-xy-10y^2=(2x-5y)\ (x+2y).$



b. $x^6-y^{12}=x^6-(y^2)^6=[x^3+(y^2)^3]\ [x^3-(y^2)^3]=$
$(x+y^2)\ (x^2-xy^2+y^4)\ (x-y^2)\ (x^2+xy^2+y^4).$

c. $1-a^8=(1+a^4)\ (1-a^4)=(1+a^4)\ (1+a^2)\ (1-a^2)=$
$(1+a^4)\ (1+a^2)\ (1+a)\ (1-a).$

d. $4a^{2m}-4a^mb^n+b^{2n}=(2a^m-b^n)\ (2a^m-b^n).$

e. $a^4+a^2b^2+b^4=a^4+2a^2b^2+b^4-a^2b^2=(a^2+b^2)^2-a^2b^2=$
$(a^2+b^2+ab)\ (a^2+b^2-ab).$

77. *a.* $\overline{a+b}\sqrt{\frac{a-b}{a+b}}=\overline{a+b}\sqrt{\frac{(a+b)\ (a-b)}{(a+b)^2}}=$

$\frac{a+b}{a+b}\sqrt{a^2-b^2}=\sqrt{a^2-b^2}.$

b. $\frac{2b}{a}\sqrt[4]{\frac{a^4}{8b^3}}=2b\sqrt[4]{\frac{1}{8b^3}}=2b\sqrt[4]{\frac{1}{16b^4}}.2b=\frac{2b}{2b}\sqrt[4]{2b}=\sqrt[4]{2b}.$

79. $\frac{4x^2+3x-10}{4x^3+7x^2-3x-15}=\frac{(4x-5)\ (x+2)}{(4x-5)\ (x^2+3x+3)}=\frac{x+2}{x^2+3x+3}.$

80. $x+y=17$
$x^2-y^2=119.$

85. $4x^2+\frac{2x}{a}=\frac{2}{a^2}$

$4a^2x^2+2ax=2$

$x^2+\frac{2a}{4a^2}x=\frac{2}{4a^2}$

$x^2+\frac{1}{2a}x=\frac{1}{2a^2}$

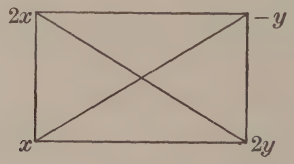
86. *a.* $16+4x^2+x^4=16+8x^2+x^4-4x^2=(4+x^2)^2-4x^2=$
$(4+x^2+2x)\ (4+x^2-2x)$.

b. $27a^3-b^3=3^3a^3-b^3=(3a-b)\ (9a^2+3ab+b^2)$.

c. $a^4-b^4=(a^2+b^2)\ (a^2-b^2)=(a^2+b^2)\ (a+b)\ (a-b)$.

d. $x^{3m}+y^{3n}=(x^m+y^n)\ (x^{2m}-x^my^n+y^{2n})$.

e. $2x^2+3xy-2y^2=(2x-y)\ (x+2y)$.



87. *a.* $\frac{1}{2a+b}\sqrt{4a^2b+ab^2+b^3}=\frac{\sqrt{b(4a^2+4ab+b^2)}}{2a+b}=$
$\sqrt{\frac{b(2a+b)\ (2a+b)}{(2a+b)^2}}=\sqrt{b}$.

b. $2\sqrt[3]{16a^3b^4c^5}=2\cdot 2abc\sqrt[3]{2bc^2}=4abc\sqrt[3]{2bc^2}$.

c. $\frac{a}{b}\sqrt[5]{\frac{32a}{b^3}}=\frac{a}{b}\cdot\frac{2}{1}\sqrt[5]{\frac{a}{b^3}}=\frac{2a}{b}\sqrt[5]{\frac{ab^2}{b^5}}=\frac{2a}{b}\cdot\frac{1}{b}\sqrt[5]{ab^2}$.

d. $\sqrt{a}\times\sqrt[3]{a}=a^{\frac{1}{2}}\times a^{\frac{1}{3}}=a^{\frac{3}{6}}\times a^{\frac{2}{6}}=\sqrt[6]{a^3\cdot a^2}=\sqrt[6]{a^5}$.

89. $\left(\frac{a}{2}-\frac{2}{b}\right)^5=\left(\frac{a}{2}\right)^5-5\left(\frac{a}{2}\right)^4\cdot\frac{2}{b}+10\left(\frac{a}{2}\right)^3\cdot\left(\frac{2}{b}\right)^2-10\left(\frac{a}{2}\right)^2\cdot\left(\frac{2}{b}\right)^3+$
$5\frac{a}{2}\cdot\left(\frac{2}{b}\right)^4-\left(\frac{2}{b}\right)^5=\frac{a^5}{32}-\frac{5a^4}{8b}+\frac{5a^3}{b^2}-\frac{20a^2}{b^3}+\frac{40a}{b^4}-\frac{32}{b^5}$.

91. $x-y=2$

$x^2+y^2=100$.

92. $\sqrt{x+1}+\sqrt{x-2}=\sqrt{2x+3}$

$x+1+2\sqrt{(x+1)\ (x-2)}+x-2=2x+3$

$2\sqrt{(x+1)\ (x-2)}=3-1+2=4$

$\sqrt{(x+1)\ (x-2)}=2$.

93. $x = vy$

$2x^2 = 2v^2y^2$

$xy = vy^2$

$3xy = 3vy^2$

$4xy = 4vy^2$

$\therefore 3y^2 - 4vy^2 = 16$

$y^2 = \dfrac{16}{3-4v}$

$2v^2 + 3vy^2 = 32$

$y^2 = \dfrac{32}{2v^2+3v}$

$\dfrac{32}{2v^2+3v} = \dfrac{16}{3-4v}$

$6 - 8v = 2v^2 + 3v$

$2v^2 + 11v = 6$

$v^2 + \frac{11}{2}v = 3$

$v^2 + \frac{11}{2}v + (\frac{11}{4})^2 = 3 + \frac{121}{16} = \frac{169}{16}$

$v + \frac{11}{4} = \pm \frac{13}{4}$

$v = -\frac{11}{4} \pm \frac{13}{4} = \frac{2}{4}$ or $\dfrac{-24}{4} = \frac{1}{2}$ or -6

$x = 2$

$y = 4.$

or,

$2x^2 + 3xy = 32$

$6y^2 - 8xy = 32$

$2x^2 + 3xy = 6y^2 - 8xy$

$2x^2 - 6y^2 + 11xy = 0$

$(6y^2 + x)\,(y - 2x) = 0$

$y - 2x = 0$

$y = 2x$

$2x^2 + 6x^2 = 32$

$8x^2 = 32$

$x^2 = 4$

$x = \pm 2$

$y = \pm 4.$

94. $\left.\begin{aligned} 2x + 2y &= 220 \\ xy &= 2925 \end{aligned}\right\}$

$x + y = 110$

$y = \dfrac{2925}{x}.$

96.

$$a^4 - a^3 + \frac{a^2}{4} + 4a - 2 + \frac{4}{a^2}\left(a^2 - \frac{a}{2} + \frac{2}{a}\right.$$

$$\underline{a^4}$$

$$2a^2 - \frac{a}{2}\Big) - a^3 + \frac{a^2}{4} + 4a - 2 + \frac{4}{a^2}$$

$$\underline{-a^3 + \frac{a^2}{4}}$$

$$2a^2 - a + \frac{2}{a}\Big) 4a - 2 + \frac{4}{a^2}$$

$$\underline{4a - 2 + \frac{4}{a^2}}$$

$$0.$$

97. $\left.\begin{aligned} x+\frac{2yx}{300}&=496\\ x+\frac{5yx}{400}&=510.\end{aligned}\right\}$

98. $\left(\frac{x}{2}-2y^2\right)^7=\frac{x^7}{2^7}-7\frac{x^6}{2^6}\cdot 2y^2+21\cdot\frac{x^5}{2^5}\cdot(2y^2)^2-35\frac{x^4}{2^4}\cdot(2y^2)^3-35\frac{x^3}{2^3}\cdot(2y^2)^4.$

99. $x-a+\sqrt{x^2-2ax}=b$

$\sqrt{x^2-2ax}=b-x+a$

$x^2-2ax=b^2+x^2+a^2-2bx+2ab-2ax.$

100. $\left.\begin{aligned} x^2-y^2&=\tfrac{35}{4}\\ xy+y^2&=\tfrac{7}{4}\end{aligned}\right\}$

$y(x+y)=\frac{7}{4}$

$\frac{x^2-y^2}{y(x+y)}=\frac{35}{4}\times\frac{4}{7}=5$

$\frac{x-y}{y}=5$

$x-y=5y$

$x=6y.$

101. *a.* $\sqrt{x^3-2x^2y+xy^2}=\sqrt{x(x^2-2xy+y^2)}=(x-y)\sqrt{x}.$

b. $5\sqrt[3]{\tfrac{3}{5}}=5\sqrt[3]{\tfrac{1}{25}\times 15}=\sqrt{15}.$

c. $\sqrt[6]{8a^3b^6}=\sqrt[6]{2^3a^3b^6}=2^{\frac{3}{6}}a^{\frac{3}{6}}b^{\frac{6}{6}}=2^{\frac{1}{2}}a^{\frac{1}{2}}b=b\sqrt{2a}.$

d. $\sqrt{a^2b-a^3}=\sqrt{a^2(b-a)}=a\sqrt{b-a}.$

e. $\sqrt[3]{16a^4b^5-8a^6b^3}=\sqrt[3]{8a^3b^3(2ab^2-a^3)}=2ab\sqrt[3]{2ab^2-a^3}.$

102. $x^{\frac{1}{4}}+y^{\frac{1}{4}})x-y\quad(x^{\frac{3}{4}}-x^{\frac{1}{2}}y^{\frac{1}{4}}+x^{\frac{1}{4}}y^{\frac{1}{2}}-y^{\frac{3}{4}}$

$x+x^{\frac{3}{4}}y^{\frac{1}{4}}$

$-x^{\frac{3}{4}}y^{\frac{1}{4}}-y$

$-x^{\frac{3}{4}}y^{\frac{1}{4}}-x^{\frac{1}{2}}y^{\frac{1}{2}}$

$x^{\frac{1}{2}}y^{\frac{1}{2}}-y$

$x^{\frac{1}{2}}y^{\frac{1}{2}}+x^{\frac{1}{4}}y^{\frac{3}{4}}$

$-x^{\frac{1}{4}}y^{\frac{3}{4}}-y$

$-x^{\frac{1}{4}}y^{\frac{3}{4}}-y$

0.

103. $x-y=3$
$x^3-y^3=513$
$x^2+xy+y^2=171$
$x^2-2xy+y^2=9$
$x^2+xy+y^2=171$
$+3xy=+162$
$y=\frac{162}{3x}=\frac{54}{x}.$

105. *b.* $\frac{50-n}{50}=\frac{2(50-n)}{100}=2(50-n)\ \%.$

107. *a.* $xy-2my+2mn-nx$
$xy-2my-nx+2mn$
$y(x-2m)-n(x-2m)$
$(x-2m)\ (y-n).$

112. $x-y=10$
$x^2+y^2=12850$
$x^2-2xy+y^2=100$
$x^2+y^2=12850.$

111. $\frac{x}{3}-4=\frac{20x}{60}-4$
$\frac{x}{4}+2=\frac{15x}{60}+2$
$\frac{x}{5}+3=\frac{12x}{60}+3$
$\frac{47x}{60}+1$
$+25$
$\frac{47x}{60}+26=x.$

114. *a.* $\sqrt{24a^6b^4c^3n}=\sqrt{4a^6b^4c^2\cdot 6cn}=2a^3b^2c\sqrt{6cn}$
b. $\sqrt{\frac{3}{5}}=\sqrt{\frac{15}{25}}=\frac{1}{5}\sqrt{15}$
c. $\sqrt{75}+\sqrt{12}=\sqrt{25\cdot 3}+\sqrt{4\cdot 3}=5\sqrt{3}+2\sqrt{3}=7\sqrt{3}.$

115. $\$b\times a=\$ab$
$\$c\times d=\$cd$
$\$p\times m=\$mp$
$\}\ \$mp-(ab+cd)=s.$

117. *b.* $2ac+bc+2ad+bd=c(2a+b)+d(2a+b)=(2a+b)\ (c+d).$

119. $\frac{x}{a}+\frac{y}{b}=2$
$bx-ay=0$
$\}\ ay+bx=2ab$
$bx-ay=0$
$\}\ 2bx=2ab\ \therefore\ x=\frac{2ab}{2b}=a.$

120. x = share of the youngest
$x+20$
$x+40$
$x+60$
$x+80$
$5x+200=1000.$

121. $\left.\begin{aligned}10x+4y+3z&=29\\9x+8y+6z&=36\\7x+6y+4z&=27.\end{aligned}\right\}$

122. $\left.\begin{aligned}xy&=126\\\frac{x}{y}&=3\tfrac{1}{2}.\end{aligned}\right\}$

124. *a.* $\sqrt{\tfrac{5}{7}}=\sqrt{\tfrac{35}{49}}=\tfrac{1}{7}\sqrt{35}$

b. $5a^{-2}\times\frac{1}{2}=5\sqrt{\frac{x}{a^2}}=\frac{5}{a}\sqrt{x}$

c. $(-3a^{\frac{1}{2}}b^2)^4=81a^2b^8.$

125. *a.* $a^6-12a^5+35a^4=a^4(a^2-12a+35)=a^4(a-5)\,(a-7).$

b. $16y^4-8y^2+1=(4y^2-1)\,(4y^2-1)=$
$(2y+1)\,(2y-1)\,(2y+1)\,(2y-1).$

d. $x^3+2x^2-x-2=x^2(x+2)-(x+2)=(x+2)\,(x^2-1).$

126. x and $x+1$ are the numbers
$$\frac{x+1}{5}=\frac{x}{3}-\frac{x}{8}.$$

128. x = B.'s age
$x+4$ = A.'s age
$x^2+(x+4)^2=976.$

127. $\left.\begin{aligned}\tfrac{1}{5}(3x-2y)+\tfrac{1}{3}(4x-3y)&=x\\\frac{4x-3y}{2}+\tfrac{2}{3}x-y&=1+y\end{aligned}\right\}$

$\left.\begin{aligned}\frac{3x-2y}{5}+\frac{4x-3y}{3}&=x\\12x-9y+4x-6y&=6+6y.\end{aligned}\right\}$

129. *a.* $\sqrt{a}.\sqrt[3]{a}=a^{\frac{1}{2}}.a^{\frac{1}{3}}=a^{\frac{3}{6}}.a^{\frac{2}{6}}=\sqrt[6]{a^3}.\sqrt[6]{a^2}=\sqrt[6]{a^5}.$

b. $\sqrt{54am^9}=\sqrt{9m^8.6am}=3m^4\sqrt{6am}.$

c. $\dfrac{\sqrt{a^2-b^2}}{\sqrt{a-b}}=\sqrt{a+b}.$

d. $\left(-\dfrac{3a^2}{x^{\frac{1}{2}}y^{-n}}\right)^4=\dfrac{81}{x^2y^{-4n}}$

UNIVERSITY OF PENNSYLVANIA.

130. *a.* $p^2-x^2+4x-4=$
$p^2-(x^2-4x+4)=$
$p^2-(x-2)^2=$
$(p+x-2)\ (p-x+2).$

b. $a^4-2ab^3-b^4+2a^3b=$
$a^4-b^4+2a^3b-2ab^3=$
$a^4-b^4+2ab\ (a^2-b^2)=$
$(a^2+b^2)\ (a+b)\ (a-b)+2ab\ (a+b)\ (a-b)=$
$(a+b)\ (a-b)\ (a^2+2ab+b^2)=$
$(a+b)\ (a-b)\ (a+b)\ (a+b).$

131. *a.* $x^2-3x \quad =x\ (x-3)$
$x^2-9 \quad =(x+3)\ (x-3)$
$x^2-6x+9=(x-3\ (x-3)$
H. C. F. $=x-3.$

b. $x^3-5x^2+\ 9x-9=(x-3)\ (x^2-2x+3)$
$x^4-4x^2+12x-9=(x^2+2x-3)\ (x^2-2x+3)$
H. C. F. $=x^2-2x+3.$

132. $\dfrac{1-2x}{1-x^3}\div\dfrac{1-2x+\ x^2-2x^3}{1+2x+2x^2+\ x^3}=$
$\dfrac{1-2x}{1-x^3}\times\dfrac{1+2x+2x^2+\ x^3}{1-2x+\ x^2-2x^3}=$
$\dfrac{1-2x}{1-x^3}\times\dfrac{(1+x)\ (1+x+x^2)}{(1-2x)\ (1+x^2)}=\dfrac{1}{1+x^2}.$

133. 1. $\dfrac{x+a}{x-b}=\dfrac{(2x+a)^2}{(2x-b)^2}.$

2. $(x+a)\ (2x-b)^2=(x-b)\ (2x+a)^2.$

3. $4x^3-4bx^2+b^2x+4ax^2-4abx+ab^2=$
$4x^3+4ax^2+a^2x-4bx^2-4abx-a^2b.$

134. $\left.\begin{aligned}\dfrac{2x+1}{5}-\dfrac{3y+2}{7}&=2y-x\\ \dfrac{3x-1}{4}+\dfrac{7y+2}{6}&=2x-y\end{aligned}\right\}$
$\left.\begin{aligned}14x+7-15y-10&=70y-35x\\ 18x-6+28y+\ 8&=48x-24y.\end{aligned}\right\}$

135. $\left.\begin{aligned}\dfrac{x}{2}-\ \dfrac{x}{3}&=10\\ 3x\ -\ 2x&=60.\end{aligned}\right\}$

136. *a.* $\sqrt{1-x}\times\sqrt{x-1}=\sqrt{2x-1-x^2}=\sqrt{-(x^2-2x+1)}=$
$\sqrt{-(x-1)^2}=\sqrt{(x-1)^2\times(-1)}=(x-1)\sqrt{-1}.$

b. $\dfrac{3\sqrt{5}-2\sqrt{2}}{2\sqrt{5}-3\sqrt{2}}=\dfrac{(3\sqrt{5}-2\sqrt{2})\ (2\sqrt{5}+3\sqrt{2})}{(2\sqrt{5}-3\sqrt{2})\ (2\sqrt{5}+3\sqrt{2})}=$

$\dfrac{30+5\sqrt{10}-12}{20-18}=\dfrac{18+5\sqrt{10}}{2}=9+\frac{5}{2}\sqrt{10}.$

137. *a.* $\left(\dfrac{2x^{\frac{4}{5}}}{3a^{-2}b^2}\right)^{-5}=\left(\dfrac{3a^{-2}b^2}{2x^{\frac{4}{5}}}\right)^{5}=\dfrac{243a^{-10}b^{10}}{32x^4}=\dfrac{243b^{10}}{32a^{10}x^4}.$

b. $(a^{\frac{1}{3}}+x^{-2})\ (a^{\frac{1}{3}}-x^{-2})=a^{\frac{2}{3}}-x^{-4}=$

$a^{\frac{2}{3}}-\dfrac{1}{x^4}=\sqrt[3]{a^2}-\dfrac{1}{x^4}.$

138. *a.* $(2x+1)(x+2)=3x^2-4$

$2x^2+5x+2=3x^2-4$

$x^2-5x=+6$

$x^2-5x-6=0$

$(x-6)\ (x+1)=0$

$x=6$

$x=-1.$ }

b. $x^2-(3a+2b)x+6ab=0$

$x^2-(3a+2b)x=-6ab$

$x^2-(3a+2b)x+\left(\dfrac{3a+2b}{2}\right)^2=-6ab+\left(\dfrac{3a+2b}{2}\right)^2$

$x-\dfrac{3a+2b}{2}=\pm\sqrt{-6ab+\dfrac{9a^2+12ab+4b^2}{4}}$

$x-\dfrac{3a+2b}{2}=\pm\sqrt{\dfrac{9a^2-12ab+4b^2}{4}}$

c. $\dfrac{x-1}{\sqrt{x}+1}=4+\dfrac{\sqrt{x}-1}{2}.$

$\sqrt{x}-1=4+\dfrac{\sqrt{x}-1}{2}.$

139. *a.* $ax^2+bx+c=0$

$x^2+\dfrac{b}{a}x+\dfrac{c}{a}=0$

$\therefore r_1+r_2=\dfrac{-b}{a}.$

b. $(x-7)\ (x+3)=x^2-4x-21$

$\therefore$ the equation is $x^2-4x=21.$

c. $(1+\sqrt{7})\,(1-\sqrt{7})=1-7=-6$

$(1+\sqrt{7})+(1-\sqrt{7})=2$

$\therefore$ the equation is $x^2-2x=6.$

141. $\left\{x+\frac{1}{x^2}\right\}^5 = x^5+5x^4.\frac{1}{x^2}+10x^3.\left(\frac{1}{x^2}\right)^2+10x^2.\left(\frac{1}{x^2}\right)^3+5x.\left(\frac{1}{x^2}\right)^4+\left(\frac{1}{x^2}\right)^5 =$

$x^5+5x^2+10x^{-1}+10x^{-4}+5x^{-7}+x^{-10}.$

144. $\sqrt{x+\sqrt{x}}+\sqrt{x+\sqrt{x}} = \frac{a\sqrt{x}}{\sqrt{x+\sqrt{x}}}$

$x+\sqrt{x}+x+\sqrt{x} = a\sqrt{x}$

$2x+2\sqrt{x} = a\sqrt{x}$

$2\sqrt{x}+2 = a$

$2\sqrt{x} = a-2$

$\sqrt{x} = \frac{a-2}{2}$

$x = \frac{(a-2)^2}{4} = \left(\frac{a-2}{2}\right)^2.$

145. $x^{\frac{1}{3}}-1+\frac{x^{\frac{1}{3}}}{x^{\frac{1}{3}}-1}+\frac{1}{x^{\frac{1}{3}}+\frac{1}{x^{\frac{1}{3}}+1}} = \frac{x^{\frac{2}{3}}-2x^{\frac{1}{3}}+1+x^{\frac{1}{3}}}{x^{\frac{1}{3}}-1}+\frac{x^{\frac{1}{3}}+1}{x^{\frac{2}{3}}+x^{\frac{1}{3}}+1} =$

$\frac{x^{\frac{2}{3}}-x^{\frac{1}{3}}+1}{x^{\frac{1}{3}}-1}+\frac{x^{\frac{1}{3}}+1}{x^{\frac{2}{3}}+x^{\frac{1}{3}}+1} = \frac{x^{\frac{4}{3}}+2x^{\frac{2}{3}}}{x-1}.$

146. $\frac{\sqrt{xa^{-1}}}{b^{\frac{1}{2}}y^{-\frac{3}{4}}} \div \frac{xa^{-\frac{3}{2}}}{b^2y^{-\frac{1}{2}}} = \frac{x^{\frac{1}{2}}a^{-\frac{1}{2}}}{b^{\frac{1}{2}}y^{-\frac{3}{4}}} \times \frac{b^2y^{-\frac{1}{2}}}{xa^{-\frac{3}{2}}} = \frac{ab^{\frac{3}{2}}y^{\frac{1}{4}}}{x^{\frac{1}{2}}} = ab^{\frac{3}{2}}x^{-\frac{1}{2}}y^{\frac{1}{4}}.$

147. Let x = right hand digit

7 = left hand digit

$10y+x$ = the number by first condition

$10y+x = 7y+7x$ (1)

$10x+y$ = the number by second condition

$10x+y = 4x+4y$ (2)

$3y-6x = 0$ from (1)

$6x-3y = 0$ from (2)

Hence $0x = 0$

and $x = \frac{0}{0}$

$0y = 0$

and $y = \frac{0}{0}$, the symbol of indetermination

Any two digits, one of which is twice the other, will satisfy the conditions of the problem.

148. $\left.\begin{aligned}\frac{1}{x}+\frac{1}{y}&=2\\ xy+\frac{1}{x}+\frac{1}{y}&=8\end{aligned}\right\}$ $\left.\begin{aligned}\frac{1}{x}+\frac{1}{y}&=2\\ \frac{1}{x}+\frac{1}{y}&=8-xy\end{aligned}\right\}$

$8-xy=2$

$xy=6$

$y=\frac{6}{x}.$

$\frac{1}{x}+\frac{x}{6}=2$

$6+x^2=12x$

$x^2-12x=-6$

$x=\frac{12\pm\sqrt{144-24}}{2}=\frac{12\pm\sqrt{120}}{2}=$

$\frac{12\pm2\sqrt{30}}{2}=6\pm\sqrt{30}$

$y=\frac{6}{6\pm\sqrt{30}}=\frac{1}{1\pm\frac{1}{6}\sqrt{30}}.$

149. $a-2d,\ a-d,\ a,\ a+d$

$(a-2d)^2+(a-d)^2+a^2+(a+d)^2=120$

$(a-d)\ a-\ (a-2d)\ (a+d)=\ 8$

$\left.\begin{aligned}&a^2-4ad+4d^2\\ &a^2-2ad+\ d^2\\ &a^2\\ &a^2+2ad+\ d^2\end{aligned}\right\}$

$4a^2-4ad+6d^2=120$

$a^2-ad-a^2+ad+2d^2=\ 8$

$2d^2=+8$

$d^2=+4$

$d=\ 2.$

150. $(a+b+c)^3-3(a+b)\ (b+c)\ (c+a)=$

$a^3+b^3+c^3+3ab^2+3ac^2+3ba^2+3bc^2+3ca^2+3cb^2+6abc-$

$6abc-3b^2c-3ac^2-3bc^2-3a^2b-3ab^2-3a^2c$

$a^3+b^3+c^3+2ab^2+2ac^2+2a^2b+2bc^2+2a^2c+2b^2c+4abc.$

152. *a.* $x^2-2a^2-ax=(x-2a)\ (x+\ a)$

$x^2-4a^2\ \ \ \ \ =(x+2a)\ (x-2a)$

$x^2-6a^2+ax=(x+3a)\ (x-2a)$

H. C. F. $=\ x-2a.$

b. $x^3+x^2+5x+3)2x^3+7x^2-\ 9\ \ \ \ (2$

$\underline{2x^3+2x^2-10x+\ 6}$

$5x^2+10x-15$

H. C. F. $=x^2+\ 2x-\ 3)x^3+\ x^2-5x+3(x-1$

$\underline{x^3+2x^2-3x}$

$-\ x^2-2x+3$

$\underline{-\ x^2-2x+3}$

$0.$

154. $\frac{1}{1-\frac{x}{x-1}}-\frac{1}{\frac{x}{x+1}-1}=\frac{1}{\frac{x-1-x}{x-1}}-\frac{1}{\frac{x-x-1}{x+1}}=\frac{1}{\frac{-1}{x-1}}-\frac{1}{\frac{-1}{x+1}}=$

$\frac{x-1}{-1}-\frac{x+1}{-1}=1-x-1+x=0.$

157. $ax^2+bx+c=0$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

If $a=0$, then

$$x=\frac{-b\pm\sqrt{b^2}}{0}=\frac{-b\pm b}{0}=\frac{0}{0}\text{ and }\frac{-2b}{0}=\infty.$$

158. *a.* $ax^2+a=a^2x+x$

$$x^2-x\left(\frac{a^2-1}{a}\right)=-1$$

$$x=\frac{\frac{a^2-1}{a}\pm\sqrt{\left(\frac{a^2-1}{a}\right)^2-4}}{2}$$

$$x=\frac{a^2-1}{2a}\pm\frac{a^2+1}{2a}=\frac{2a^2}{2a}=a,$$

or $\frac{-2}{2a}=-\frac{1}{a}$.

160.

$$\left.\begin{aligned}\frac{1}{x}-\frac{1}{y}&=\tfrac{1}{2}\\ xy+\frac{1}{x}-\frac{1}{y}&=2\tfrac{1}{2}\end{aligned}\right\}$$

$$xy=2$$

$$y=\frac{2}{x}$$

$$\frac{1}{x}-\frac{x}{2}=\tfrac{1}{2}$$

$$x=\frac{-1\pm\sqrt{1+8}}{2}=1\text{ or }-2.$$

161. $\dfrac{1}{\sqrt{2}+\sqrt{3}+\sqrt{5}}\times\dfrac{\sqrt{2}+\sqrt{3}-\sqrt{5}}{\sqrt{2}+\sqrt{3}-\sqrt{5}}=\dfrac{(\sqrt{2}+\sqrt{3}-\sqrt{5})\sqrt{6}}{12}.$

162. $(2a-3a^2)^4=$

$(2a^4-4.(2a)^3.3a^2+6.(2a)^2.(3a^2)^2-4.2a.(3a^2)^2+(3a^2)^4=$

$16a^4-96a^5+216a^6-216a^7+81a^8.$

163. $\frac{3}{4}$ of $12=9$ $\qquad l=ar^{n-1}=12\times(\frac{3}{4})^4=2\frac{43}{64}$

$\frac{3}{4}$ of $9=\frac{27}{4}=6\frac{3}{4}$ $\qquad lr=2\frac{43}{64}\times\frac{3}{4}=7\frac{29}{256}$

$\frac{3}{4}=r$, $n=5$, $a=12$

$$S=\frac{a-lr}{1-r}=\frac{12-7\frac{29}{256}}{\frac{1}{4}}=\frac{2\frac{43}{64}}{\frac{1}{4}}=2\tfrac{43}{64}=36\tfrac{39}{64}.$$

165. $a^3-b^3-a\,(a^2-b^2)+b\,(a-b)^2$

$(a-b)\,[a^2+ab+b^2-a\,(a+b)+b\,(a-b)]$

$(a-b)\,(a^2+ab+b^2-a^2-ab+ab-b^2)$

$ab\,(a-b).$

166. H. C. F. $= a^2 - 3a + 2$

L. C. Dd. $= (a^2 - 3a + 2)\ (a^2 + 2a + 4)\ (a^2 + 3a - 1)$.

167. $\dfrac{\dfrac{m^2+n^2}{n} - m}{\dfrac{1}{n} - \dfrac{1}{m}} \times \dfrac{m^2 - n^2}{m^3 + n^3} = \dfrac{n}{mn} = m.$

168. $5\frac{5}{11} \times 3 = 16\frac{4}{11}$ minutes past 12 o'clock.

170. $x + y = 25\frac{1}{4} = \frac{101}{4}$

$\sqrt{xy} = \frac{5}{2}$

$4x^2 - 101x = -25$

$x = \dfrac{101 \pm \sqrt{101^2 - 400}}{8} = \dfrac{101 \pm 99}{8} = 25 \text{ or } \frac{1}{4}.$

171. $\sqrt{\left\{\left(x^{\frac{1}{a-b}} . x^{\frac{1}{a+b}}\right)^{a^4-b^4}\right\}^{\frac{1}{a+\frac{b^2}{a}}}} =$

$\sqrt{\left\{\left(x^{\frac{1}{a^2-b^2}}\right)^{a^4-b^4}\right\}^{\frac{1}{a+\frac{b^2}{a}}}} =$

$\sqrt{\left\{x^{2a(a^2+b^2)}\right\}^{\frac{1}{a+\frac{b^2}{a}}}} =$

$\sqrt{\left\{x^{2a(a^2+b^2)}\right\}^{\frac{a}{a^2+b^2}}} = \sqrt{x^{2a^2}} = x^{a^2}.$

172. $\sqrt[2]{x^{-\frac{2}{3}} y^{\frac{1}{2}}} . \sqrt[3]{xy^2} . \sqrt[11]{y^{12}} = x^{-\frac{1}{3}} y^{\frac{1}{4}} . x^{\frac{1}{3}} y^{\frac{2}{3}} . y^{\frac{12}{11}} = y^{\frac{1}{4}+\frac{2}{3}+\frac{12}{11}}.$

173. $\dfrac{m^n}{m^n} = 1 = m^{n-n} = m^0$

$\therefore m^0 = 1.$

See No. 573.

174. $\left(2a^{\frac{1}{2}} - b^{\frac{2}{3}}a\right)^4 = \left(2a^{\frac{1}{2}}\right)^4 - 4.\left(2a^{\frac{1}{2}}\right)^3 . b^{\frac{2}{3}}a + 6.\left(2a^{\frac{1}{2}}\right)^2 . \left(b^{\frac{2}{3}}a\right)^2 -$

$4.2a^{\frac{1}{2}} . \left(b^{\frac{2}{3}}a\right)^3 + \left(b^{\frac{2}{3}}a\right)^4 = 16a^2 - 32a^{\frac{5}{2}}b^{\frac{2}{3}} + 24a^3 b^{\frac{4}{3}} - 8a^{\frac{7}{2}}b^2 + b^{\frac{8}{3}}a^4.$

175. $?+?+1+?+(-3) = \text{series}$

$l = a + (n-1)\,d$

$-3 = a + (5-1)\ (-2)$

$a = 5$

$S = \dfrac{a+l}{2} . n = \dfrac{5-3}{2} \times 5 = 5.$

176. $x^3 - 27 = (x-3)\ (x^2+3x+9)$

$x^2 - 15x + 36 = (x-3)\ (x-12)$

$x^3 - 3x^2 - 2x + 6 = x^2(x-3) - 2\,(x-3) = (x-3)\ (x^2-2)$

L. C. Dd. $= (x-3)\ (x^2+3x+9)\ (x-12)\ (x^2-2)$.

177. $\frac{(8x^3-1)(x^2-x-2)}{(6-3x)(4x^2-1)} = \frac{(2x-1)(4x^2+2x+1)(x-2)(x+1)}{3(2-x)(2x+1)(2x-1)} =$

$\frac{(4x^2+2x+1)(x-2)(x+1)}{3(2-x)(2x+1)}$.

178. $\frac{2a^2c}{\frac{2ac}{a+c}-a} + \frac{2ac^2}{a-c} = \frac{2ac}{\frac{2c}{a+c}-1} + \frac{2ac^2}{a-c} = \frac{2ac}{\frac{2c-a-c}{a+c}} + \frac{2ac^2}{a-c} =$

$\frac{2ac(a+c)}{c-a} + \frac{2ac^2}{a-c} = \frac{-2ac(a+c)}{a-c} + \frac{2ac^2}{a-c} = -\frac{2a^2c}{a-c}$, Ans.

179. $\frac{x-4+\frac{6}{x+1}}{x-\frac{6}{x-1}} \times \frac{1-\frac{x+5}{x^2-1}}{(x-1)(x-2)} = \frac{\frac{x^2-3x-4+6}{x+1}}{\frac{x^2-x-6}{x-1}} \times \frac{x^2-1-x-5}{(x-1)(x-2)} =$

$\frac{x^2-3x+2}{x+1} \times \frac{x-1}{x^2-x-6} \times \frac{x^2-x-6}{x^2-1} \times \frac{1}{(x-1)(x-2)} =$

$\frac{1}{(x+1)(x+1)} = \frac{1}{(x+1)^2}$.

180. $\frac{x-y}{x+y}-1+\frac{y}{y-x}+\frac{x^2}{x^2-y^2} = \frac{x-y}{x+y}-1+\frac{-y}{x-y}+\frac{x^2}{x^2-y^2} =$

$\frac{(x-y)(x-y)-x^2+y^2-xy-y^2+x^2}{x^2-y^2} = \frac{x^2-3xy+y^2}{x^2-y^2}$.

181. $\frac{3}{x+1} - \frac{x+1}{x-1} = \frac{x^2}{1-x^2}$

$\frac{3}{1+x} + \frac{1+x}{1-x} = \frac{x^2}{1-x^2}$

$3(1-x)+(1+x)(1+x)=x^2$.

182. x^2+60 = the number of men

$(x+5)(x-3)-1$ = the number of men

$x^2+60=(x+5)(x-3)-1$.

184. *a.* $\frac{1}{(a-b)-(a-b)x} + \frac{1}{(a+b)+(a+b)x} =$

$\frac{1}{(a-b)(1-x)} + \frac{1}{(a+b)(1+x)} =$

$\frac{(a+b)(1+x)+(a-b)(1-x)}{(a-b)(a+b)(1-x)(1+x)} = \frac{2a+2bx}{(a^2-b^2)(1-x^2)} =$

$\frac{2(a+bx)}{(a^2-b^2)(1-x^2)}$.

184. *b.* $\frac{x^5-y^5}{x^4-y^4}-\frac{1}{x+y}=\frac{(x-y)\ (x^2+xy+y^2)}{(x^2+y^2)\ (x+y)\ (x-y)}-\frac{1}{x+y}=$

$\frac{x^2+xy+y^2}{(x^2+y^2)\ (x+y)}-\frac{1}{x+y}=\frac{x^2+xy+y^2-x^2-y^2}{(x^2+y^2)\ (x+y)}=$

$\frac{xy}{(x^2+y^2)\ (x+y)}$

$\frac{1}{2}\left\{\frac{(x+y)^2-x^2-y^2}{(x^2+y^2)(x+y)}\right\}=\frac{1}{2}\left[\frac{2xy}{(x^2+y^2)\ (x+y)}\right]=\frac{xy}{(x^2+y^2)(x+y)}$

$\frac{xy}{(x^2+y^2)\ (x+y)}-\frac{xy}{(x^2+y^2)\ (x+y)}=0.$

185. *a.* $a^3+a^2b-ab^2-b^3=a^2(a+b)-b^2(a+b)=$

$(a+b)\ (a^2-b^2)=(a+b)\ (a+b)\ (a-b).$

b. $(p+q)x^3+(p-q)x^2-(p+q)x-(p-q)=$

$(p+q)\ (x^3-x)+(p-q)\ (x^2-1)=$

$(p+q)\ (x+1)\ (x-1)\ x+(p-q)\ (x+1)\ (x-1)=$

$(x+1)\ (x-1)\ ([p+q]x+p-q)=$

$(x+1)\ (x-1)\ (px+qx+p-q).$

c. $(a^2-2b^2-c^2)^2-4(b^2-c^2)^2=$

$[a^2-2b^2-c^2+2(b^2-c^2)]\ [a^2-2b^2-c^2-2(b^2-c^2)]=$

$(a^2-2b^2-c^2+2b^2-2c^2)\ (a^2-2b^2-c^2-2b^2+2c^2)=$

$(a^2-3c^2)\ (a^2-4b^2+c^2).$

186. *a.*

$$\left.\begin{array}{l}\frac{9}{x}-\frac{4}{y}=2\\ \frac{18}{x}+\frac{8}{y}=10\end{array}\right\}$$

$$\frac{\frac{18}{x}-\frac{8}{y}=4}{\frac{16}{y}=6.}$$

b.

$$\left\{\begin{array}{l}ax+by=0\\ (a-b)x+(a+b)y=2c\end{array}\right.$$

$ax-bx+ay+by=2c$

$ax-bx+ay-ax=2c$

$by=-ax$

$y=-\frac{ax}{b}$

$ay=-\frac{a^2x}{b}.$

187. $\frac{x}{x-a}-\frac{x+2b}{x+a}=\frac{a^2+b^2}{x^2-a^2}$

$\frac{x\ (x+a)}{x^2-a^2}-\frac{(x+2b)\ (x-a)}{x^2-a^2}=\frac{a^2+b^2}{x^2-a^2}.$

189. $(a-b)\ (a+b-c)+(b-c)\ (b+c-a)+(c-a)\ (c+a-b)=0$

$a^2-b^2-ac+bc+b^2-c^2-ab+ac+c^2-a^2-bc+ab=0$

$0=0.$

190. $x^{16}-a^{16}.$

192. $a^3+b^3=(a+b)(a^2-ab+b^2)$

$a^4+a^2b^2+b^4=a^4+2a^2b^2+b^4-a^2b^2=(a^2+b^2)^2-a^2b^2=$
$(a^2+ab+b^2)(a^2-ab+b^2)$.

$a^2-b^2=(a+b)(a-b)$

L. C. M. $=(a+b)(a^2-ab+b^2)(a^2+ab+b^2)(a-b)$.

193. *a.* $\left(\frac{1+x}{1-x}-\frac{1-x}{1+x}\right)\div\left(\frac{1+x}{1-x}-1\right)=$

$\frac{1+2x+x^2-1+2x-x^2}{1-x^2}\div\frac{1+x-1+x}{1-x}=$

$\frac{4x}{1-x^2}\div\frac{2x}{1-x}=\frac{4x}{1-x^2}\times\frac{1-x}{2x}=\frac{2x}{1+x}$.

b. $\frac{a^2c^2}{b}\times\frac{a}{b^2c^2}=\frac{a^3}{b^3}$

$\frac{a^2b^2}{c}\times\frac{b^3}{a^3}=\frac{b^5}{ac}$.

194. $\frac{(x+2)(x-2)(x-2)(x-1)}{(x-2)(x-2)(x+2)(x-1)}=1$.

197. $x+y=a$

$\frac{x}{y}=b+\frac{c}{y}$.

205. *a.* $\frac{3-x+1-x}{\sqrt{1-x}}=\frac{4-2x}{\sqrt{1-x}}$

$\frac{1}{\sqrt{2}-\sqrt{x}}=\frac{\sqrt{2}+\sqrt{x}}{2-x}$

$\frac{4-2x}{\sqrt{1-x}}\times\frac{\sqrt{2}+\sqrt{x}}{2-x}=\frac{2(\sqrt{2}+\sqrt{x})}{\sqrt{1-x}}$.

b. $\frac{\sqrt{1\div.25^{-4}+2\times125^{-\frac{4}{3}}}}{\frac{9^3.18^4}{54^5}}=\frac{\sqrt{(\frac{1}{4})^4+\frac{2}{125^{\frac{4}{3}}}}}{\frac{1}{6}}=\frac{1}{16}+\frac{2}{6\,2\,5}\times\frac{6}{1}=$

$\frac{625+32}{10000}\times\frac{6}{1}=\frac{3\,9\,4\,2}{1\,0\,0\,0\,0}=.3942$.

207. $\left(\sqrt{a}-\frac{1}{\sqrt{a}}\right)^5=\left(a^{\frac{1}{2}}-\frac{1}{a^{\frac{1}{2}}}\right)^5=a^{\frac{5}{2}}-\frac{5a^2}{a^{\frac{1}{2}}}+\frac{10a^3}{a}$, etc. $=$

$a^{\frac{5}{2}}-5a^{\frac{3}{2}}+10a^2$, etc.

208. $2\sqrt{\frac{1}{8}}-4\sqrt{2}-\sqrt{4\frac{1}{2}}+3\frac{1}{2}\sqrt{8}-8\sqrt{\frac{1}{2}}+6\sqrt{\frac{2}{9}}=$

$\frac{1}{2}\sqrt{2}+7\sqrt{2}+2\sqrt{2}-4\sqrt{2}-\frac{3}{2}\sqrt{2}-4\sqrt{2}=9\frac{1}{2}\sqrt{2}-9\frac{1}{2}\sqrt{2}=0$.

209. $\frac{1}{\sqrt{2}-2\sqrt{3}-\sqrt{5}} \times \frac{\sqrt{2}+2\sqrt{3}-\sqrt{5}}{\sqrt{2}+2\sqrt{3}-\sqrt{5}} = \frac{\sqrt{2}+2\sqrt{3}-\sqrt{5}}{9+4\sqrt{6}}$, and

$\frac{\sqrt{2}+2\sqrt{3}-\sqrt{5}}{9+4\sqrt{6}} \times \frac{9-4\sqrt{6}}{9-4\sqrt{6}} = \frac{(\sqrt{2}+2\sqrt{3}-\sqrt{5})(9-4\sqrt{6})}{81-96} = -15$.

210. $\frac{x-\sqrt{x}}{x+\sqrt{x}} = \frac{4}{x^2-x}$

$\frac{x^2-x}{(x+\sqrt{x})^2} = \frac{4}{x^2-x}$

$4(x+\sqrt{x})^2 = (x^2-x)^2$

$2(x+\sqrt{x}) = x^2-x$

$x+\sqrt{x} = \frac{x^2-x}{2}$

$1 = \frac{x-\sqrt{x}}{2}$

$x-\sqrt{x} = 2$

$\sqrt{x} = \frac{+1 \pm \sqrt{1+8}}{2} = \frac{+1 \pm 3}{2} = 2 \text{ or } -1$

Hence $x = 4$ or 1.

211. $\left.\begin{aligned} x^2+y^2 &= 5 \\ x+y &= \tfrac{3}{2}xy \end{aligned}\right\}$

$x^2+2xy+y^2 = \frac{9}{4}x^2y^2$

$5+2xy = \frac{9}{4}x^2y^2$

$20+8xy = 9x^2y^2$

$9x^2y^2-8xy = 20$

$xy = \frac{+8 \pm \sqrt{64+720}}{18} = \frac{+8 \pm 28}{18} = \frac{36}{18} + 2 \text{ or } -\frac{20}{18} = -\frac{10}{9}$

$\left.\begin{aligned} x+y &= \tfrac{3}{2} \text{ of } 2 = 3 \\ xy &= 2 \end{aligned}\right\}$

$y = \frac{2}{x}$

$x + \frac{2}{x} = 3$

$x^2+2 = 3x$

$x^2-3x = -2.$

212. $x^2-(3m-1)x+2m$

x^2-2x+1 is exactly divisible by $x-1$

$\therefore -(3m-1)x+2m = -2x+1$

$-3m+x+2m = -2x+1$

$3m-x-2m = 2x-1$

$m = 3x-1.$

214. $2x = \text{sum}$

$\frac{5x}{100} = \text{interest on } \frac{1}{2} \text{ sum}$

$\frac{4\frac{1}{2}x}{100} = \text{interest on } \frac{1}{2} \text{ sum}$

$\frac{5x}{100} - \frac{4\frac{1}{2}x}{100} = 60.$

215. *b.* $abx^2-(a^2+b^2)x+ab=0$

$$x=\frac{a^2+b^2\pm\sqrt{(a^2+b^2)^2-4a^2b^2}}{4ab}=$$

$$\frac{a^2+b^2\pm\sqrt{(a^4-2a^2b^2+b^4)}}{4ab}=\frac{a^2+b^2+a^2-b^2}{4ab}=\frac{2a^2}{4ab}=\frac{a}{2b}$$

$$x=\frac{a^2+b^2-a^2+b^2}{4ab}=\frac{2b^2}{4ab}=\frac{b}{2a}.$$

216. $(1+2\sqrt{-3})\ (1-2\sqrt{-3}=13$

$1+2\sqrt{-3}+1-2\sqrt{-3}=\ 2$

$\therefore\ x^2-2x+13=0.$

218. $(x^m+536^m)^2=6'55'36'00^{qm}$

$x^m+536^m\ =2560$

$x=2560-536=2024^m.$

219. *a.* $\left(\frac{1}{\frac{a^{-5}b^{-2}c^2}{a^{-3}b^3c^4}}\right)^2=\frac{1}{\frac{a^{-10}b^{-3}c^4}{a^{-6}b^6c^8}}=\frac{1}{a^{-4}b^{-10}c^{-4}}=a^4b^{10}c^4.$

b. $\frac{x^{\frac{1}{2}}y^{-\frac{3}{2}}}{x^{\frac{2}{3}}y^{\frac{1}{4}}}=\frac{1}{x^{\frac{1}{6}}y^{\frac{7}{4}}}=\frac{1}{x^{\frac{1}{12}}y^{\frac{21}{12}}}=\frac{1}{\sqrt[12]{x^2y^{21}}}=\frac{\sqrt[12]{x^2y^{21}}}{x^2y^{21}}.$

220. $\left.\begin{aligned}\sqrt{5-\sqrt{24}}&=\sqrt{x}-\sqrt{y}\\ \sqrt{5+\sqrt{24}}&=\sqrt{x}+\sqrt{y}\end{aligned}\right\}$

$\left.\begin{aligned}\sqrt{25-24}\ &=x-y\\ x-y&=1\end{aligned}\right\}$

$5-\sqrt{24}=x-2\sqrt{xy}+y$

$\left.\begin{aligned}x+y&=5\\ x-y&=1\end{aligned}\right\}$

$2x=6,\ x=3$

$2y=4,\ y=2$

$\sqrt{x}-\sqrt{y}=\sqrt{3}-\sqrt{2}.$

221. $\frac{7+2\sqrt{6}}{9-3\sqrt{6}}\times\frac{9+3\sqrt{6}}{9+3\sqrt{6}}=\frac{(7+2\sqrt{6})\ (9+3\sqrt{6})}{27}.$

224. $1+(1+b)+(1+2b)+(1+3b)+\ .\ .\ .\ .\ .+(1+nb)$

$d=b=\ 2$

$n=11$

$l=a+(n-1)\ d$

$l=1+(10)\ 2=21$

$S=\frac{a+l}{2}.n=\frac{1+21}{2}.11=121.$

226. *b.* $(x-a)\ (y-b)-(x-b)\ (y-a)$

$xy-bx-ay+ab-xy+ax+by-ab$

$ax-by-ay+by$

$x\ (a-b)-y\ (a-b)$

$(a-b)\ (x-y).$

227. *a.* $\frac{2x-4}{x-3} - \frac{1-x}{2+x} = 3$

$(2x-4)\ (2+x) - (x-3)\ (1-x) = 3\ (x-3)\ (2+x)$

$2x^2 - 8 - x + x^2 - 3x + 3 = 6x - 18 + 3x^2 - 9x.$

c. $(x-5)^2 = 4\ (x-3)^2$

$x^2 - 10x + 25 = 4x^2 - 24x + 36$

$3x^2 - 14x + 11 = 0$

$x = \frac{14 \pm \sqrt{196 - 132}}{6} = \frac{14 \pm 8}{6} = 3\frac{1}{3}$ or 1.

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229. $x^4 - 10x^2 + 1 = 0$

$x^2 = \frac{10 \pm \sqrt{100 - 4}}{2} = \frac{10 \pm \sqrt{96}}{2} = \frac{10 \pm 4\sqrt{6}}{2} = 5 \pm 2\sqrt{6}$

$x = \sqrt{5 \pm 2\sqrt{6}}$

$x = \sqrt{3} + \sqrt{2}$

or $x = \sqrt{3} - \sqrt{2}.$

230. $x^6 - y^6 = (x^3 + y^3)\ (x^3 - y^3) =$

$(x+y)\ (x^2 - xy + y^2)\ (x-y)\ (x^2 + xy + y^2).$

231. $\frac{-(1+x^2)\ 2x - (1-x^2)\ 2x}{(1+x^2)^2} = \frac{-2x - 2x^3 - 2x + 2x^3}{(1+x^2)^2} =$

$\frac{-4x}{(1+x^2)^2}$

$\sqrt{\frac{4x^2}{(1+x^2)^2}}$

$\frac{2x}{1+x^2}$

$\frac{-4x}{(1+x^2)^2} \times \frac{1+x^2}{2x} = -\frac{2}{1+x^2}.$

233. $x + \frac{1}{x} = a + \frac{1}{a}$

$ax^2 - a^2x - x = -a$

$ax^2 - x\ (a^2+1) = -a$

$x = \frac{a^2+1 \pm \sqrt{(a^2+1)^2 - 4a^2}}{2a} = \frac{a^2+1 \pm \sqrt{a^4 \pm 2a^2 + 1 - 4a^2}}{2a} =$

$\frac{a^2+1 \pm (a^2-1)}{2a} = \frac{a^2+1+a^2-1}{2a}$ or $\frac{a^2+1-a^2+1}{2a} = a$ or $\frac{1}{a}.$

234. x = miles per hour

$x-5$ = miles per hour

$\frac{280}{x}$ = hours

$\frac{280}{x-5}$ = hours

$\frac{280}{x-5}-\frac{280}{x}=1.$

235. $x^2+xy+y^2=-21$ A.

$2xy+y^2=-15$ B.

$x^2-xy=-6$ C.

$y=vx$

$y^2=v^2x^2$

$xy=vx^2$

Substituting in B. and C.,

$2vx^2+v^2x^2=-15$

$x^2-vx^2=-6.$

236. x = rate in still water

y = velocity of stream

$x+y$ = rate with stream

$x-y$ = rate against stream

$\frac{8}{x+y}+\frac{8}{x-y}=\frac{3}{2}$ hours

$\frac{16x}{x^2-y^2}=\frac{3}{2}$

1. $3(x^2-y^2)=32x$

$\frac{12}{x+y}+\frac{6}{x-y}=\frac{3}{2}$

$\frac{18x-6y}{x^2-y^2}=\frac{3}{2}.$

2. $x^2-y^2=12x-4y$

$x^2-y^2=\frac{32x}{3}$

$\frac{32x}{3}=12x-4y$

$32x=36x-12y$

$x=3y$

$\frac{48y}{9y^2-y^2}=\frac{3}{2}$

$y=4$ miles

$x=3y=12$ miles per hour for crew.

240. $\frac{x^6-y^6}{x^2-xy+y^2}=\frac{(x^3+y^3)(x^3-y^3)}{x^2-xy+y^2}=$

$\frac{(x+y)(x^2-xy+y^2)(x-y)(x^3+xy+y^2)}{x^2-xy+y^2}=$

$(x+y)(x-y)(x^2+xy+y^2).$

244. $\frac{\sqrt{a}\sqrt[3]{b}}{\sqrt[3]{a}\sqrt[5]{b}}=\frac{a^{\frac{1}{2}}b^{\frac{1}{3}}}{a^{\frac{1}{3}}b^{\frac{1}{5}}}=\frac{a^{\frac{3}{6}}b^{\frac{5}{15}}}{a^{\frac{2}{6}}b^{\frac{3}{15}}}=a^{\frac{1}{6}}b^{\frac{2}{15}}=\sqrt[6]{a}\sqrt[15]{b^2}.$

245. $\frac{\sqrt{x}+\sqrt{x-3}}{\sqrt{x}-\sqrt{x-3}}=\frac{3}{x-3}$

$(x-3)\sqrt{x}+(x-3)\sqrt{x-3}=3\sqrt{x}-3\sqrt{x-3}$

$\sqrt{x^3}-3\sqrt{x}+x\sqrt{x-3}-3\sqrt{x-3}=3\sqrt{x}-3\sqrt{x-3}$

$\sqrt{x^3}-6\sqrt{x}=-x\sqrt{x-3}$

$x^3-12x^2+36x=x^3-3x^2$

$9x^2=36x$

$9x=36$

$x=4.$

246. A. $x+2y=7$
$x^2+3xy+y^2=31$
A. $x^2+4xy+4y^2=49$
$xy+3y^2=18$
A. $x=7-2y$
$xy=7y-2y^2$
$7y-2y^2+3y^2=18$
$y^2+7y=18.$

247. $x^6-3x^3=40$
$x^3=\frac{3\pm\sqrt{9+160}}{2}.$

249. A. + B. $=\frac{1}{12}$ in one day
A. $=\frac{1}{20}$ in one day
B. $=\frac{1}{x}$ in one day
$\frac{1}{20}+\frac{1}{x}=\frac{1}{12}$
$\frac{1}{x}=\frac{1}{12}-\frac{1}{20}=\frac{5-3}{60}=\frac{2}{60}=\frac{1}{30}$
$x=30.$

250.
$x=$ less
$y=$ greater
$y+a=mx$
$x+b=ny$
$y-mx=-a$
$x-ny=-b$
$ny-mnx=-na$
$-ny+x=-b$
$x-mnx=-b-na$
$x(1-mn)=-b-na$
$x=\frac{-b-na}{1-mn}=\frac{b+an}{mn-1}.$

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251. $(a+b)^3(a-b)-a^3(b+a+b)+b^3(a+a+b)$
$(a+b)^3(a-b)-a^3(2b+a)+b^3(2a+b)$
$(a^3+3a^2b+3ab^2+b^3)(a-b)-2a^3b-a^4+2ab^3+b^4$
$a-b$
$a^4+3a^3b+3a^2b^2+ab^3$
$-a^3b-3a^2b^2-3ab^3-b^4$
$a^4+2a^3b-2ab^3-b^4-2a^3b-a^4+2ab^3+b^4=0.$

252. $\frac{x^3+27}{x+3}+\frac{x^3-27}{x-3}=x^2-3x+9+x^2+3x+9=2x^2+18.$

253. I. $(x^4-1)(x^3+1)=(x^2+1)(x+1)(x-1)(x+1)(x^2-x+1).$
II. $(a^2+b^2)^2(a^2+c^2)-(a^2+b^2)(a^2+c^2)^2=$
$(a^2+c^2)[(a^2+b^2)^2-(a^2+b^2)(a^2+c^2)]=$
$(a^2+c^2)(a^2+b^2)(a^2+b^2-a^2-c^2)=(a^2+c^2)(a^2+b^2)(b^2-c^2)=$
$(a^2+c^2)(a^2+b^2)(b+c)(b-c).$

254. I. $(4x+1)^2 = 4x^2+4x+9$

$3x^2+x-2=0$

$(3x-2)\ (x+1)=0$

$x+1=0$

$x=-1$

$3x-2=0$

$3x=2$

$x=\frac{2}{3}$.

255. $x+y=56$

$xy=720$

$y=\frac{720}{x}$

$x+\frac{720}{x}=56$

$x^2+720=56x$

$x^2-56x=-720$

$x=\frac{56\pm\sqrt{56^2-2880}}{2}=\frac{56\pm16}{2}=\frac{72}{2}$ or $\frac{40}{2}=36$ or 20

$y=\frac{720}{36 \text{ or } 20}=20$ or 36.

256. $S=a+ar+ar^2 \ . \ . \ . \ +ar^{n-1}$

Multiply by r

$rS=ar+ar^2 \ . \ . \ . \ ar^{n-1}+ar^n$

Subtract 1st equation from 2d

All the terms disappear except

ar^n and a

Subtraction gives us

ar^n-a

or $a\ (r^n-1)$

Hence $rS-S=a\ (r^n-1)$

$S\ (r-1)=a\ (r^n-1)$

$S=\frac{a\ (r^n-1)}{r-1}$

Since $l=ar^{n-1}$, by substituting l for ar^{n-1}, we have $lr-a$ after subtraction,

and $S=\frac{lr-a}{r-1}$.

257. $3.1 : .002 = 9.9 : x$

$3.1x=.0198$

$x=\frac{.0198}{3.1}=.00638+$.

258. If $a:b=b:c$, then $a:c=a^2:b^2$

Proof:

1. $ac=b^2$
2. $\frac{a^2}{ac}=\frac{a^2}{b^2}$
3. $\frac{a}{c}=\frac{a^2}{b^2}$
4. $a:c=a^2:b^2$

If $a:b=b:c$, then $a:c=b^2:c^2$

1. $ac=b^2$
2. $\frac{ac}{c^2}=\frac{b^2}{c^2}$
3. $\frac{a}{c}=\frac{b^2}{c^2}$
4. $a:c=b^2:c^2$

Since $b^2=ac$,

$b=\sqrt{ac}$.

259. *a.* $4^{-3} = \frac{1}{4^3} = \frac{1}{64}$.

b. $(.02)^{-1} = \frac{1}{.02} = \frac{1}{1\frac{2}{0}\,0} = 1\frac{0}{2}\,0 = 50.$

c. $\frac{1}{(\sqrt[3]{2})^{-6}} = \frac{1}{(2^{\frac{1}{3}})^{-6}} = \left(2^{\frac{1}{3}}\right)^6 = 2^{\frac{6}{3}} = 2^2 = 4$

$4\sqrt{123} = 4 \times 11.0905 = 44.362.$

260. $x = \frac{-p \pm \sqrt{p^2 - 4q}}{2}$

When $p^2 = 4q$,

$\sqrt{p^2 - 4q} = 0$, and

$$\left.\begin{aligned} x &= \frac{-p+0}{2} = \frac{-p}{2} \\ x &= \frac{-p-0}{2} = \frac{-p}{2} \end{aligned}\right\}$$

When $4q$ is numerically greater than p^2, the two roots will be imaginary.

263. $20 = 2 \times 2 \times 5$

$25 = 5 \times 5$

G. C. D. $= 5$

L. C. M. $= 5 \times 5 \times 2 \times 2 = 100$

$100 \times 5 = 500$

$25 \times 20 = 500.$

264. $2a^2x + 3y^2x + 2x^2y + 3a^2y$

$2x^2y - 3a^2y + 2a^2x - 3y^2x$

$2x\,(a^2 + xy) + 3y\,(xy + a^2) = (a^2 + xy)\,(2x + 3y)$

$2x\,(xy + a^2) - 3y\,(a^2 + xy) = (a^2 + xy)\,(2x - 3y)$

H. C. D. $= a^2 + xy.$

266. $S = \frac{a+l}{2} \cdot n = \frac{1+365}{2} \cdot 365 = \frac{3\,6\,6}{2} \cdot 365 = 183 \times 365 = \$667.95.$

267. $x + y = 11$

$x^2 - y^2 + 47 = 20y.$

268. $-2 + \sqrt{5}$ add $-2 - \sqrt{5}$ gives -4

$x^2 + 4x = +1.$

$-2 + \sqrt{5}$ multiply $-2 - \sqrt{5}$ gives $4 - 5 = -1$

269. (1) $\frac{a}{b}=\frac{c}{d}$

(2) $ad=bc$

(3) $3a\,.\,5d=5b\,.\,3c$

(4) $3a:5b=3c:5d$

(5) $3a+5b:5b=3c+5d:5d$

(6) $3a+5b:3c+5d=5b:5d=b:d$

(7) $ad=bc$

(8) $5a\,.\,3d=3b\,.\,5c$

(9) $5a:3b=5c:3d$

(10) $5a+3b=5c+3d:3d$

(11) $5a+3b:5c+3d=3b:3d=b:d$

Comparing (6) and (11),

(12) $3a+5b:3c+5d=5a+3b:5c+3d.$

270. 1. $a+ar+ar^2=52.$

2. $a+ar^2=\frac{10}{3}ar$

$3a+3ar+3ar^2=156$

$3a-10ar+3ar^2=000$

$13ar=156$

$ar=12$

$r=\frac{12}{a}$

$r^2=\frac{144}{a^2}.$

1. $a+ar^2=40$

$a+\frac{144}{a}=40$

$a^2-40a+144=0$

$a=\frac{40\pm\sqrt{1600-576}}{2}=\frac{40\pm32}{2}=\frac{72}{2}$ or $\frac{8}{2}=36$ or 4

$r=\frac{12}{4}=3$

$a=4$

$ar=12$

$ar^2=36.$

271. $\left(a^{\frac{1}{4}}+a^{-\frac{1}{4}}\right)^4=\left(a^{\frac{1}{4}}+\frac{1}{a^{\frac{1}{4}}}\right)^4=$

$\left(a^{\frac{1}{4}}\right)^4+4.\left(a^{\frac{1}{4}}\right)^3.\left(\frac{1}{a^{\frac{1}{4}}}\right)+6.\left(a^{\frac{1}{4}}\right)^2.\left(\frac{1}{a^{\frac{1}{4}}}\right)^2+4.\left(a^{\frac{1}{4}}\right).\left(\frac{1}{a^{\frac{1}{4}}}\right)^3+\left(\frac{1}{a^{\frac{1}{4}}}\right)^4=$

$a+\frac{4a^{\frac{3}{4}}}{a^{\frac{1}{4}}}+\frac{6a^{\frac{1}{2}}}{a^{\frac{1}{2}}}+\frac{4a^{\frac{1}{4}}}{a^{\frac{3}{4}}}+\frac{1}{a}=a+4a^{\frac{1}{2}}+6+\frac{4}{a^{\frac{1}{2}}}+\frac{1}{a}.$

h. c.

272. $x + y = 450$

$\frac{1}{20}x + \frac{1}{25}y = 45$

$x = \$300,\ y = \$150.$

273. II.

$$\begin{aligned} x + y &= 2 \\ x^3 + y^3 &= 2 \\ x^2 - xy + y^2 &= 1 \\ x^2 + 2xy + y^2 &= 4 \\ 3xy &= 3 \\ xy &= 1 \\ x &= \frac{1}{y} \end{aligned}$$

$$\begin{aligned} \frac{1}{y} + y &= 2 \\ 1 + y^2 &= 2y \\ y^2 - 2y^2 &= -1 \\ y &= \frac{2 \pm \sqrt{4-4}}{2} \\ y &= \tfrac{2}{2} = 1 \\ x &= \tfrac{1}{1} = 1. \end{aligned}$$

276. $\frac{a}{b} = \frac{la}{lb} = \frac{mc}{md} = \frac{ne}{nf}$

$a : b = la : lb = mc : md = ne : nf$

$a.lb = la.b$

$a.md = b.mc$

$a.nf = b.ne$

$a\,(lb + md + nf) = b\,(la + mc + ne)$

$\frac{a}{b} = \frac{la + mc + ne}{lb + md + nf}.$

277. $\frac{a\,(a+d)}{c\,(b+c)} =$ [Expanding]

$\frac{a^2 + ad}{c^2 + bc} =$ [Multiplying by $a^{n-1}\,d^{n-1}$]

$\frac{a^{n+1} + a^n d^n}{a^{n-1}d^{n-1}c^2 + a^{n-1}d^{n-1}bc} =$ [Substituting bc for ad, and ad for bc, since $ad = bc$]

$\frac{a^{n+1} + b^n c^n}{a^{n-1}c^2 d^{n-1} + a^n d^n}.$

278. I. $l = a - (n-1)d$

$l = 201 - (100)2 = 201 - 200 = 1$

$S = \frac{a+l}{2} \cdot n = \frac{201+1}{2} \times 101 = 101^2 = 10201.$

II. $S = \frac{a}{1-2} = \frac{\frac{1}{2}}{1-\frac{1}{2}} = \frac{1}{2} \times \frac{2}{1} = 1.$

279. I. $a^m = a'.a.a.a \ldots\ldots$

$a^n = a\ .a.a.a \ldots\ldots$

$a.a.a.a \ldots\ldots \times a.a.a.a \ldots\ldots = a^{m+n}$.

II. $a^{\frac{1}{2}} = \sqrt{a}$

$\left(a^{\frac{1}{2}}\right)^{\frac{1}{3}} = a^{\frac{1}{6}} = \sqrt[6]{a}$

$a^0 = 1$

$(a^{-2})^{-3} = \dfrac{1}{(a^{-2})^3} = \dfrac{1}{a^{-6}} = a^6$

$\dfrac{1}{a^2} \cdot \dfrac{1}{a^3} = \dfrac{1}{a^5}$.

280. $\left\{\dfrac{\sqrt{a+b}}{\sqrt{a-b}} - \dfrac{\sqrt{a-b}}{\sqrt{a+b}}\right\} \dfrac{\sqrt{a^3+b^3}}{\sqrt{a^2-ab+b^2}} =$

$\left\{\dfrac{a+b-a+b}{\sqrt{a^2-b^2}}\right\} \dfrac{\sqrt{a^3+b^3}}{\sqrt{a^2-ab+b^2}} =$

$\dfrac{2b}{\sqrt{a^2-b^2}} \times \dfrac{\sqrt{a^3+b^3}}{\sqrt{a^2-ab+b^2}} = \dfrac{2b}{\sqrt{a^2-b^2}} \sqrt{a+b} = \dfrac{2b}{\sqrt{a-b}}$.

282. $3^3 = 27$

$4^3 = 64$

91

$\sqrt[3]{91} = 4.5$ very nearly.

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283. *a.* $x^2 - 9xy + 20y^2 = (x-5y)\,(x-4y)$.

b. $a^3 + a + b^3 + b = (a^3+b^3) + (a+b) =$

$(a+b)\,(a^2-ab+b^2) + (a+b) = (a+b)\,(a^2-ab+b^2+1)$.

c. $a^2 + x^4 - 1 - 2ax^2 = (x^4 - 2ax^2 + a^2) - 1 = (x^2-a)^2 - 1 =$

$(x^2-a+1)\,(x^2-a-1)$.

d. $(a+3b)^3 - (3a+b)^3 = [(a+3b) - (3a+b)]\ [(a+3b)^2 +$

$(a+3b)\,(3a+b) + (3a+b)^2] = (a+3b-3a-b)$

$(a^2+6ab+9b^2+9a^2+6ab+b^2+3a^2+10ab+3b^2) =$

$2\,(b-a)\,(13a^2+22ab+13b^2)$.

284.

$x^3 + y^3 = 72$

$x^2 - xy + y^2 = 12$

$x + y = 6$

$x^2 + 2xy + y^2 = 36$

$x^2 - xy + y^2 = 12$

$3xy = 24$

$xy = 8$

$y = \dfrac{8}{x}$

$x + \dfrac{8}{x} = 6$

$x^2 + 8 = 6x$

$x^2 - 6x + 8 = 0$

$x = \dfrac{6 \pm \sqrt{36-32}}{2} = \dfrac{6 \pm 2}{2} - \dfrac{8}{2} = 4$ or 2

$y = \dfrac{8}{4 \text{ or } 2} = 2$ or 4.

285. $2x^2+\frac{1}{3}x+5=x$

$2x^2+\frac{1}{3}x-x+5=0$

$2x^2-\frac{2}{3}x+5=0$

$6x^2-2x+15=0$

$$x=\frac{2\pm\sqrt{4-360}}{12}$$

$$x=\frac{2\pm\sqrt{-356}}{12}=\frac{2\pm2\sqrt{-89}}{12}=\frac{1\pm\sqrt{-89}}{6}.$$

286. $(x-1)\,(x^2+x+1)=0.$

287. $$\frac{1}{\sqrt{2}-\sqrt{3}}\times\frac{\sqrt{2}+\sqrt{3}}{\sqrt{2}+\sqrt{3}}=\frac{\sqrt{2}+\sqrt{3}}{2-3}=\frac{\sqrt{2}+\sqrt{3}}{-1}=$$

$$\frac{1.732+1.414+}{-1}=\frac{3.146\ldots}{-1}=-3.146\ldots.$$

288. $ax+by=c$

$a'x+b'y=c'$

$aa'x+a'by=a'c$

$aa'x+ab'y=ac'$

$a'by-ab'y=a'c-ac'$

$y\,(a'b-ab')=a'c-ac'$

$$y=\frac{a'c-ac'}{a'b-ab'}$$

$ab'x+bb'y=b'c$

$a'bx+bb'y=bc'$

$ab'x-a'bx=b'c-bc'$

$x\,(ab'-a'b=b'c-bc'$

$$x=\frac{b'c-bc'}{ab'-a'b}.$$

289. $$(2\sqrt{x}-x^{-1})^4=16x^2-32\sqrt{x}+\frac{24}{x}-\frac{8}{\sqrt{x^5}}+\frac{1}{x^4}.$$

290. $\sqrt{3x+7}-2-\sqrt{x+1}=0$

$\sqrt{3x+7}-\sqrt{x+1}=2$

$3x+7-2\sqrt{(3x+7)\,(x+1)}+x+1=4$

$\sqrt{(3x+7)\,(x+1)}=2x+2$

$3x^2+10x+7=4x^2+8x+4$

$x^2-2x=3$

$x-1=\pm2$

$x=3$ or -1.

Page 268.

291. *a.* $x^2y+3xy^2-3x^3-y^3=(x^2y-y^3)-(3x^3-3xy^2)=$
$y(x^2-y^2)-3x(x^2-y^2)=(x^2-y^2)(y-3x)=$
$(x+y)(x-y)(y-3x)$.

b. $y^2-c^2+2cx-x^2=y^2-(c^2-2cx+x^2)=y^2-(c-x)^2=$
$(y+c-x)(y-c+x)$.

c. $81a^4-16b^4=(9a^2+4b^2)(3a+2b)(3a-2b)$.

292. *a.* $\left(\dfrac{a^{-\frac{1}{2}}}{4c^2}\right)^{-\frac{1}{2}}=\dfrac{a^{\frac{1}{4}}}{2^{-1}c^{-1}}=2a^{\frac{1}{4}}c$.

b. $(\sqrt[b]{x^b}\div\sqrt[a]{x})^{\frac{1}{1-a}}=\left(x\div x^{\frac{1}{a}}\right)^{\frac{1}{1-a}}=\left(x^{1-\frac{1}{a}}\right)^{\frac{1}{1-a}}=\left(x^{\frac{a-1}{a}}\right)^{\frac{1}{1-a}}=$
$x^{\frac{a-1}{a}\cdot\frac{1}{1-a}}=x^{\frac{a-1}{a(1-a)}}=x^{\frac{1-a}{-a(1-a)}}=x^{-a}=\dfrac{1}{x^a}$.

c. $\dfrac{\sqrt{7}+\sqrt{2}}{9+2\sqrt{14}}=\dfrac{\sqrt{7}+\sqrt{2}}{9+2\sqrt{14}}\times\dfrac{9-2\sqrt{14}}{9-2\sqrt{14}}=\dfrac{5\sqrt{7}-5\sqrt{2}}{81-56}=$
$=\dfrac{\sqrt{7}-\sqrt{2}}{5}$.

293.

$$
\begin{array}{l|l}
4x^n-16x^{\frac{n}{2}}+28-24x^{-\frac{n}{2}}+9x^{-n} & 4x^{\frac{n}{2}}-4+3x^{-\frac{n}{2}} \\
\hline
4x^n & \\
\hline
-16x^{\frac{n}{2}}+28-24x^{-\frac{n}{2}}+9x^{-n} & \\
-16x^{\frac{n}{2}}+16 & \\
\hline
12-24x^{-\frac{n}{2}}+9x^{-n} & \\
12-24x^{-\frac{n}{2}}+9x^{-n} & \\
\hline
0. &
\end{array}
$$

294. *b.*
$$\left.\begin{array}{l}\sqrt{x+y}+\sqrt{x-y}=4\\ x^2-y^2=9\end{array}\right\}$$

$x+y+2\sqrt{x^2-y^2}+x-y=16$
$2x+6=16$
$2x=10$
$x=5$
$\therefore y=4,$

296. *a.* 1. $\frac{a^2}{b^2} = \frac{c^2}{d^2}$ and $\frac{a^2}{c^2} = \frac{b^2}{d^2}$.

2. $\frac{a^2}{ab} = \frac{c^2}{cd}$.

3. $a^2 : ab = c^2 : cd$.

4. $a^2 + ab : a^2 = c^2 + cd : c^2$.

5. $a^2 + ab : c^2 + cd = a^2 : c^2$

6. $\frac{a^2+ab}{c^2+cd} = \frac{a^2}{c^2}$.

7. $\frac{ab}{b^2} = \frac{cd}{d^2}$.

8. $\frac{2ab}{b^2} = \frac{2cd}{d^2}$.

9. $b^2 : d^2 = 2ab : 2cd$.

10. $b^2 : 2ab = d^2 : 2cd$.

11. $b^2 - 2ab : b^2 = d^2 - 2cd : d^2$.

12. $b^2 - 2ab : d^2 - 2cd : b^2 : c^2$.

13. $\frac{b^2-2ab}{d^2-2cd} = \frac{b^2}{d^2}$.

14. $\therefore \frac{a^2+ab}{c^2+cd} = \frac{b^2-2ab}{d^2-2cd}$.

b. If $a : b = b : c = c : d$ A

Show $a+b : b+c = b+c : c+d$

$(a+b)(c+d)(b+c)^2$

$ac+ad+bc+bd = b^2+2bc+c^2$ B

From A, $ad = bc$

$b^2 = ac$

$c^2 = ab$

Substituting in B,

$b^2+2bc+c^2 = b^2+2bc+c^2$.

297. If circle A has a radius of 5 ft.
If circle B has a radius of 4 ft.
If circle C has a radius of 3 ft.
then A : B + C = $5^2 : 3^2 + 4^2$.
that is, A : B + C = 25 : 9 + 16.
or A : B + C = 25 : 25.

298. *a.* $\left.\begin{array}{l} a = a \\ d = a+3d \end{array}\right\}$ $a+d = 2a+3d$

$\left.\begin{array}{l} b = a+d \\ c = a+2d \end{array}\right\}$ $b+c = 2a+3d$

$\therefore a+d = b+c$.

b. $\left.\begin{array}{l} a + ar = 72 \\ ar^2 + ar^3 = 8 \end{array}\right\}$

$r^2 = 9$

$r = 3$

$\frac{1}{r} = \frac{1}{3}$

$S = \frac{a-ar^n}{1-\frac{1}{r}}$

$72 = \frac{a-\frac{1}{3}a}{\frac{2}{3}}$

$4a = 216$.

$a = 54$.

299. *a.* $250(a-b)^3+2 = 2[5^3(a-b)^3+1] =$
$2[5(a-b)+1][5^2(a-b)^2-5(a-b)+1] =$
$2[5a-5b+1][25a^2-50ab+25b^2-5a+5b+1]$.

b. $1-a^2x^2 = b^2y^2+2abxy = 1-(a^2y^2-2abxy+b^2y^2) =$
$1-(ax-by)^2 = (1+ax-by)(1-ax+by)$.

c. $x^4+y^4-18x^2y^2 = x^4-2x^2y^2+y^4-16x^2y^2 =$
$(x^2-y^2+4xy)(x^2-y^2-4xy)$.

Page 269.

300. *a.* $\left(a^{n2-1}\right)^{\frac{n}{n+1}} + \frac{\sqrt[n]{a^{2n}}}{a} = a^{\frac{n(n2-1)}{(n+1)}} + \frac{a^2}{a} = a^{n(n-1)} + a.$

b. $\frac{3+\sqrt{5}}{\sqrt{5}-2} = \frac{(3+\sqrt{5})(\sqrt{5}+2)}{(\sqrt{5}-2)(\sqrt{5}+2)} = \frac{5\sqrt{5}+11}{5-4} = 5\sqrt{5}+11.$

c. $\frac{2^n}{2^{n+1}}\frac{(2^{n-1})^n}{2^{n-1}} + \frac{1}{4^{-n}} = \frac{2^n \times 2^{n(n-1)}}{2^{2n}} + \frac{1}{4^{-n}} = \frac{2^{n(n-1)}}{2^n} + \frac{1}{4^{-n}} =$

$\frac{2^{n2-n}}{2^n} + \frac{1}{4^{-n}} = 2^{n2-2n} + 4^n = 2^{n2-2n} + 2^{2n}.$

301.

$$25x^{\frac{4}{3}} - 30x + 16 - 24x^{\frac{1}{3}} + 49x^{\frac{2}{3}} \Big| 5x^{\frac{2}{3}} - 3x^{\frac{1}{3}} + 4$$

$25x^{\frac{4}{3}}$

$10x^{\frac{2}{3}} - 3x^{\frac{1}{3}}$ | $-30x + 16 - 24x^{\frac{1}{3}} + 49x^{\frac{2}{3}}$

$-30x + 9x^{\frac{2}{3}}$

$10x^{\frac{2}{3}} - 6x^{\frac{1}{3}} + 4$ | $40x^{\frac{2}{3}} + 16 - 24x^{\frac{1}{3}}$

$40x^{\frac{2}{3}} + 16 - 24x^{\frac{1}{3}}$

$0.$

302. *a.*

$$\left.\begin{array}{l} ax = by \\ bx + ay = c \end{array}\right\}$$

$abx = b^2y$

$abx = -a^2y + ac$

$b^2y + a^2y = ac$

$y(b^2+a^2) = ac$

$y = \frac{ac}{a^2+b^2}$

$x = \frac{bc}{a^2+b^2}.$

b. $\sqrt{x^2-1} = 2 - y$

$x^2 - 1 = 4 - 4y + y^2$ A

$x + 1 - 2\sqrt{x^2-1} + x - 1 = y$

$2x - 2\sqrt{x^2-1} = y$

$2\sqrt{x^2-1} = 2x - y$

$4x^2 - 4 = 4x^2 - 4xy + y^2$ B

$y^2 - 4xy = -4$ B

$x^2 - y^2 + 4y = 5$ A

B. $y = \frac{4x \pm \sqrt{16x^2-16}}{2} = 2x \pm 2\sqrt{x^2-1}$

$y^2 = 4x^2 \pm 8\sqrt{x^2-1} + 4x^2 - 4$

$y^2 = 8x^2 \pm 8\sqrt{x^2-1} - 4$

A. $x^2 - (4x^2 \pm 8\sqrt{x^2-1} + 4x^2 - 4) + 8x \pm 8\sqrt{x^2-1} = 5$

$x^2 - 4x^2 \mp 8\sqrt{x^2-1} - 4x^2 + 4 + 8x \pm 8\sqrt{x^2-1} = 5$

$x^2 - 8x^2 + 4 + 8x = 5$

$7x^2 - 8x = -1$

$x = \frac{8 \pm \sqrt{64-28}}{14} = \frac{8 \pm 6}{14} = 1 \text{ or } \frac{1}{7}$

B. $y = 2 \pm 2\sqrt{1-1} = 2.$

303. $\frac{x-a}{x+a}+\frac{x+a}{x-a}=5$

$3x^2=7a^2$

$x^2=\frac{7}{3}a^2$

$x=\pm\sqrt{\frac{7}{3}a^2}=\pm\frac{a}{3}\sqrt{21}.$

304. $6x-13y=1$ A

$6x=1+13y$

$x=0+2y+\frac{1+y}{6}$

$\therefore\ x-2y=\frac{1+y}{6}$

$\frac{1+y}{6}=\text{integer}$

$\frac{7+7y}{6}=\text{integer}$

$1+y+\frac{1+y}{6}=\text{integer}$

$\frac{1+y}{6}=\text{integer}$

Let $\frac{1+y}{6}=m$, an integer

$1+y=6m$, or $y=6m-1$ B

Substituting in A,

$6x-78m+13=1$

$6x=78m-12$

$x=13m-2$ C

If $m=1$

then $x=13-2=11$

and $y=6m-1=6-1=5$.

305. See text-book, page 206.

306. $\frac{a}{b}=\frac{c}{d}$

$\frac{ab}{b^2}=\frac{dc}{d^2}$

$ab:b^2=dc:d^2$

By alternation:

$ab:dc=b^2:d^2$

By composition and division:

$\frac{ab+dc}{ab-dc}=\frac{b^2+d^2}{b^2-d^2}$

By squaring:

$\frac{a^2}{b^2}=\frac{c^2}{d^2}$

By alternation:

$a^2:c^2=b^2:d^2$

By composition and division:

$\frac{a^2+c^2}{a^2-c^2}=\frac{b^2+d^2}{b^2-d^2}$

$\therefore\ \frac{ab+dc}{ab-dc}=\frac{a^2+c^2}{a^2-c^2}.$

307. $d=\frac{l-a}{n-1}=\frac{-\frac{32}{5}-\left(-\frac{36}{5}\right)}{17}=\frac{-\frac{32}{5}+\frac{36}{5}=\frac{4}{5}}{17}=\frac{4}{85}$

$-\frac{36}{5}+\frac{4}{85}=\frac{-612+4}{85}=-\frac{608}{85}=-7\frac{13}{85}.$

308. $S=\frac{a}{1-r}=\frac{\frac{1}{2}}{1-\frac{1}{2}}=\frac{1}{2}\times\frac{2}{1}=1.$

309. $\sqrt{(a+b)\ (c+d)}=b+c$

$(a+b)\ (c+d)=(b+c)^2$

A. $ac+bc+ad+bd=b^2+2bc+c^2$

$\frac{b}{a}=\frac{c}{b}$ $\quad$ $\frac{b}{a}=\frac{d}{c}$ $\quad$ $\frac{c}{b}=\frac{d}{c}$

$b^2=ac$ $\quad$ $ad=bc$ $\quad$ $c^2=bd$

Substituting these values in A,

$b^2+bc+bc+c^2=b^2+2bc+c^2$

$\therefore\ \sqrt{(a+b)\ (c+d)}=b+c.$

310. Five terms of $(x-x^{-1})^8 =$

$$x^8-8x^7(x^{-1})+28x^6(x^{-1})^2-56x^5(x^{-1})^3+70x^4(x^{-1})^4=$$

$$x^8-\frac{8x^7}{x}+\frac{28x^6}{x^2}-\frac{56x^5}{x^3}+\frac{70x^4}{x^4}=x^8-8x^6+28x^4-56x^2+70.$$

311. *a.* $28x^4y+64x^3y-60x^2y=4x^2y(7x^2+16x-15)=$

$$4x^2y(7x-5)(x+3).$$

b. $1-x^2-2xy-y^2=1-(x+y)^2=(1+x+y)(1-x-y).$

c. $(a^3+b^3)+(a+b)=(a+b)(a^2-ab+b^2+1).$

Page 270.

312. $\left\{\frac{a^{p-q}}{\sqrt[q]{a^{q^2-pq}}}\times a^{2(p-q)}\right\}^n=\left\{\frac{a^{p-q}}{a^{q-p}}\times a^{2(p-q)}\right\}^n=$

$$\{a^{2p-2q}\times a^{2p-2q}\}^n=(a^{4p-4q})^n=a^{n(4p-q)}=a^{4n(p-q)}.$$

313.

$$\begin{array}{l} 9x-12x^{\frac{1}{2}}+10-4x^{-\frac{1}{2}}+x^{-1}\left(3x^{\frac{1}{2}}-2+x^{-\frac{1}{2}}\right. \\ \underline{9x} \\ 6x^{\frac{1}{2}}-2\Big| \begin{array}{l} -12x^{\frac{1}{2}}+10-4x^{-\frac{1}{2}}+x^{-1} \\ \underline{-12x^{\frac{1}{2}}+\ 4} \end{array} \\ 6x^{\frac{1}{2}}-4+x^{-\frac{1}{2}}\Big| \begin{array}{l} 6-4x^{-\frac{1}{2}}+x^{-1} \\ \underline{6-4x^{-\frac{1}{2}}+x^{-1}} \end{array} \\ \qquad\qquad 0. \end{array}$$

314. *a.*

$$\left.\begin{array}{l} qx-rb=ap-py \\ q\frac{x}{a}+r=p+\frac{py}{b} \end{array}\right\}$$

$$q\frac{x}{a}-\frac{rb}{a}=p-\frac{py}{a}$$

$$q\frac{x}{a}+r=p+\frac{py}{b}$$

$$r+\frac{rb}{a}=\frac{py}{a}+\frac{py}{b}$$

$$abr+rb^2=bpy+apy$$

$$y(bp+ap)=abr+rb^2$$

$$y=\frac{abr+rb^2}{bp+ap}=\frac{br(a+b)}{p(a+b)}=\frac{br}{p}$$

$$\therefore x=\frac{ap}{q}.$$

b.

$$\left.\begin{array}{l} \frac{1}{x^2}+\frac{1}{y^2}=\frac{45}{4} \\ \frac{1}{x}-\frac{1}{y}=\frac{3}{2} \end{array}\right\}\begin{array}{l}\text{A.}\\ \text{B.}\end{array}$$

$$\frac{1}{x^2}-\frac{2}{xy}+\frac{1}{y^2}=\frac{9}{4} \quad \text{B.}$$

$$\frac{2}{xy}=+\frac{36}{4} \quad \text{A. B.}$$

$$\frac{1}{xy}=\frac{18}{4}=\frac{9}{2}$$

$$9xy=2$$

$$xy=\tfrac{2}{9}$$

$$y=\frac{2}{9x}$$

$$\frac{1}{x}-\frac{9x}{2}=\frac{3}{2} \quad \text{B.}$$

$$2-9x^2=3x$$

$$9x^2+3x=2$$

$$x=\frac{-3\pm\sqrt{9+72}}{18}=$$

$$\frac{-3\pm 9}{18}=\tfrac{1}{3} \text{ or } -\tfrac{2}{3}$$

$$y=\frac{2}{9x}=\frac{2}{\frac{9}{3}}=\tfrac{2}{3} \text{ or } -\tfrac{1}{3}.$$

315. $\sqrt{4+8k}=0$ when $k=-\frac{1}{2}$.

316. $8x-21y=33$ A.

$8x=33+21y$

$x=4+2y+\dfrac{1+5y}{8}$

$x-2y-4=\dfrac{5y+1}{8}=\text{integer}$

$\dfrac{25y+5}{8}=\text{integer}$

$3y+\dfrac{y+5}{8}=\text{integer}$

$\dfrac{y+5}{8}=m$

$y+5=8m$

$y=8m-5$

$8x-168m+105=33$ A.

$8x=168m-72$

$x=21m-9$

If $m=1$

then $x=12$

$y=8-5=3.$

317. $a:b=b:c=c:d$

$b^2=ac$

$c^2=bd$

$ad=bc$

$\sqrt{(a+b)\ (c+d)}=b+c$

$ac+ad+bc+bd=b^2+2bc+c^2$

$b^2+bc+bc+c^2=b^2+2bc+c^2$

$b^2+2bc+c^2=b^2+2bc+c^2.$

318. $r=\sqrt[5]{\frac{5}{160}}=\sqrt[5]{\frac{1}{32}}=\frac{1}{2}.$

$m\ \ m\ \ m\ \ m$

Series 160, 80, 40, 20, 10, 5.

319. $l=a-(n-1)d$

$l=42-(n-1)3$

$l=45-3n$

$S=\dfrac{l+a}{2}.n$

$S=\dfrac{45-3n+42}{2}.n$

$315=\dfrac{87-3n}{2}.n$

$3n^2-87n=-630$

$n=15.$

320. Four terms of $(2ax^2-x^3)^5=$

$(2ax^2)^5-5.(2ax^2)^4.x^3+10.(2ax^2)^3.(x^3)^2-10.(2ax^2)^2.(x^3)^3=$

$32a^5x^{10}-80a^4x^{11}+80a^3x^{12}-40a^2x^{13}.$

321. $p^{\frac{1}{10}}-q^{\frac{1}{5}})p^{\frac{1}{2}}-q\quad(p^{\frac{2}{5}}+p^{\frac{3}{10}}q^{\frac{1}{5}}+p^{\frac{1}{5}}q^{\frac{2}{5}}+p^{\frac{1}{10}}q^{\frac{3}{5}}+q^{\frac{4}{5}}$

$p^{\frac{1}{2}}-p^{\frac{2}{5}}q^{\frac{1}{5}}$

$p^{\frac{2}{5}}q^{\frac{1}{5}}-q$

$p^{\frac{2}{5}}q^{\frac{1}{5}}-p^{\frac{3}{10}}q^{\frac{2}{5}}$

$p^{\frac{3}{10}}q^{\frac{2}{5}}-q$

$p^{\frac{3}{10}}q^{\frac{2}{5}}-p^{\frac{1}{5}}q^{\frac{3}{5}}$

etc.

323. $\left[\left(x^2\right)^{\frac{1}{7}}\left(x^3\right)^{-\frac{1}{2}}\right]^{-14}\left[\left(y^{\frac{1}{3}}\right)^{-2}\left(y^{\frac{1}{2}}\right)^{7}\right]^{-6} \div \left(x^3y^{-3}\right)^{7} =$
$\left[x^{\frac{2}{7}}x^{-\frac{3}{2}}\right]^{-14}\left[y^{-\frac{2}{3}}y^{\frac{7}{2}}\right]^{-6} \div x^{21}y^{-21} = x^{-4}x^{21}.y^{4}y^{-21} \div x^{21}y^{-21} =$
$\frac{x^{17}y^{-17}}{x^{21}y^{-21}} = \frac{y^4}{x^4}.$

324. $5(x-2y)-(x-y)=-24$
$11(2x+3y)+(2x-y)=200$
$5x-10y-x+y=-24$
$22x+33y+2x-y=200$
$4x-9y=-24$
$24x+32y=200$
$x=3$
$y=4.$

326. *a.* $lx^2+mx+n=0$
$x=\frac{-m\pm\sqrt{m^2-4ln}}{2l}$

b. $(3x-2)^2+(x-\frac{2}{3})=84$
$3(3x-2)^2+(3x-2)=252$
$3x-2=\frac{-1\pm\sqrt{1+3024}}{6}=$
$9 \text{ or } -9\frac{1}{3}$
$3x-2=9 \text{ or } -2\frac{2}{3}$
$3x=11 \text{ or } -2\frac{2}{3}$
$x=3\frac{2}{3} \text{ or } -\frac{2}{9}=-2\frac{4}{9}.$

328. $a=5,\ l=205,\ n=41$
$S=\frac{a+l}{2}.n=\frac{5+205}{2}.41=4305.$

329. $(1-\frac{2}{3}x)^{\frac{3}{2}}=1^{\frac{3}{2}}-\frac{3}{2}.\frac{2}{3}x+\frac{\frac{3}{2}.\frac{1}{2}}{1.2}(\frac{2}{3}x)^2-\frac{\frac{3}{2}.\frac{1}{2}.(-\frac{1}{2})}{1.2.3}(\frac{2}{3}x)^3, \text{ etc.} =$
$1-x+\frac{1}{6}x^2-\frac{1}{54}x^3, \text{ etc.}$

331. $\frac{1-a^{\frac{1}{2}}-ba^{\frac{1}{2}}+ab}{ba^{\frac{1}{2}}-b}=\frac{(1-a^{\frac{1}{2}})-ba^{\frac{1}{2}}(1-a^{\frac{1}{2}})}{b(a^{\frac{1}{2}}-1)}=\frac{(1-a^{\frac{1}{2}})\ (1-ba^{\frac{1}{2}})}{b(a^{\frac{1}{2}}-1)}=$
$\frac{-1(1-ba^{\frac{1}{2}})}{b}=\frac{ba^{\frac{1}{2}}-1}{b}=a^{\frac{1}{2}}-\frac{1}{b}=\sqrt{a}-\frac{1}{b}.$

Page 271.

333. $\left(\sqrt{\frac{x+\sqrt{x^2-a}}{2}}+\sqrt{\frac{x-\sqrt{x^2-a}}{2}}\right)^2=\frac{x+\sqrt{x^2-a}}{2}+2\sqrt{\frac{x^2-x^2+a}{4}}+$
$\frac{x-\sqrt{x^2-a}}{2}=\frac{2x}{2}+\sqrt{a}=x+\sqrt{a}.$

335. $S=\frac{a}{1-r}=\frac{-3}{1-(-\frac{1}{6})}=\frac{-3}{1+\frac{1}{6}}=\frac{-3}{\frac{7}{6}}=-3x\frac{6}{7}=-1\frac{8}{7}=-2\frac{1}{7}.$

345. $l=ar^{n-1}$
$r=\frac{3}{4}$
12
$S=\frac{a-lr}{1-r}=2\frac{2}{3}\frac{0}{3}\frac{3}{3}\frac{0}{2}\frac{9}{1}\frac{5}{6}.$

AMHERST COLLEGE.

347. $y + \frac{1}{y} = 4.$

$y^2 - 4y = -1.$

$y = \frac{4 \pm \sqrt{16-4}}{2} = \frac{4 \pm 2\sqrt{3}}{2} = 2 \pm \sqrt{3}$, the two values that make $y + \frac{1}{y} = 4$. Every quadratic equation has two roots.

Verification: $2 \pm \sqrt{3} + \frac{1}{2 \pm \sqrt{3}} = 4$

$4 \pm 4\sqrt{3} + 3 + 1 = 8 \pm 4\sqrt{3}$

$8 \pm 4\sqrt{3} = 8 \pm 4\sqrt{3}.$

351. $l = ar^{n-1} = 2\left(-\frac{243}{32}\right) = -\frac{243}{16}$

$S = \frac{a - lr}{l - r} = \frac{2 - \left(-\frac{243}{16}\right)\left(-\frac{3}{2}\right)}{1 - \left(-\frac{3}{2}\right)} = -8\frac{5}{16}.$

Page 273.

352. The sum of the roots is $\frac{-2c}{a}$; the product of the roots is $\frac{c^2 - c^2 + ab}{a^2}$, which equals $\frac{b}{a}$.

Therefore the equation is $x^2 + \frac{2c}{a}x = -\frac{b}{a}$.

356. $\left[\left(\frac{1}{x^{a-b}}\right)^{a-\frac{b^2}{a}}\right]^{\frac{a}{a+b}} = \left[\left(\frac{1}{x^{a-b}}\right)^{\frac{a^2-b^2}{a}}\right]^{\frac{a}{a+b}} = \left[x^{\frac{a+b}{a}}\right]^{\frac{a}{a+b}} = x.$

357. $5x$ = one number, $3x$ = the other

$25x^2 - 9x^2 = 400$

$5x = 25$

$3x = 15.$

358. See text-book, page 174.

359. $3x^2 + 5x + 2 = 0$

$x = \frac{-5 \pm \sqrt{25-24}}{6} = \frac{-5 \pm 1}{6} = \frac{-4}{6}$ or $\frac{-6}{6} = -\frac{2}{3}$ or $-1.$

These are the only two roots of the equation.

360. $S = \frac{a}{1 - r} = \frac{\frac{3}{4}}{1 - \frac{2}{3}} = \frac{3}{4} \times \frac{3}{1} = \frac{9}{4} = 2\frac{1}{4}.$

363. $3x$ = the hound's leaps

$4x$ = the hare's leaps

$3a$ = feet in a hound leap

$2a$ = feet in a hare leap

$9ax$ = feet passed over by hound

$8ax$ = feet passed over by hare

$60a$ = feet of start hare has

$\therefore 9ax = 8ax + 60a$

$9x = 8x + 60$

$x = 60$

$3x = 180$

$4x = 240.$

Page 274.

YALE COLLEGE.

364. $\frac{4\sqrt{12.5}-2\sqrt{.5}}{6\sqrt{18}}=\frac{4\sqrt{125}-2\sqrt{5}}{6\sqrt{180}}=\frac{20\sqrt{5}-2\sqrt{5}}{36\sqrt{5}}=\frac{18}{36}=\frac{1}{2}.$

365. *a.* $a-2b^5$ is obviously one factor.

b. $81x^4-y^8z^{12}$ is the difference of two squares, and is easily factored.

366. $\frac{\sqrt{1-\frac{1}{2}\sqrt{2}}}{\sqrt{1-\frac{\sqrt{5}}{3}}}=\frac{\sqrt{(1-\frac{1}{2}\sqrt{2})\ (1+\frac{1}{3}\sqrt{5})}}{\sqrt{1-\frac{5}{9}}}=$

$\frac{\sqrt{(1-\frac{1}{2}\sqrt{2})\ (1+\frac{1}{3}\sqrt{5})}}{\sqrt{\frac{4}{9}}}=\frac{\sqrt{(1-\frac{1}{2}\sqrt{2})\ (1-\frac{1}{3}\sqrt{5})}}{\frac{2}{3}}=$

$\frac{3\sqrt{(1-\frac{1}{2}\sqrt{2})(1+\frac{1}{3}\sqrt{5})}}{2}.$

368. $\frac{b}{\sqrt{a}}\cdot\sqrt[3]{ac}\cdot\frac{\sqrt[4]{c^3}}{\sqrt{b}}\cdot\frac{\sqrt{b^{-1}}}{a^{-\frac{1}{6}}}=\frac{b}{a^{\frac{1}{2}}}\cdot a^{\frac{1}{3}}c^{\frac{1}{3}}\cdot\frac{c^{\frac{3}{4}}}{b^{\frac{1}{2}}}\cdot\frac{a^{\frac{1}{6}}}{b^{\frac{1}{2}}}=$

$c^{\frac{1}{3}+\frac{1}{4}}=c^{\frac{13}{12}}=\sqrt[12]{c^{13}}.$

371. $\left.\begin{array}{l} a+ar=\frac{8}{3} \\ \frac{a}{1-r}=\frac{25}{6} \end{array}\right\}$

$\left.\begin{array}{l} a=\frac{5}{3} \\ r=\frac{3}{5}. \end{array}\right\}$

372. x = old price per photo.

y = new price per photo.

$\left.\begin{array}{l} 12y-12x=3 \\ \frac{5}{x}-\frac{5}{y}=10 \end{array}\right\}$

$x=\frac{1}{4}$, making \$3 per dozen

$y=\frac{1}{2}$, making \$6 per dozen.

Page 275.

373. *a.* $3x^3-12x^2y^2-4y^2+1=(3x^3+1)-4y^2\ (3x^3+1)=$

$(3x^3+1)\ (1-4y^2)=(3x^3+1)\ (1+2y)\ (1-2y).$

b. $3a+3b-6\sqrt{ab}=3\ (a-2\sqrt{ab}+b)=3\ (\sqrt{a}-\sqrt{b})\ (\sqrt{a}-\sqrt{b}).$

c. $8+2\sqrt{15}=5+2\sqrt{15}+3=(\sqrt{5}+\sqrt{3})\ (\sqrt{5}+\sqrt{3}).$

374. $\frac{1}{2\sqrt{3}+\sqrt{-2}-\sqrt{6}}\times\frac{2\sqrt{3}-\sqrt{-2}+\sqrt{6}}{2\sqrt{3}-\sqrt{-2}+\sqrt{6}}=$

$\frac{2\sqrt{3}-\sqrt{-2}+\sqrt{6}}{8+4\sqrt{3}}\times\frac{8-4\sqrt{3}}{8-4\sqrt{3}}=$

$\frac{(2\sqrt{3}-\sqrt{-2}+\sqrt{6})\ (8-4\sqrt{3})}{64-16\ (+3)}=$

$\frac{(2\sqrt{3}-\sqrt{-2}+\sqrt{6})\ (8-4\sqrt{3})}{16}.$

375. $2\sqrt{\frac{8}{3}} = 2\sqrt{\frac{24}{9}} = 2\sqrt{\frac{4}{9}\times 6} = 2\times\frac{2}{3}\sqrt{6} = \frac{4}{3}\sqrt{6}$, etc.

376. $$\frac{1}{\sqrt{a-x}+\sqrt{a}} + \frac{1}{\sqrt{a-x}-\sqrt{a}} = \frac{\sqrt{a}}{x}$$

$$\frac{\sqrt{a-x}-\sqrt{a}+\sqrt{a-x}+\sqrt{a}}{a-x-a} = \frac{\sqrt{a}}{x}$$

$$\frac{2\sqrt{a-x}}{-x} = \frac{\sqrt{a}}{x}$$

$$\frac{4a-4x}{x^2} = \frac{a}{x^2}$$

$$x = \tfrac{3}{4}a.$$

377. He walked x miles

He walked $\frac{x}{4}$ hours

He rode $20-x$ miles

He rode $\frac{20-x}{10}$ hours

$\frac{x}{4} + \frac{20-x}{10} = 2+1\frac{1}{2} = \frac{7}{2}$ hours

$\therefore x = 10$ miles.

378. $x-3y+y-3x=16$

A. $x+y=-8$

$\frac{12x+45y}{30x-220y} = -\frac{7}{30}$

B. $570x-190y=-\ 0$

A. $190x+190y=-1520$

$760x\ \ =-1520$

$x=-2$

A. $-2+y=-8$

$y=-6.$

Page 276.

379. $\sqrt{x+1} : \sqrt{2x} = \frac{1}{x} : \frac{2}{x+4}$

$x+1 : 2x = \frac{1}{x^2} : \frac{4}{(x+4)^2}$

$\frac{2x+2}{(x+4)^2} = \frac{1}{x}$

$x^2-6x=16$

$x=8$ or $-5.$

380. $(x^{-2}+\frac{1}{4})^{-3} = 27$

$(x^{-2}+\frac{1}{4})^{-1} = 3$

$x^{-2}+\frac{1}{4} = \frac{1}{3}$

$x^{-2} = \frac{1}{3}-\frac{1}{4} = \frac{1}{12}$

$x^2 = 12$

$x = \pm 2\sqrt{3}.$

381. $a = 60+\frac{1}{3}$ of $60 = 80$, $r = \frac{1}{3}$

$S = \frac{a}{1-r} = \frac{80}{\frac{2}{3}} = \frac{80}{1}\times\frac{3}{2} = 120.$

382. $\frac{10\times 9\times 8}{1\times 2\times 3} = \frac{720}{6} = 120.$

383. A. $\frac{x^2}{y}+\frac{y^2}{x} = \frac{35}{6}$

B. $x+y=5$

$x = my$

$x^2 = m^2y^2$

A. $\frac{m^2y^2}{y}+\frac{y^2}{my} = \frac{35}{6}$

B. $my+y=5$

A. $6\ m^3y+6y=35m$

A. $6y(m^3+1)=35m$

B. $y(m\ +1)=\ 5$

Dividing A. by B.

$6(m^2-m+1)=7m$

$m=\frac{2}{3}$ or $\frac{3}{2}$.

B. $y(\frac{2}{3}+1)=5$

$\frac{5}{3}y=5$

$y=3$

$\therefore\ x=2.$

Page 277.

386. $2\sqrt{\frac{3}{2}}+3\sqrt{\frac{2}{3}}-(\frac{2}{3}\sqrt[3]{\frac{1}{5}}\times\sqrt[3]{\frac{4}{2^2 5}})\ (\frac{1}{2}\sqrt{6}-\sqrt{24})$

$\sqrt{6}+\sqrt{6}-(\frac{2}{3}\sqrt[3]{\frac{4}{1^2 5}})\ (\frac{1}{2}\sqrt{6}-2\sqrt{6})$

$2\sqrt{6}-(\frac{2}{3}.\frac{1}{5}\sqrt[3]{4})\ (-\frac{3}{2}\sqrt{6})$

$2\sqrt{6}-(\frac{2}{1^2 5}\sqrt[3]{4})\ (-\frac{3}{2}\sqrt{6})$

$2\sqrt{6}+\frac{6}{3^2 0}\sqrt[3]{4}.\sqrt{6}$

$2\sqrt{6}+\frac{1}{5}.4^{\frac{1}{3}}.6^{\frac{1}{2}}$

$2\sqrt{6}+\frac{1}{5}.16^{\frac{1}{6}}.216^{\frac{1}{6}}$

$2\sqrt{6}+\frac{1}{5}\sqrt[6]{216\times 16}$

$2\sqrt{6}+\frac{1}{5}\sqrt[6]{3456}$

$2\sqrt{6}+\frac{1}{5}\sqrt[6]{64\times 54}$

$2\sqrt{6}+\frac{1}{5}.2\sqrt[6]{54}$

$2\sqrt{6}+\frac{2}{5}\sqrt[6]{54}.$

387. *b.* $x^{\frac{1}{a}}-\frac{5}{x^{\frac{1}{a}}}=4$

$x^{\frac{2}{a}}-4x^{\frac{1}{a}}=5$

$x^{\frac{1}{a}}=5 \text{ or } -1$

$x=5^a \text{ or } -1^a.$

390. *a.* See text-book, page 174.

391. *b.* $\left(x^{1+\frac{a}{b}}\right)^{-\frac{a}{a+b}}\div\sqrt[a]{\frac{x^{2a}}{(x^{-1})^{-a}}}=\left(x^{\frac{a+b}{b}}\right)^{-\frac{a}{a+b}}\div\frac{x^2}{(x^{-1})^{-1}}=x^{\frac{a}{b}}\div$

$\frac{x^2}{x}=x^{-\frac{a}{b}}\div x=x^{-\frac{a}{b}-1}=x^{-\frac{a+b}{b}}.$

392. $x:y::ab$

$\frac{x}{y}=\frac{a}{b}$

$\frac{mx}{y}=\frac{ma}{b}$

$mx:y::ma:b$

By comp.

$mx+y:y::ma+b:b$

$mx+y:ma+b::y:b$

In like manner,

$pa+y:pa+b::y:b.$

$\therefore mx+y:ma+b::pa+y:pa+b.$

393. $\frac{\sqrt{-3}+\sqrt{3}}{\sqrt{-4}-2\sqrt{3}}\times\frac{\sqrt{-4}+2\sqrt{3}}{\sqrt{-4}+2\sqrt{3}}=\frac{\sqrt{3}\sqrt{-1}+3}{2\sqrt{-1}-2\sqrt{3}}\times\frac{2\sqrt{-1}+2\sqrt{3}}{2\sqrt{-1}+2\sqrt{3}}=$

$\frac{4\sqrt{3}+12\sqrt{-1}}{-4-12}=\frac{\sqrt{3}+3\sqrt{-1}}{-4}.$

Page 278.

394. *a.* $2x^2-3\sqrt{x^2+2x+14}+4x-49=0$

$2(x^2+2x+14)-3\sqrt{x^2+2x+14}=77$

$\sqrt{x^2+2x+14}=\frac{3\pm\sqrt{9+616}}{4}=\frac{3\pm 25}{4}=7$

$x^2+2x+14=49$

$x=\frac{-2\pm\sqrt{4+140}}{2}=\frac{-2\pm 12}{2}=5 \text{ or } -7.$

394. *b.* $x^{\frac{1}{2}}+y^{\frac{1}{2}}=5$
$6(x^{-\frac{1}{2}}+y^{-\frac{1}{2}})=5$
$\sqrt{x}+\sqrt{y}=5$
$\frac{6}{\sqrt{x}}+\frac{6}{\sqrt{y}}=5$

$\frac{6}{5-\sqrt{y}}+\frac{6}{\sqrt{y}}=5$
$y-5\sqrt{y}=-6$
$\sqrt{y}=2$ or 3
$y=4$ or 9
$\therefore x=9$ or 4.

395. $S=\frac{a}{1-r}=\frac{6}{1-(-\frac{2}{3})}=\frac{6}{1+\frac{2}{3}}=6\times\frac{3}{5}=\frac{18}{5}=3\frac{3}{5}$.

397. *a.* $2\sqrt[3]{3}(\sqrt[3]{9}-2\sqrt[3]{\frac{8}{3}}+4\sqrt[3]{\frac{1}{3}}-3\sqrt[3]{2})=$
$2\sqrt[3]{27}-\frac{8}{3}\sqrt[3]{27}+\frac{8}{3}\sqrt[3]{27}-6\sqrt[3]{6}=$
$6-8+8-6\sqrt[3]{6}=$
$6-6\sqrt[3]{6}$.

b. $\frac{b^2}{a^8c^2}\times\frac{a^{-1}b^{-2}}{ab^{-3}}\times\frac{ab^{2-1}c^{-2}}{b^4}\times\left(\frac{a^2}{a^{-2}b^{-1}}\right)^2=$
$\frac{b^2}{a^8c^2}\times\frac{b}{a^2}\times\frac{a}{b^3c^2}\times\frac{a^8b^2}{1}=\frac{b^2}{ac^4}$.

Page 279.

405. The length of time is 10 days of 15 hours each $=150$ hours
If $150x$ gallons ran in in 150 hours,
$150x+2250$ gallons were drawn out in same time
But $300x$ gallons were drawn out
$\therefore 150x+2250=300x$
$150x=2250$
$x=15$.

408. $x^2+y^2+x+y=18$
$xy=6$
$2xy=12$
$x^2+2xy+y^2+x+y=30$
$(x+y)^2+(x+y)=30$
$x+y=\frac{-1\pm\sqrt{1+120}}{2}=\frac{10}{2}=5$

$y=\frac{6}{x}$
$x+\frac{6}{x}=5$
$x^2-5x=-6$
$x=3$
$\therefore y=2$.

409. $x^{\frac{1}{4}}-x^{-\frac{1}{4}}=\frac{3}{2}$
$x^{\frac{1}{4}}-\frac{1}{x^{\frac{1}{4}}}=\frac{3}{2}$
$x^{\frac{1}{2}}-1=\frac{3}{2}x^{\frac{1}{4}}$
$2x^{\frac{1}{2}}-2=3x^{\frac{1}{4}}$

$2x^{\frac{1}{2}}-3x^{\frac{1}{4}}=2$
$x^{\frac{1}{4}}=\frac{3\pm\sqrt{9+16}}{4}=$
$\frac{3\pm5}{4}=\frac{8}{4}$ or $\frac{-2}{4}=2$ or $-\frac{1}{2}$
$\therefore x=16$ or $\frac{1}{16}$.

411. $a : b = c : d$

$a^2 : b^2 = c^2 : d^2$

$a^2 + b^2 : a^2 = c^2 + d^2 : c^2$

$a^2 + b^2 : c^2 + d^2 = a^2 : c^2$

Also $(a+b)^2 : (c+d)^2 = a^2 : c$

$\therefore a^2 + b^2 : c^2 + d^2 = (a+b)^2 : (c+d)^2$

Page 280.

VASSAR COLLEGE.

413. $(a + b\sqrt{-1})(a - b\sqrt{-1}) = a^2 - b^2(-1) = a^2 + b^2$

$(a+b+\sqrt{2ab})(a+b-\sqrt{2ab}) = (a+b)^2 - (\sqrt{2ab})^2 =$

$a^2 + 2ab + b^2 - 2ab = a^2 + b^2.$

414. Clearing of fraction, we obtain $3x^2 - 8x = 0$

$3x - 8 = 0$

$3x = 8$

$x = \frac{8}{3}.$

415. $a : b = c : d$

$a : c = b : d$

$\frac{a}{c} = \frac{b}{d}$

$a+b : a = c+d : c$

$a+b : c+d = a : c$

$\frac{a+b}{c+d} = \frac{a}{c}$

By division:

$\frac{a+b}{c-d} = \frac{a}{c}$

$\therefore \frac{a-b}{c+a} = \frac{a-b}{c-d} = \frac{a}{c} = \frac{b}{d}.$

417. $l = a + (n-1)d$

$l = 1 + (2n-1)2 = 1 + 4n - 2 = 4n - 1.$

Page 281.

OHIO.

423. *b.* $8(5x-3)^{-\frac{4}{5}} - 6(5x-3)^{-\frac{2}{5}} = -1$

$(5x-3)^{-\frac{2}{5}} = \frac{6 \pm \sqrt{36-32}}{16} = \frac{6 \pm 2}{16} = \frac{1}{2}$ or $\frac{1}{4}$

Taking the latter value,

$(5x-3)^{\frac{2}{5}} = 4$

$(5x-3)^{\frac{1}{5}} = \pm 2$

$5x - 3 = (\pm 2)^5 = \pm 32$

$5x = 35$ or -29

$x = 7$ or $-\frac{29}{5}$.

425. A. $x^2-xy=27y$
B. $xy-y^2=3x$

$x=my$
$xy=my^2$
$y^2=m^2y^2$

A. $m^2y^2-my^2=27y$
$m^2y-my=27$
$y(m^2-m)=27$
$y=\frac{27}{m^2-m}$

B. $my^2-y^2=3my$
$y^2(m-1)=3my$
$y(m-1)=3m$
$y=\frac{3m}{m-1}$

$\therefore \frac{27}{m^2-m}=\frac{3m^2}{m^2-m}$
$3m^2=27$
$m^2=9$
$m=\pm 3$

$y=\frac{\pm 9}{\pm 3-1}=\frac{\pm 9}{2 \text{ or } -4}=$
$\pm 4\frac{1}{2}$ or $\mp 2\frac{1}{4}$
$x=\pm 3x(\pm\frac{1}{2})=\frac{27}{2}=13\frac{1}{2}$
$x=\pm 3(\mp\frac{9}{4})=-\frac{27}{4}=-6\frac{3}{4}$.

426. $xy+x-y=26$
$x^2+y^2-(x-y)=50$
Wherefore $x=6$, $y=4$.

427. Had x eggs
Sold $\frac{1}{2}x+\frac{1}{2}$
Had left $\frac{1}{2}x-\frac{1}{2}$
Sold $\frac{1}{4}x+\frac{1}{4}$
Had left $\frac{1}{4}x-\frac{3}{4}$
Sold $\frac{1}{8}x+\frac{1}{8}$
Had left $\frac{1}{8}x-\frac{7}{8}=0$
Hence $x=7$.

429. A. $a:b::(a+c)^2:(b+c)^2$
B. $c^2=ab$

A. $ab^2+2abc+ac^2=a^2b+2abc+bc^2$
$ab^2+ac^2=a^2b+bc^2$
Substituting ab for c^2, $ab^2+a^2b=a^2b+ab^2$
Since the substitution preserves the equality in A., B. is a true equation, and c is a mean proportional between a and b.

Page 282.

430. *a.* $\frac{b+c}{(a-b)(a-c)}+\frac{a+c}{(b-c)(b-a)}+\frac{a+b}{(c-a)(c-b)}=$
$\frac{b+c}{(a-b)(a-c)}-\frac{a+c}{(b-c)(a-b)}+\frac{a+b}{(a-c)(b-c)}=$
$\frac{b^2-c^2-(a^2-c^2)+a^2-b^2}{(a-b)(a-c)(b-c)}=\frac{b^2-c^2-a^2+c^2+a^2-b^2}{(a-b)(a-c)(b-c)}=0$.

b. $\frac{1}{1-x}+\frac{1}{1+x}+\frac{2}{1+x^2}+\frac{4}{1+x^4}=\frac{2}{1-x^2}+\frac{2}{1+x^2}+\frac{4}{1+x^4}=$
$\frac{4}{1-x^4}+\frac{4}{1+x^4}=\frac{8}{1-x^8}$.

431. Rate, x miles per hour

$x+4$, conditional rate

$$\frac{105}{x}-\frac{105}{x+4}=9\tfrac{1}{8}$$

$$x^2+4x=45$$

$$x=9 \text{ or } -5.$$

438. x sheep were bought

$\frac{468}{x}$ = cost of each

$$\therefore \left(\frac{468}{x}+1\right)(x-8)-468=12$$

$$x^2-20x=3744$$

$$x=72.$$

439. x is the sum of money

$\frac{x}{3}-4$ = A.'s share

$\frac{x}{4}+2$ = B.'s share

$\frac{x}{5}+3$ = C.'s share

25 = D.'s share

The sum of these shares $= x$

$$13x=1560$$

$$x=120$$

$\frac{x}{3}-4=36$, A.'s share,

$\frac{x}{4}+2=32$, B.'s share,

and so on.

442. $$\frac{1}{a(a-b)(a-c)}+\frac{1}{b(b-a)(b-c)}+\frac{1}{c(c-a)(c-b)}=$$

$$\frac{1}{a(a-b)(a-c)}+\frac{-1}{b(a-b)(b-c)}+\frac{1}{c(a-c)(b-c)}=\frac{1}{abc}.$$

443. All together in x days

A.+B.+C. can do $\frac{1}{x}$ in 1 day

A.+C. can do $\frac{1}{6}$ in 1 day

B.+C. can do $\frac{1}{8}$ in 1 day

A.+C. $=\frac{1}{6}$

B.+C. $=\frac{1}{8}$

A.$-$B. $=\frac{1}{6}-\frac{1}{8}=\frac{1}{24}$

$\frac{3}{2}$B.$-$B. $=\frac{1}{24}$

B. $=\frac{1}{12}$

$\therefore$ A. $=\frac{1}{8}$, B. $=\frac{1}{12}$, C. $=\frac{1}{24}$

Hence, $\frac{x}{12}+\frac{x}{8}+\frac{x}{24}=1$

and $x=4$.

Page 283.

444. $$2\sqrt[3]{3x}+5\sqrt[6]{9x^2}-\sqrt{27x^3}=2(3x)^{\frac{2}{6}}+5(9x^2)^{\frac{1}{6}}-3x\sqrt{3x}=$$

$$7\sqrt[6]{9x^2}-3x\sqrt{3x}=7\sqrt[3]{3x}-3x\sqrt{3x}.$$

446. $$\left(\frac{6}{x}+x\right)^2+\left(\frac{6}{x}+x\right)=30$$

$$\frac{6}{x}+x=\frac{-1\pm\sqrt{1+120}}{2}=\frac{-1\pm 11}{2}=\frac{10}{2} \text{ or } \frac{-12}{2}=5 \text{ or } -6$$

$$\frac{6}{x}+x=5$$

$$x^2-5x=-6$$

$$x=2 \text{ or } 3$$

$$\frac{6}{x}+x=-6$$

$$x^2+6x=-6$$

$$x=-3\pm\tfrac{1}{2}\sqrt{2}.$$

448. He rode x miles per hour
He walked $x-4$ miles per hour
$\therefore \frac{10}{x-4} - \frac{10}{x} = \frac{5}{4}$
and $x=8$.

Page 284.

453. x days, working together
A. will do $\frac{x}{12}$ and B. $\frac{x}{24}$
$\therefore \frac{x}{12} + \frac{x}{24} = 1$
and $x=8$ days.

458. $x : y = y : 12$
$x^2+y^2=45$
Wherefore, $x=3$, $y=6$.

456. $x+2=\sqrt{4+x\sqrt{64+x^2}}$
$x^2+4x+4=4+x\sqrt{64+x^2}$
$x+4=\sqrt{64+x^2}$
$x^2+8x+16=64+x^2$
$8x=48$
$x=6$.

Page 285.

465. $2\sqrt{-1}+2\sqrt{-9}-3\sqrt{-4}=$
$2\sqrt{-1}+6\sqrt{-1}-6\sqrt{-1}=2\sqrt{-1}$.

467. $9x$ = A.'s money,
$5x$ = B.'s money,
$5x+4\frac{1}{2}x-9x=2$
Wherefore $x=4$
$9x=36$
$5x=20$.

468. $l=a+(n-1)\,d$
$S=\frac{l+a}{2}.n$
$a=1$, $d=1$, $S=9870$
Wherefore $l=n$
and $n=140$.

474. $(ax^2)^{\frac{1}{3}} \div (xy)^{\frac{1}{7}} = \frac{a^{\frac{7}{21}}x^{\frac{14}{21}}}{x^{\frac{3}{21}}y^{\frac{3}{21}}} = \frac{a^{\frac{7}{21}}x^{\frac{11}{21}}}{y^{\frac{3}{21}}} = \sqrt[21]{\frac{a^7x^{11}}{y^3}}$.

477. The first travelled x hours
The second travelled $x-2$ hours
$\therefore 13\frac{2}{3}(x-2)=10\frac{3}{5}x$, and $x=6\frac{21}{23}$ hours.

Page 287.

484. $m+n-\frac{2mn+n^2}{m-n} = \frac{(m+n)(m-n)-(2mn+n^2)}{m-n} =$
$\frac{m^2-n^2-2mn-n^2}{m-n} = \frac{m^2-2mn-2n^2}{m-n}$.

485. The house cost x dollars

$$\frac{10000-x}{3}\times\frac{6}{100}+\frac{2\,(10000-x)}{3}\times\frac{5}{100}=320$$

From which $x=\$4000$.

486. $\frac{x}{y}$ is the fraction

$$\left.\begin{aligned}\frac{x+3}{y}&=\tfrac{1}{3}\\ \frac{x}{y-1}&=\tfrac{1}{5}\end{aligned}\right\}\quad\begin{aligned}&\text{Whence } x=4,\ y=21\\ &\text{The fraction}=\tfrac{4}{21}.\end{aligned}$$

487. Each having given away $\frac{2}{5}$ had $\frac{3}{5}$ left; hence we have

$$\left.\begin{aligned}\tfrac{3}{5}x+\tfrac{1}{5}y+\tfrac{1}{5}z&=740\\ \tfrac{3}{5}y+\tfrac{1}{5}x+\tfrac{1}{5}z&=580\\ \tfrac{3}{5}z+\tfrac{1}{5}x+\tfrac{1}{5}y&=380\end{aligned}\right\}\quad\begin{aligned}&\text{Whence } x=1000,\ y=600\\ &\qquad\quad z=100.\end{aligned}$$

488. A. has x dollars, B. has y dollars, and C. has z dollars

	A.	B.	C.
After first distribution	$x-y-z$	$2y$	$2z$
After second distribution	$2x-2y-2z$	$3y-x-z$	$4z$
After third distribution	$4x-4y-4z$	$6y-2x-2z$	$7z-x-y$

$$\text{Hence}\left\{\begin{aligned}4x-4y-4z&=8\\ 6y-2x-2z&=8\\ 7z-x-y&=8\end{aligned}\right\}\quad\text{Whence } x=13,\ x=7,\ z=4.$$

491. $x-ay+a^2z=a^3$ **Page 288.**

$x-by+b^2z=b^3$

$x-cy+c^2z=c^3$

1. $x=a^3+ay-a^2z$
2. $x=b^3+by-b^2z$
3. $x=c^3+cy-c^2z$

$$a^3+ay-a^2z=b^3+by-b^2z$$
$$a^3+ay-a^2z=c^3+cy-c^2z$$
$$ay-by+b^2z-a^2z=b^3-a^3$$
$$ay-cy+c^2z-a^2z=c^3-a^3$$
$$y\,(a-b)+z\,(b^2-a^2)=b^3-a^3$$
$$y\,(a-c)+z\,(c^2-a^2)=c^3-a^3$$
$$-y+z\,(b+a)=b^2+ab+a^2$$
$$-y+z\,(c+a)=c^2+ac+a^2$$
$$z\,(b+a-c-a)=b^2-c^2+ab-ac$$
$$z\,(b-c)=(b^2-c^2)+a\,(b-c)$$
$$z=(b+c)+a$$
$$z=a+b+c.$$

Page 289.

492. A. can do $\frac{1}{x}$ work in 1 day

B. can do $\frac{1}{y}$ work in 1 day

1. $\frac{1}{x}+\frac{1}{y}=\frac{1}{15}$

A. + B. did $\frac{6}{15}$ or $\frac{2}{5}$ in 6 days

B. did the remaining $\frac{3}{5}$ in 24 days

2. $\frac{24}{y}=\frac{3}{5}$

$3y=120$

$y=40$

1. $\frac{1}{x}+\frac{1}{40}=\frac{1}{15}$

$\therefore x=24.$

493. $\frac{3}{x-1}=\frac{15}{x+3}-\frac{4}{x-3}$

$\frac{3}{x-1}=\frac{11x-57}{x^2-9}$

$2x^2-17x=-21$

$x=7$ or $1\frac{1}{2}$.

494. $\frac{x-b}{x-a}-\frac{x-a}{x-b}=\frac{2(a-b)}{x-a-b}$

$\frac{x^2-2bx+b^2-x^2+2ax-a^2}{x^2-bx-ax+ab}=\frac{2(a-b)}{x-a-b}$

$\frac{(a-b)\,(2x-a-b)}{x^2-bx-ax+ab}=\frac{2(a-b)}{x-a-b}$

$\frac{2x-a-b}{x^2-bx-ax+ab}=\frac{2}{x-a-b}$

$2x^2-2bx-2ax+2ab=2x^2-3ax-3bx+2ab+a^2+b^2$

$bx+ax=a^2+b^2$

$x(a+b)=a^2+b^2$

$x=\frac{a^2+b^2}{a+b}$.

495. Speed of faster train x miles per hour

Speed of slower train y miles per hour

Then $x-y=5$

And $\frac{30}{y}-\frac{30}{x}=\frac{1}{5}$

Hence $x=30$, $y=25$.

496. $2:8=64:x$

$x=256.$

497. $l=1+(n-1)\frac{1}{2}$

$45=\frac{1+\frac{1+n}{2}}{2}\cdot n$

$n=12.$

Page 289.

501. $\sqrt[4]{\frac{a}{b}}\div\sqrt[2]{\frac{a}{b}}=\left(\frac{a}{b}\right)^{\frac{1}{4}}\div\left(\frac{a}{b}\right)^{\frac{2}{4}}=\sqrt[4]{\frac{a}{b}\times\frac{b^2}{a^2}}=\sqrt[4]{\frac{b}{a}}.$

502. $\left(x\sqrt[mn]{y}\right)^n = \left(xy^{\frac{1}{mn}}\right)^n = x^n y^{\frac{n}{mn}} = x^n y^{\frac{1}{m}} = x^n\sqrt[m]{y}.$

504. $9\sqrt{-1} - 2\sqrt{-4} = 9\sqrt{-1} - 4\sqrt{-1} = 5\sqrt{-1}.$

506. See text-book, page 205, III.

508. Since $-a$ taken b times is $-ab$, $-a$ taken $-b$ times is $+ab$, a result diametrically opposite in kind. See text-book, pages 8 and 9.

513. $(x-5)2 = y+5$
$(y-5)3 = x+5$
$x = 13,\ y = 11.$

Page 291.

519. *a.* $x^{-3} = \frac{1}{x^3}.$

b. $(a-b)\ (a+b) = a^2 - b^2.$

c. $(x-y)^3 = x^3 - 3x^2y + 3xy^2 - y^3.$

d. $a-[b-\{c-(d-e-f)\}] = a-[b-\{c-d+e+f\}] =$
$a-[b-c+d-e-f] = a-b+c-d+e+f.$

e. $\sqrt[6]{\frac{a}{3}\sqrt{\frac{a}{3}}} = \frac{a}{3}\left[\left(\frac{a}{3}\right)^{\frac{1}{2}}\right]^{\frac{1}{3}} = \left(\frac{a}{3}\right)^{\frac{1}{3}}\left(\frac{a}{3}\right)^{\frac{1}{6}} = \left(\frac{a}{3}\right)^{\frac{2}{6}}\left(\frac{a}{3}\right)^{\frac{1}{6}} = \left(\frac{a^2}{9}\right)^{\frac{1}{6}}\left(\frac{a}{3}\right)^{\frac{1}{6}} =$
$\sqrt[6]{\frac{a^3}{27}} = \sqrt{\frac{a}{3}} = \sqrt{\frac{3a}{9}} = \frac{1}{3}\sqrt{3a}.$

522. x = body, 10 = head, $\frac{x}{2}$ + 10 = tail

$\therefore\ x = \frac{x}{2} + 20$

and $x = 40$
head $= 10$
tail $= 30$
$\Big\} = 80.$

526. $\frac{4}{5}x.\frac{4}{10} + \frac{1}{5}x.\frac{5}{10} = 2940$
Wherefore $x = 70000.$

533. $\frac{94}{x}$ = prime cost of each

$\frac{94}{x}(x-7) = 20$

$x = 47.$

534. The wheel revolves in x seconds. $1\frac{7}{8}$ miles = 600 rods
conditionally in $x+1$ seconds. 1 hour = 3600 seconds

$\therefore\ \frac{3600}{x} - \frac{3600}{x+1} = 600$

Whence $x = 2$ seconds

Hence rate per hour = 1800 rods = $5\frac{5}{8}$ miles.

537. x = entire distance
y = rate per hour
$\frac{x}{y}$ = schedule time

A. 1 hour + 1 hour + $\frac{x-y}{\frac{3}{4}y} = \frac{x}{y} + 3$ hours

Whence $x = 7y$

B. 1 hour + 1 hour + $\frac{50}{y} + \frac{x-y-50}{\frac{3}{4}y} = \frac{x}{y} + 1\frac{1}{4}$

Whence $y = 9\frac{1}{2}\frac{1}{1}$ miles

$\therefore x$ or $7y = 66\frac{2}{3}$ miles.

538. x = time down
$2x$ = time up
$3x = 8$
$x = 2\frac{2}{3}$ hours
$2\frac{2}{3} \times 6 = 16$ miles.

Page 294.

539. $x^4 + x^2y^2 + y^4 = x^4 + 2x^2y^2 + y^4 - x^2y^2 = (x^2+y^2)^2 - x^2y^2 =$
$(x^2+y^2+xy)\ (x^2+y^2-xy)$.

541. $2x^4 - 3x^2 - 20 = 0$

$$x^2 = \frac{3 \pm \sqrt{9+160}}{4} = \frac{3 \pm 13}{4} = \frac{16}{4} \text{ or } \frac{-10}{4} = 4 \text{ or } -\tfrac{5}{2}$$

$x = \sqrt{4}$ or $\sqrt{-\frac{5}{2}} = \pm 2$ or $\pm \frac{1}{2}\sqrt{-10}$.

542. $y = 5 - 3x$, $y^2 = 25 - 3cx + 9x^2$, $xy = 5x - 3x^2$
$\therefore 5x^2 - 20x + 12x^2 + 25 - 30x + 9x^2 = 1$
$13x^2 - 25x = -12$
$x = 1$ or $\frac{1}{1}\frac{2}{3}$, $y = 2$ or $\frac{2}{1}\frac{9}{3}$.

543. $S = \frac{ar^n - a}{r-1} = \frac{3(\frac{1}{2})^{10} - 3}{\frac{1}{2} - 1} = \frac{3069}{512} = 5\frac{5}{5}\frac{0}{1}\frac{9}{2}$.

545. *a.* $5a^{m+2} + 15a^{m+1}b^n - 10a^m b^{n+2} = 5.2^5 + 15.2^4.5 - 10.2^3.5^3 =$
$160 + 1200 - 10000 = -8640$.

547. $x + y = 75$, $x - y = 35$.

557. $\frac{m^2+n^2-mn}{n} \times \frac{mn}{m+n} \times \frac{m^2-n^2}{m^3+n^3} = \frac{1}{n} \times \frac{mn}{m+n} \times \frac{m^2-n^2}{m+n} =$
$\frac{1}{n} \times \frac{mn}{m+n} \times \frac{m-n}{1} = \frac{m(m-n)}{m+n}$.

558. $2\frac{1}{3} : 3\frac{1}{2} = 2 : 3$ $\quad \therefore x = 2$ hours
If $2x$ = time down $\quad 2x = 4$ hours
$3x$ = time up $\quad 3\frac{1}{3} \times 4 = 14$ miles.
and $5x = 10$

Page 296.

h. t. u.

560. $\begin{cases} x+y+z=21 \\ x+y=z+3 \\ 100x+10y+z+198=100z+10y+x. \end{cases}$

561. *a.* $\left(\dfrac{a^{-\frac{1}{2}}b^{\frac{1}{2}x}}{x^{\frac{1}{n}}y^{-\frac{n}{2}}}\right)^{-2n} = \dfrac{a^{\frac{2n}{2}}b^{\frac{-2n}{2x}}}{x^{\frac{-2n}{n}}y^{\frac{2n^2}{2}}} = \dfrac{a^n b^{-\frac{n}{x}}}{x^{-2}y^{n^2}} = \dfrac{a^n x^2}{b^{\frac{n}{x}}y^{n^2}}.$

b. $\sqrt[n]{a^{\frac{1}{n}}b^{-n^2}x^{-3n}} = a^{\frac{1}{n^2}}b^{\ n}x^{-3} = \dfrac{a^{\frac{1}{n^2}}}{b^n x^3}.$

568. $l=a+(n-1)d=1\frac{1}{2}+9\times 3-28\frac{1}{2}$

$S=\left(\dfrac{l+a}{2}\right)n=\left(\dfrac{28\frac{1}{2}+1\frac{1}{2}}{2}\right)10=150.$

569. See text-book, page 225.

572. $\dfrac{10.9.8.7.6.5}{1.2.3.4.5.6}\left(\dfrac{x}{2}\right)^4\left(\dfrac{y}{3}\right)^6=\dfrac{35x^4y^6}{1944}.$

573. Assume $a^n = aaa\ldots$

Dividing by a, $a^{n-1}=aaa\ldots$

" " $a^{n-2}=aaa\ldots$

" " $a^2=aa$

" " $a^1=a$

" " $a^0=1$

Dividing by a, $a^{-1}=\dfrac{1}{a}$

" " $a^{-2}=\dfrac{1}{a^2}$

" " $a^{-n}=\dfrac{1}{a^n}.$

Page 297.

575. $S=\dfrac{a}{1-r}$

$1=\dfrac{\frac{1}{3}}{1-r}$

$1-r=\frac{1}{3}$

$r=1-\frac{1}{3}=\frac{2}{3}$

$l=ar^{n-1}$

$0=\frac{1}{3}\cdot(\frac{2}{3})^{n-1}$

$\therefore (\frac{2}{3})^{n-1}=0$, and

$n=\infty.$

576. $\dfrac{\sqrt{3}+\sqrt{-5}}{-\sqrt{-5}-\sqrt{-3}}\times\dfrac{-\sqrt{-5}+\sqrt{-3}}{-\sqrt{-5}+\sqrt{-3}}=\dfrac{\sqrt{-15}-3\sqrt{-1}+\sqrt{15}-5}{2}.$

577. See method of page 174, text-book.

580. $x^2+3x+2=(x+1)^3$

$(x+2)\ (x+1)=(x+1)^3$

$x+2+(x+1)^2=x^2+2x+1$

$x^2+x=1$

$x=\dfrac{-1\pm\sqrt{1+4}}{2}=\dfrac{-1\pm\sqrt{5}}{2}.$

581. Extract the square root, and the cube root of the result.

582. He rode x miles

He rode $\frac{x}{m}$ hours

He walked $\frac{x}{n}$ hours

$\therefore \frac{x}{m} + \frac{x}{n} = t$

and $x = \frac{mnt}{m+n}$.

583. $\sqrt{x} - 3 = \frac{2\sqrt{10}}{\sqrt{x}+3}$

$x - 9 = 2\sqrt{10}$

$x^2 - 18x + 81 = 40$

$x - 9 = \pm 2\sqrt{10}$

$x = 9 \pm 2\sqrt{10}$.

587. $\left.\begin{aligned}(x+1)\ (y+1) &= xy+14\\ (x-1)\ (y+\tfrac{1}{2}) &= xy.\end{aligned}\right\}$

592. x is the sum, y is the rate

$\left.\begin{aligned}x + \frac{mxy}{100} &= a\\ x + \frac{nxy}{100} &= b.\end{aligned}\right\}$

593. *a.* $(2x+5)^{-\frac{5}{2}} = \frac{-31 \pm \sqrt{31^2+128}}{2} = \frac{-31 \pm 33}{2} = 1 \text{ or } -32$

$\therefore (2x+5)^{-\frac{1}{2}} = +1 \text{ or } -2$

$(2x+5)^{-1} = 1 \text{ or } 4$

$2x+5 = 1 \text{ or } \frac{1}{4}$

$\therefore x = -2 \text{ or } -2\frac{3}{8}$.

Page 299.

595. x = length, y = breadth

$\left.\begin{aligned}(x+14)\ (y+14) &= 18696\\ xy &= 5000.\end{aligned}\right\}$

598.

First integer	$=$	$x-3d$
Second integer	$=$	$x-d$
Third integer	$=$	$x+d$
Fourth integer	$=$	$x+3d$
Sum,		$4x = 24$
		$x = 6$

Product, $(x-3d)\ (x-d)\ (x+d)\ (x+3d) = 945$

Substituting 6 for x, and simplifying, we find $d = 1$

Hence the integers are 3, 5, 7, 9.

604. See No. 573.

606. See text-book, page 206, III.

607. A turkey cost x, a duck y

$\left.\begin{aligned}6x+2y &= 15\\ \frac{14}{y} - \frac{9}{x} &= 4.\end{aligned}\right\}$

608. Each horse cost x

Realized $\frac{x^2}{100}$

$\therefore \frac{x^2}{100} = 75 - x$

Hence $x = 50$.

371.3M R198.2
Key